FUNCTION

Let A and B be two given sets. If each element $a \in A$ is linked with a unique element $b \in B$ under a rule f , then this relation is termed a **function.**

Here b, is referred to as the image of a and a is called the pre- image of b under f.

Note:
1. Every element of A should be associated with B but vice-versa is not essential.
2. Every element of A should be linked with a unique (one and only one) element of B, while any element of B can have two or more relations in A.

Definition of Function

A function can be defined as a distinctive relation that associates each element of set A with precisely one element from set B. It is essential that both sets A and B are non-empty. A function assigns a specific output to a given input. Therefore, if $f: A \rightarrow B$, it signifies that for any $a \in A$, there exists a unique element $b \in B$ such that $(a, b) \in f$.

Function as a Special Type of Relation

In mathematics, a function is indeed a special type of relation. Both functions and relations are ways of associating elements from one set (called the domain) to elements in another set (called the codomain). However, there are key distinctions between functions and general relations.

A relation is any set of ordered pairs, where each pair consists of an element from the first set (domain) and an element from the second set (codomain). Relations don't have strict rules about whether each element in the domain has a unique corresponding element in the codomain.

On the other hand, a function is a special type of relation with additional restrictions. In a function, each element in the domain is associated with exactly one element in the codomain. This means that for any given input, there is a unique output. In other words, if (a, b) is part of the function, and (a, c) is also part of the function, then b must be equal to c.

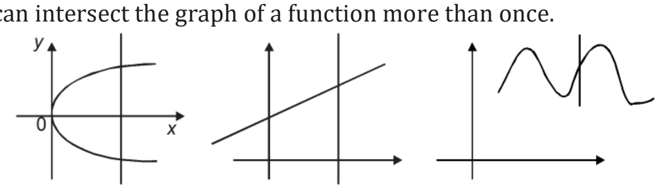
Functions are often denoted by $f:A \rightarrow B$, where A is the domain and B is the codomain. The elements in A are mapped to elements in B in such a way that each element in A is associated with exactly one element in B.

The vertical line test is a visual way to determine if a relation is a function. If no vertical line intersects a graph more than once, then the relation is a function. This corresponds to the idea that each input (x-value) has only one output (y-value).

In summary, while all functions are relations, not all relations are functions. Functions have the special property of assigning a unique output to each input.

The Graph of a Function

The plot of a function $y=f(x)$ comprises all points $(x, f(x))$ in the Cartesian plane. Due to the definition of a function, where each x corresponds to exactly one y, it can be concluded that no vertical line can intersect the graph of a function more than once.



not a function of *x*   a function of *x*   a function of *x*

Ex. Determine the domain of the function $f(x) = \cos^{-1}\left(\frac{|x|-2}{5}\right)$

Sol.

$$f(x) \text{ exist if } -1 \leq \frac{|x|-2}{5} \leq 1$$
$$-5 < |x| - 2 \leq 5$$
$$-3 \leq |x| \leq 7$$
$$|x| \geq -3 \text{ true " } x \in R$$
$$|x| \leq 7$$
$$x \in [-7,7]$$

Ex. Determine the domain and range of the function $f(x)=\frac{x^2+x+1}{x^2-x+1}$

Sol. $x^2-x+1\neq 0$ for anv value of $x(b^2-4ac<0)$ So domain of f(x) is R Range

$$f(x)=y$$
$$\frac{x^2+x+1}{x^2-x+1}=y$$
$$x^2(1-y)+x(1+y)+(1-y)=0$$

But x is real so $b^2-4ac\geq 0$

$$(1+y)^2-4(1-y)^2\geq 0$$
$$3y^2-10y+3\leq 0$$
$$(y-3)(3y-1)\leq 0$$
$$y\leq[\frac{1}{3},3]$$

So range of $f(x)[\frac{1}{3},3]$.

Ex. Determine the domain of the function f(x), given by $f(x)=\sqrt{\log_{0.5}x}$

Sol.
$$f(x)=\sqrt{\log_{0.5}x}$$

Now, it is established that f(x) exists when $\log_{0.5}x\geq 0x>0$

(as the logarithm is undefined for zero and negative numbers).

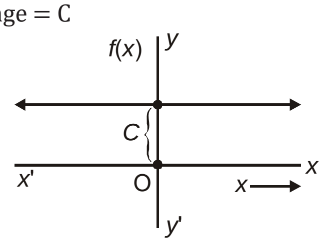
$$\log_{0:5}x\geq 0$$
$$x<(0.5)^{\circ}$$
$$x\leq 1=x\in(-\infty,11$$
$$x>0$$
$$x\in(0,1]$$

SOME FUNCTIONS AND THEIR GRAPHS

Constant Function

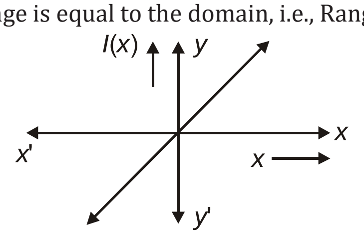
A function represented as $f(x)=C$ (where $C\in R$) is referred to as constant function

Domain = R Range = C



Identity Function [I(x)] :

A function that is paired with itself is termed an identity function, denoted by $I(x)=x$. Given that x can assume any value, the domain of this function is R, and the corresponding values of I(x) are also in R. Therefore, the range is equal to the domain, i.e., Range = Domain = R.



Ex. Determine whether the given pair of functions are identical or not.

1. $ff(x)=\frac{x^2-1}{x-1}$ and $g(x)=x+1$
2. $f(x)=\sin^2x+\cos^2x$ and $g(x)=\sec^2x-\tan^2x$

Sol. 1. No, since the domain of f(x) is R – {1}
While domain of g(x) is R
2. No, as domain are not same. Domain of f(x) is R
While that of g(x) is $R - \{(2n+1)\frac{\pi}{2}; n \in 1\}$

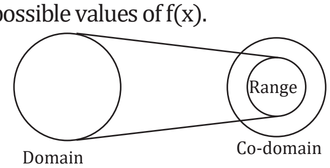
Domain, Co-domain and Range:

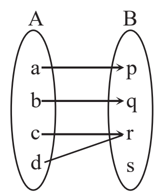
If a function f is defined from a set A to set B, then for f : A→B, set A is referred to as the domain of function f, and set B is known as the co-domain of function f. The collection of f-images of the elements in A is termed the range of function f.

In simpler terms, we can express:

Domain = All possible values of x, for which f(x) exists.

Range = For all values of x, all possible values of f(x).





Domain = {a, b, c, d} = A
Co-domain = {p, q, r, s} = B
Range = {p, q, r}

Ex. Determine the domain of the given functions:
1. $f(x) = \sqrt{x^2 - 5}$
2. $\sin(x^3 - x)$

Sol. 1. $f(x) = \sqrt{x^2 - 5}$ is real iff $x^2 - 5 \geq 0$

$$|x| \geq \sqrt{5}$$

$$x \leq -\sqrt{5} \text{ or } x \geq \sqrt{5}$$

The domain of f is $(-\infty, -\sqrt{5}] \cup [\sqrt{5}, \infty)$

2. $x^3 - x \in R$

Domain is $x \in R$