Equivalence Relation

A relation R on a non-empty set A is termed an equivalence relation if and only if it possesses the properties of being reflexive, symmetric, and transitive.
Examples of equivalence relations include "is parallel to," "is equal to," "is congruent to," and the "identity relation."
While every identity relation is inherently an equivalence relation, it's important to note that not every equivalence relation necessarily functions as an identity relation.

Ex.: Examine the provided relations to determine whether they exhibit reflexivity, symmetry, and transitivity, and identify any that meet the criteria for being equivalence relations.

1. a Rb if $\le$ b; a, b $\in$ set of real numbers.
2. a R b if $a < b$; a, b $\in$ N.
3. a Rb if $>$; a, b $\in$ R.
4. aRb if a divides b; a, b $\in$ N.
5. aRb if $(a - b)$ is divisible by n; a, b $\in$ I, n is a fixed positive integer.

Sol: **1.** Not reflexive, not symmetric but transitive
Suppose, $a = -2$ and$b = 3 - 2,3) \in R$.
Since ≤ 3 is true
And –2= 2 –2 is valid, the relation is not reflexive.
However as ≤ -2 is wrong the relation is not symmetric
Now, consider a, b, c be three real numbers such that $\le$ b and $\le$ c

$$\le b \Rightarrow b \ge 0$$
$$\le c \Rightarrow b \le c$$

Hence, $\le$ C is true so the given relation is transitive.

2. Not reflexive, not symmetric but transitive.
Since no natural number is less than itself the relation is not reflexive,
Additionally, If $a < b$ then $b < a$ is false, making it non–symmetric.
However, If $a < b$ then $b < c$, it is evident that $a < c$. demonstrating transitive

3. Not reflexive, symmetric, not transitive.
Therefore, it is not reflexive. It is symmetric.
If $a = 1, b = -1$ then $(b, c) \in R$
Hence R is not a transitive relation.

4. Reflexive, not symmetric, transitive
Given that 1 divides any number, hence R is reflexive.
If a divides b, it does not imply that b divides a (unless $(a = b)$ making the relation is not Symmetric (but anti-symmetric).
If a divides b and b divides c then it is clear that a will divide c. Indicating transitive.

5. Reflexive, symmetric and transitive, thus, it is an equivalence relation.
As 0 is divisible by n the given relation is reflexive
If a – b is divisible by n, then (b – a) will also be divisible by n, establishing Symmetry.
If $a - b = nI_1$ and $b - c = nI_2$,
Where, I_1, I_2 are integer.
Then, $a - c = (a - b) + (b - c) = n(I_1 + I_2)$
So, a – c is also divisible by n, hence transitive.