TYPES OF THE MATRICES

In this section, we will explore various types of matrices:

i. Column Matrix

A matrix is classified as a column matrix if it consists of only one column.

For example, $A = \begin{bmatrix} 0 \\ \sqrt{3} \\ -1 \\ \frac{1}{2} \end{bmatrix}$

is a column matrix of order 4×1.

ii. Row Matrix

A matrix is defined as a row matrix if it contains only one row.
For instance, A matrix is considered a row matrix if it possesses only one row.
For example, $B = [-\frac{1}{2}\sqrt{5} \quad 23]_{1\times 4}$
is a row matrix.
In general, $B = [b_{ij}]_{1 \times n}$ is a row matrix of order $1 \times n$.

iii. Square Matrix

A matrix in which the number of rows are equal to the number of columns, is termed a square matrix. Thus an $m \times n$ matrix is a square matrix if $m = n$ and is referred to as a square matrix of order 'n'.

For example, $A = \begin{bmatrix} 3 & -1 & 0 \\ \frac{3}{2} & 3\sqrt{2} & 1 \\ 4 & 3 & -1 \end{bmatrix}$ is a square matrix of order 3.

In general, $A = [a_{ij}]_{m \times m}$ is a square matrix of order m.

iv. Diagonal Matrix

A square matrix $A = [a_{ij}]_{n \times n}$ is considered a diagonal matrix if all the elements, excluding those in the leading diagonal, are zero denoted as $a_{ij} = 0$ for all $i \neq j$.
A diagonal matrix of order $n \times n$ having $d_1, d_2, \ldots, d_n$ as diagonal elements is Symbolized as diag $[d_1, d_2, \ldots, d_n]$.

For example, the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is an example of a diagonal matrix, represented as $A = \text{diag}[1, 2, 3]$.

v. Scalar Matrix

A diagonal matrix is classified as a scalar matrix if its diagonal elements are identical. In other words, a square matrix $B = [b_{ij}]_{n \times n}$ is designated a scalar matrix
if $b_{ij} = 0$,
when $i \neq j$.
$b_{ij} = k$,
when $i = j$, for some constant k.
For example

$$A = [3],\ B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix},\ C = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{3} & 0 \\ 0 & 0 & \sqrt{3} \end{bmatrix}$$

are examples of scalar matrices of order 1, 2 and 3, respectively.

vi. Identity Matrix

An identity matrix is a square matrix in which all the elements on the diagonal are 1, and the rest are 0. In formal terms, a square matrix $A = [a_{ij}]_{n \times n}$ is deemed an identity matrix if

$$a_{ij} = \begin{cases} 0 & \text{if } i = j \\ 1 & \text{if } i \neq j \end{cases}$$

The identity matrix of order n is symbolized as I_n and when the order is evident from the context, it is commonly represented as I.
For example, matrices

$$I_1 = [1], I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

are identity matrices of order 1, 2, and 3, respectively. It is important to note that a scalar matrix is considered an identity matrix only when k=1. However, every identity matrix is unequivocally a scalar matrix.

vii. Zero Matrix

A matrix is identified as a zero matrix or null matrix when all its elements are zero. For instance, matrices

$$O_1 = [0], O_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, O_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

are examples of zero matrices. We symbolize the zero matrix as O, and its order is usually evident from the context.

viii. Upper Triangular Matrix

An upper triangular matrix, denoted as A = [aij], is characterized by having aij = 0 for all i > j. In simpler terms, in an upper triangular matrix, all elements below the main diagonal are zero. An example of an upper triangular matrix is given by:

$$A = \begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & 5 & 1 & 3 \\ 0 & 0 & 2 & 9 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

ix. Lower Triangular Matrix

A lower triangular matrix, denoted as A = [aij], is characterized by having elements aij = 0 for all i < j. This means that in a lower triangular matrix, all the entries above the main diagonal are zero. For instance,

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 3 & 2 & 0 \\ 4 & 5 & 3 \end{bmatrix}$$

consider the matrix of order 3. Additionally, a triangular matrix A = [aij] of size n × n is termed strictly triangular if its diagonal elements aii are all zero for i = 1, 2, ..., n.

Equality of matrices

Definition 2 states that two matrices, A = [aij] and B = [bij], are considered equal under the following conditions:

i. They must be of the same order.

ii. Each element of matrix A must be identical to the corresponding element of matrix B, denoted by aij = bij for all i and j.

For instance, matrices

$$\begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$$

are equal, satisfying both criteria. On the other hand, matrices

$$\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$$

do not meet these conditions and are therefore not equal. Symbolically, the equality of matrices A and B is denoted by

A = B.

Ex. Determine the values of a, b, c, and d in the given equation.

$$\begin{bmatrix} 2a + b & a - 2\ b \\ 5c - d & 4c + 3\ d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$$

Sol. Utilizing the equality of two matrices and equating corresponding elements, we establish the following system of equations:

$$2a + b = 4 \qquad 5c - d = 11$$
$$a - 2b = -3 \qquad 4c + 3d = 24$$

Solving this system, we find the values: a = 1, b = 2, c = 3 and d = 4

Ex. Consider the matrix

$$\begin{bmatrix} \sin\theta & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \cos\theta \\ \cos\theta & \tan\theta \end{bmatrix} \& B = \begin{bmatrix} \frac{1}{\sqrt{2}} & \sin\theta \\ \cos\theta & \cos\theta \\ \cos\theta & -1 \end{bmatrix}$$

Determine the value of q such that $A = B$

Sol. According to the definition of equality for matrices, two matrices, in this case, A and B, are considered equal if they share the same order, and all corresponding elements are identical.

Therefore, we have $\sin\theta = \frac{1}{\sqrt{2}}$

$\cos\theta = -\frac{1}{\sqrt{2}}$

& $\tan\theta = -1$

$\Rightarrow$ $\theta = (2n+1)\pi - \frac{\pi}{4}$

$\theta = 2n\pi + \frac{3}{4}\pi$