

SUM AND DIFFERENCE OF ANGLES IN TERMS OF $\tan^{-1}$ For all $x, y \in [-1, 1]$

$$\tan^{-1} x + \tan^{-1} y = \begin{cases} \tan^{-1} \frac{x+y}{1-xy}, & \text{if } xy < 1 \\ \pi + \tan^{-1} \frac{x+y}{1-xy}, & \text{if } xy > 1 \text{ and } x, y > 0 \\ -\pi + \tan^{-1} \frac{x+y}{1-xy}, & \text{if } xy > 1 \text{ and } x, y < 0 \end{cases}$$

$$\tan^{-1} x - \tan^{-1} y = \begin{cases} \tan^{-1} \frac{x-y}{1+xy}, & \text{if } xy > -1 \\ \pi + \tan^{-1} \frac{x-y}{1+xy}, & \text{if } xy < -1 \text{ and } x > 0, y < 0 \\ -\pi + \tan^{-1} \frac{x-y}{1+xy}, & \text{if } xy < -1 \text{ and } x < 0, y > 0 \end{cases}$$