SUM AND DIFFERENCE OF ANGLES IN TERMS OF SIN^{-1} AND COS^{-1}

For all $x, y \in [-1,1]$

$$\sin^{-1} x + \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right), \text{ if } x^2 + y^2 \leq 1 \text{ or } xy < 0$$
$$= \pi - \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right), \text{ if } x^2 + y^2 > 1 \text{ and } x, y > 0$$
$$= -\pi - \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right), \text{ if } x^2 + y^2 > 1 \text{ and } x, y < 0$$

$$\sin^{-1} x - \sin^{-1} y = \sin^{-1}(x\sqrt{1-y^2} - y\sqrt{1-x^2}), \text{ if } x^2 + y^2 \leq 1 \text{ or } xy > 0$$
$$= \pi - \sin^{-1}(x\sqrt{1-y^2} - y\sqrt{1-x^2}), \text{ if } x^2 + y^2 > 1 \text{ and } x > 0, y < 0$$
$$= -\pi - \sin^{-1}(x\sqrt{1-y^2} - y\sqrt{1-x^2}), \text{ if } x^2 + y^2 > 1 \text{ and } x < 0, y > 0$$

$$\cos^{-1} x + \cos^{-1} y = \cos^{-1}[xy - \sqrt{1-x^2}\sqrt{1-y^2}]; \text{ if } -1 \leq x, y \leq 1, x + y \geq 0$$
$$= 2\pi - \cos^{-1}[xy - \sqrt{1-x^2}\sqrt{1-y^2}]; \text{ if } -1 \leq x, y < 1, x + y \leq 0$$

$$\cos^{-1} x - \cos^{-1} y = \cos^{-1}(xy + \sqrt{1-x^2}\sqrt{1-y^2}); \text{ if } -1 \leq x, y \leq 1, x \leq y$$
$$= -\cos^{-1}(xy + \sqrt{1-x^2}\sqrt{1-y^2}); \text{ if } -1 \leq y < 0, 0 < x \leq 1, x \geq y$$