RELATING DIFFERENT INVERSE TRIGONOMETRIC FUNCTIONS

Simplifying expression using trigonometric substitution

Property 1 : T(T $^{-1}$)

(i) $\sin(\sin^{-1} x) = x,\ -1 \le x \le 1$

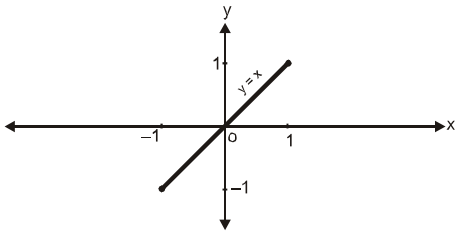
Proof:

Let $\theta = \sin^{-1}x$.

Then $x \in [-1, 1]$ & $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$\Rightarrow$ $\sin\theta = x$, by meaning of the symbol

$\Rightarrow$ $\sin(\sin^{-1} x) = x$



Similar proofs can be carried out to obtain

(ii) $\cos(\cos^{-1} x) = x,$ $-1 \le x \le 1$

(iii) $\tan(\tan^{-1} x) = x,$ $x \in R$

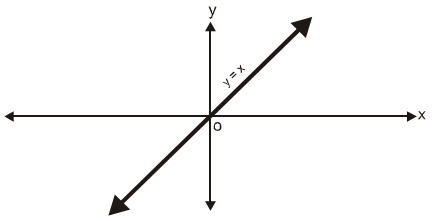
(iv) $\cot(\cot^{-1} x) = x,$ $x \in R$

(v) $\sec(\sec^{-1} x) = x,$ $x \le -1, x \ge 1$

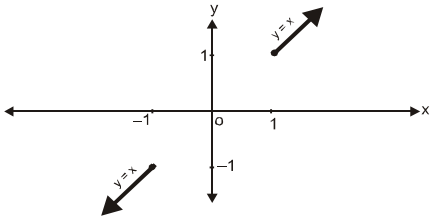
(vi) $\text{cosec}(\text{cosec}^{-1} x) = x,$ $|x| \ge 1$

The graph of $y = \sin(\sin^{-1}x) \equiv \cos(\cos^{-1}x)$

The graph of $y = \tan(\tan^{-1}x) \equiv \cot(\cot^{-1}x)$



The graph of $y = \text{cosec}(\text{cosec}^{-1}x) \equiv \sec(\sec^{-1}x)$



Property 2 : T^{-1}(T)

(i) $\sin^{-1}(\sin x) = x$; $-\frac{\pi}{2} \le x \le -\frac{\pi}{2}$

(ii) $\cos^{-1}(\cos x) = x$; $0 \le x \le \pi$

(iii) $\tan^{-1}(\tan x) = x$; $-\frac{\pi}{2} < x < \frac{\pi}{2}$

(iv) $\text{cosec}^{-1}(\text{cosec } x) = x$; $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ and $x \ne 0$

(v) $\sec^{-1}(\sec x) = x$; $0 \le x \le \pi$ and $x \ne \frac{\pi}{2}$

(vii) $\cot^{-1}(\cot x) = x$; $0 < x < \pi$

Property 3 : "$\frac{\pi}{2}$"

(i) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}, -1 \le x \le 1$

(ii) $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2},\ x \in R$

(iii) $\text{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2},\ |x| \ge 1$