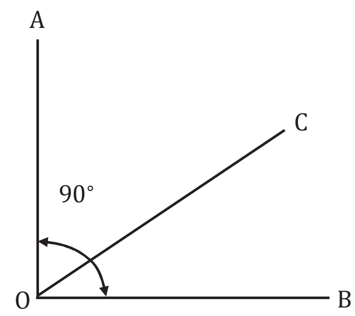
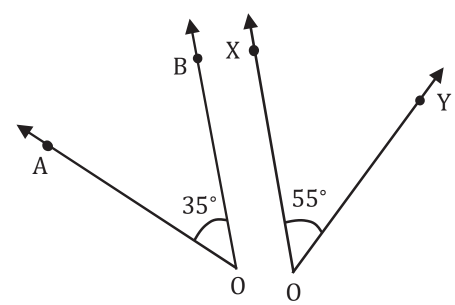
COMPLEMENTARY ANGLES

Complementary angles are angles that, when added together, result in a total of 90 degrees. We also say that these angles complement each other. For instance, examples of complementary angles include pairs such as 60° and 30°, 40° and 50°, and so on, because their sum equals 90°.



The angles AOC and BOC shown in the diagram above add up to 90°, making them complementary angles. Additionally, complementary angles don't have to be positioned next to each other. For example, these two angles also add up to 90° and are considered complementary.



Non-adjacent Complementary Angles

Based on adjacency there are two types of complementary angles.

Types of Complementary Angles

As we've mentioned before, complementary angles are any two angles that add up to form a right angle. When angles combine to make a pair of complementary angles, we say they complement each other. There are two main types of complementary angles:

1. Adjacent Complementary Angles:

These are angles that share a corner and a side. In the picture above, AOC and BOC are adjacent complementary angles because they share a side and a corner.

2. Non-adjacent Complementary Angles:

These are pairs of complementary angles that aren't next to each other. Even though they don't share a side or a corner, their total always equals 90°. For instance, the angles shown in the second figure illustrate non-adjacent complementary angles.

Trigonometric Ratios of Complementary Angles

Trigonometry is all about understanding how angles and sides of triangles relate to each other. One of the key ideas in trigonometry is complementary angles.

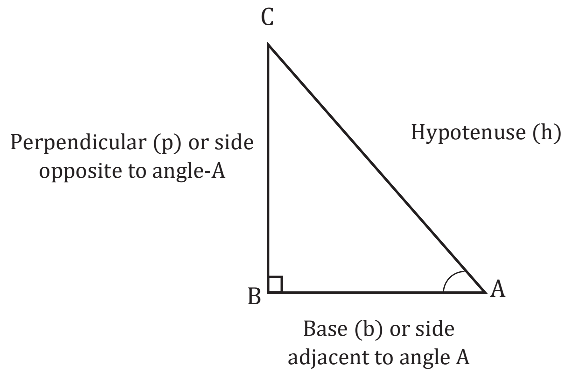
When we explore the trigonometric functions of complementary angles, we find interesting connections among the six main trigonometric functions: sine, cosine, tangent, cotangent, secant, and cosecant.

For complementary angles, the sine of one angle matches the cosine of the other angle, and vice versa. Similarly, the tangent of one angle equals the cotangent of the other angle, and vice versa. Lastly, the secant of one angle is the same as the cosecant of the other angle, and vice versa.

These connections are incredibly helpful when solving trigonometry problems.

Trigonometric Ratios of Complementary angles formula

Consider the $\triangle ABC$ shown below. We will find out the trigonometric ratios for both angles A and C.



Sine and Cosine of Complementary Angles

For $\angle$**A**	For $\angle$**C**
$\sin A = \frac{BC}{AC}$	$\sin C = \frac{AB}{AC}$
$\cos A = \frac{AB}{AC}$	$\cos C = \frac{BC}{AC}$
$\tan A = \frac{BC}{AB}$	$\tan C = \frac{AB}{BC}$
$\cot A = \frac{AB}{BC}$	$\cot C = \frac{BC}{AB}$
$\sec A = \frac{AC}{AB}$	$\sec C = \frac{AC}{BC}$
$\operatorname{cosec} A = \frac{AC}{BC}$	$\operatorname{cosec} C = \frac{AC}{AB}$

From the above table, we can see that;

$$\sin A = \cos C = \frac{BC}{AC}$$

$$\sin C = \cos A = \frac{BC}{AC}$$

$$\tan A = \cot C = \frac{BC}{AB}$$

$$\tan C = \cot A = \frac{AB}{BC}$$

$$\sec A = \operatorname{cosec} C = \frac{AC}{AB}$$

$\sec C = \text{cosec } A = \frac{AC}{BC}$

This formulae come handy while finding out the unknown sides and angles in trigonometry.
There is another way of proving this.
We know that, $\angle A + \angle C = 90^\circ$.
Thus, $\sin A = \sin(90^\circ - C)$
From the trigonometric identities we know that, $\sin(90^\circ - \theta) = \cos(\theta)$
Thus, $\sin A = \sin(90^\circ - C) = \cos C$.
Similarly, we can prove for all other triangles.

Relation between Trigonometric Ratios of Complementary Angles

The following formulas express the relationships between the trigonometric functions for complementary angles.

- Sine (sin): $\sin(\theta) = \cos(90^\circ - \theta)$
- Cosine (cos): $\cos(\theta) = \sin(90^\circ - \theta)$
- Tangent (tan): $\tan(\theta) = \cot(90^\circ - \theta)$
- Cotangent (cot): $\cot(\theta) = \tan(90^\circ - \theta)$
- Secant (sec): $\sec(\theta) = \csc(90^\circ - \theta)$
- Cosecant(csc): $\csc(\theta) = \sec(90^\circ - \theta)$