THE TERMS USED IN PROBABILITY

Experiment

An experiment is defined as an operation that yields well-defined outcomes.

Sample Space And Sample Points

The collection of all conceivable outcomes in a random experiment is termed the sample space, denoted by S. Each potential outcome, that is, every element within this set, is referred to as a sample point.

For example:

- In a coin toss, the sample space is denoted by S = {H, T}, where H and T represent a head and a tail, respectively.
- In a die throw, the sample space is given by S = {1, 2, 3, 4, 5, 6}, where the numbers represent the six faces as sample points.

Event

An Event, Which Is A Subset Of The Sample Space, Represents A Set Of Specific Outcomes From A Random Experiment.

Simple event

Every individual sample point within the sample space is referred to as an elementary event or simple event.

For example

- The event of obtaining a head in a coin toss is a simple event.

Sure event

The set that includes all sample points is a certain event. For example, in the throw of a die, the event of obtaining a natural number less than 7 is a certain event.

Null event

The set that does not include any sample points.

Mixed/compound event

A subset of the sample space S that comprises more than one element is referred to as a composite event or a mixed event.

Compliment of an event

Consider S as the sample space and E as an event. The complement of event E, denoted as E^cor $\overline{E}$, is a subset that includes all sample points in S not present in E. It signifies the non-occurrence of event E.

Algebra Of Events

In the context of fundamental probability laws, we require the following concepts and facts regarding events (subsets) A, B, C … of a given sample space S. The union $A \cup B$ of A and B includes all points in either A or B or both. The intersection $A \cap B$ of A and B comprises all points that belong to both A and B. If A and B have no common points, we express it as

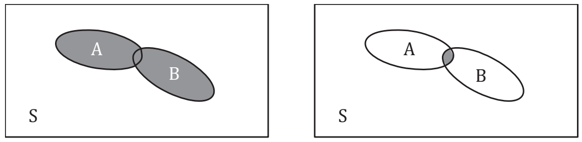
$$A \cap B = \phi$$

Where ϕ is the empty set (a set with no elements), and we refer to A and B as mutually exclusive (or disjoint) because the occurrence of A excludes that of B, and vice versa. For example, if a die turns up an odd number in a trial, it cannot turn up an even number in the same trial. Similarly, a coin cannot show both Head and Tail at the same time.

The complement A^c of A comprises all the points in S that are not in A. Thus,

$$A \cap A^C = \phi, A \cup A^C = S$$

The use and understanding of events can be depicted and enhanced through Venn diagrams, which illustrate unions, intersections, and complements, as depicted in the figure.



Union $A \cup B$ Intersection $A \cap B$

Venn diagrams illustrate two events, A and B, within a sample space S, along with their union $(A \cup B)$and intersection $(A \cap B)$depicted in colour. The concepts of union and intersection extend similarly for more events.

$$\cup_{i=1}^{m} A_j = A_1 \cup A_2 \cup \ldots \ldots \cup A_m$$

The union of events A_1... Am comprises all points that belong to at least one of the A_j. Similarly, for the union $A_1 \cup A_2 \cup \ldots$ of an infinite number of subsets $A_1, A_2, \ldots$of an infinite sample space S (meaning S contains an infinite number of points). The intersection

$$\bigcap_{j=1}^{m} A_j = A_1 \cap A_2 \cap \ldots \ldots \cap A_m$$

The intersection of A_1 ..., Am consists of the points in S that are common to each of these events. Similarly, for the intersection $A_1 \cap A_2 \cap \ldots$of an infinite number of subsets of S.

Equally Likely Events

The events are considered equally likely if there is no expectation that one of them will occur over the other.

For example

- In the toss of a fair coin, the likelihood of a head or a tail occurring is equal. Therefore, the events of a head appearing and a tail appearing are equally likely.

Mutually Exclusive Events

A collection of events is considered mutually exclusive if the occurrence of one event prevents the occurrence of any other event.

For example:

- In the roll of a die, the event of obtaining an even number and the event of obtaining an odd number are mutually exclusive.
- In the toss of a fair coin, the occurrence of a head or a tail is mutually exclusive.

Exhaustive Events

A set of events is exhaustive if the performance of the experiment results in occurrence of at least one of them.

For example:

- In the roll of a die, the event of obtaining an even number and the event of obtaining an odd number are exhaustive.
- In the toss of a fair coin, the occurrence of a head or a tail is exhaustive.
- Exhaustive events cover the whole of the sample space. Their union is equal to S.

Random Experiment

It is a process that can yield any of its well-defined outcomes, and predicting the outcome with certainty is not possible.

For example

- Certain random experiments include:
- Coin toss, resulting in either heads or tails
- Rolling a die, leading to any one of its six faces
- Drawing a card from a deck of 52 cards, resulting in any one of the 52 cards

Trial

When an experiment is conducted repeatedly under consistent conditions and does not yield the same outcome each time but can result in any of several possible outcomes, the experiment is referred to as a trial, and the outcomes are termed cases. The count of how many times the experiment is repeated is known as the number of trials.

For example:

- A single coin toss constitutes a trial when the coin is tossed five times.
- A single die throw is considered a trial when the die is thrown four times.

Definition of probability with discrete sample space

If the sample space S of an experiment comprises a finite number of outcomes (points) that are equally likely, then the probability of event A occurring is

$$P(A) = \frac{\text{Number of sample points in A}}{\text{Number of sample points in S}}$$

$$P(A) = \frac{n(A)}{n(S)}$$

In particular $P(S) = 1$ and, $0 \leq P(A) \leq 1$.

Ex. Out of a set of 100 items, ten are defective. What is the probability that three out of any four chosen items are defective?

Sol. Probability $= \frac{{}^{90}C_1 \cdot {}^{10}C_3}{{}^{100}C_4} = \frac{144}{52283}$

Ex. Seven individuals need to be seated on one side of a straight table. What is the probability that two specific persons will be seated next to each other?

Sol. Total number of ways of 7 persons being seated is ${}^7P_7 = \lfloor 7$ ways

If two are to be seated next to each other, treat them as one unit – and this one unit with the remaining 5 can be seated in $\lfloor 6$ ways – and in each one of these $\lfloor 6$ ways the two persons can be interchanged in 2 ways.

$$\text{Probability } = \frac{2\lfloor 6}{\lfloor 7} = \frac{2}{7}$$