Random Variables

A variable is a quantity that can change its value, varying with different outcomes of an experiment. If a variable's value depends on the outcome of a random experiment, it is termed a random variable and can assume any real value.

In mathematical terms, a random variable is a real-valued function defined on the sample space S of a random experiment. Denoted by capital letters such as X, Y, M, etc., it is represented by lowercase letters like x, y, z, m, etc., for specific values.

Consider the random experiment of tossing a coin 20 times, where earning Rs. 5 is associated with getting heads and losing Rs. 5 with tails. In a competition to see who can earn more money between you and your friend, the value of getting heads in the coin toss for 20 times can range from zero to twenty. If we let X denote the number of heads, then $X = \{0, 1, 2, \dots, 20\}$. The probability of getting heads is consistently $\frac{1}{2}$.

Properties of a Random Variable

- It exclusively accepts real values.
- If X is a random variable and C is a constant, then CX is likewise a random variable.
- When X_1 and X_2 are two random variables, both $X_1 + X_2$ and $X_1 X_2$ are also random.
- For any constants C_1 and C_2, the expression $C_1X_1 + C_2X_2$ is also considered a random variable.
- The absolute value of X, denoted as |X|, is a random variable.

Types of Random Variable

A random variable can be classified into two distinct types.

Discrete Random Variable

As implied by its name, this variable is not linked or continuous. It can only take on a countable number of real values, meaning the values of the discrete random sample are inherently discrete. The outcome of the random variable is contingent on chance. In simpler terms, a real-valued function defined on a discrete sample space is referred to as a discrete random variable.

Examples of discrete random variables include the number of calls a person receives in a day, the quantity of items sold by a company, the count of items manufactured, the number of accidents, and the number of gifts received on a birthday.

Continuous Random variable

A variable that takes on an infinite range of values within the sample space is termed a continuous random variable. It has the capacity to encompass all possible values within certain limits, including both integral and fractional values. Examples of continuous random variables include height, weight, age of a person, and the distance between two cities.