PROBABILITY DISTRIBUTION

We can determine the probability associated with any event in a random experiment. Similarly, for various values of a random variable, we can ascertain their respective probabilities. The combination of the values of random variables and their corresponding probabilities constitutes the probability distribution of the random variable.

Let X be a random variable, and its probability distribution is represented by the function P(X). Any function F defined for all real x as $F(x) = P(X \leq x)$ is referred to as the distribution function of the random variable X.

Properties of Probability Distribution

- The probability distribution of a random variable X is $P(X = x_i) = p_i$ for $x = x_i$ and $P(X = x_i) = 0$ for $x \neq x_i$.
- The probability distribution for all conceivable values of a random variable spans from 0 to 1, meaning $0 \leq p(x) \leq 1$.

Probability Distribution of a Discrete Random Variable

If X represents a discrete random variable with distinct values $x_1, x_2, \dots, x_n, \dots$, then the probability function is denoted as $P(x) = p_X(x)$. The distribution function is given by

$$F_X(x) = P(X \leq x_i) = \sum_i p(x_i) = p_i$$

when $x = x_i$ and is 0 for other values of x. In this context, i ranges from 1 to n and beyond.

To illustrate, consider the example of tossing two fair coins, where the potential outcomes are S = {HH, HT, TH, TT}. If X denotes a random variable representing the occurrence of tails, the possible values for X are 0, 1, and 2. The distribution function for X is expressed as $F(x) = P(X \leq x)$.

Value of x	0	1	2
$P(X = x) = p(x)$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$
$F(x) = p(x \leq x) = \sum_i p(x_i)$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{4}{4} = 1$

Probability Distribution of a Continuous Random Variable

If X is a discrete random variable with specific values $x_1, x_2, \dots, x_n, \dots$, then the probability distribution function is represented as $F(x) = p_X(x_i)$. On the other hand, for a continuous random variable

$$F_X(x) = \int p_X(x_i)\, dx$$

Where $i = 1, 2, \dots, n, \dots$

Ex. Three unbiased coins are thrown. Define X as the count of heads and Y as the count of head runs, where a 'head run' signifies the consecutive occurrence of at least two heads. Determine the probability functions for X and Y.

Sol. The possible outcomes of the experiment is

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}.

X represents the number of heads, assuming values of 0, 1, 2, and 3.

EVENTS	RANDOM VARIABLES	
	X	Y
HHH	3	1
HHT	2	1
HTH	2	0
HTT	1	0
THH	2	1
THT	1	0

TTH	1	0
TTT	0	0

- $P(\text{no head}) = p(0) = \frac{1}{8}$
- $P(\text{one head}) = p(1) = \frac{3}{8}$
- $P(\text{two heads}) = p(2) = \frac{3}{8}$
- $P(\text{three heads}) = p(3) = \frac{1}{8}$

VALU OF X,				
x	0	1	2	3
P(x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Y is the number of head runs. It takes up the values 0 and 1.

$$P(Y = 0) = p(0) = \frac{5}{8}$$

and
$$P(Y = 1) = p(1) = \frac{3}{8}$$

Value of Y, y	0	1
P(y)	$\frac{5}{8}$	$\frac{3}{8}$

Ex. Can you provide a definition for a random variable?

Sol. A variable is something that can alter its value and may fluctuate based on the outcomes of an experiment. If a variable's value relies on the result of a random experiment, it is categorized as a random variable, capable of assuming any real value.

Ex. Can you enumerate the characteristics of a random variable?

Sol. A random variable exclusively assumes real values. For example, if X is a random variable and C is a constant, then CX is also recognized as a random variable. Additionally, when X_1 and X_2 are two random variables, both X_1+X_2 and $X_1\ X_2$ are also considered random variables. Moreover, for any constants C_1 and C_2, the expression $C_1X_1 + C_2X_2$ is likewise acknowledged as a random variable. Consequently, X qualifies as a random variable.

Ex. Can you provide an explanation for the concept of probability distribution?

Sol. We can determine the probability associated with any event in a random experiment. Similarly, for various values of the random variable, one can ascertain their respective probabilities. Furthermore, the combination of the values of random variables and their corresponding probabilities constitutes the probability distribution of the random variable.