INDEPENDENT EVENTS

Dependent and Independent Events

Two events, A and B, are considered independent if the occurrence or non-occurrence of one event does not influence the probability of the occurrence or non-occurrence of the other.

(A) If the occurrence of one event influences the probability of the occurrence of the other event, then the events are termed Dependent or Contingent. For two independent events A and B: $P(A \cap B) = P(A) . P(B)$.
This relation is frequently considered as the definition of independent events.

(B) Three events A, B, and C are deemed independent if and only if the following conditions are met:

$$P(A \cap B) = P(A) . P(B)$$
$$P(B \cap C) = P(B). P(C)$$
$$P(C \cap A) = P(C) . P(A)$$
& $$P(A \cap B \cap C) = P(A) . P(B) . P(C)$$

In other words, they must exhibit pairwise as well as mutual independence. Similarly, for n events $A_1, A_2, A_3, ..., A_n$ to be independent, the number of these conditions should be equal to:

$$^nC_2 + {^nC_3} + ... + {^nC_n} = 2^n - n - 1.$$

Ex. The probability of hitting an enemy plane with the first, second, and third shots from an antiaircraft gun is 0.6, 0.7, and 0.1, respectively. The overall probability of hitting the plane is:

Sol. Consider the events of hitting the enemy plane with the first, second, and third shots as A, B, and C, respectively.

Then as given $P(A) = 0.6, P(B) = 0.7, P(C) = 0.1$

Science $P(A \cup B \cup C) = 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$

and events A,B,C are independent

$$P(\bar{A} \cap \bar{B} \cap \bar{C}) = P(\bar{A})P(\bar{B})P(\bar{C})$$

Required probability $= P(A \cup B \cup C) = 1 - P(\bar{A})P(\bar{B})P(\bar{C})$

$$= 1 - (1 - 0.6)(1 - 0.7)(1 - 0.1)$$
$$= 1 - (0.4)(0.3)(0.9)$$
$$= 1 - 0.108 = 0.892$$

Ex. A pair of fair coins is tossed, resulting in the equiprobable space
S = {HH, HT, TH, TT}. Let's define the events as follows:
A = {head on first coin} = {HH, HT},
B = {head on second coin} = {HH, TH}
C = {head on exactly one coin} = {HT, TH}
Now, we need to determine whether A, B, and C are independent events or not.

Sol.

$$P(A) = P(B) = P(C) = \frac{2}{4} = \frac{1}{2}$$

$$P(A \cap B) = \frac{1}{4} = P(A)P(B),$$

$$P(A \cap C) = \frac{1}{4} = P(A)P(C),$$

$$P(B \cap C) = \frac{1}{4} = P(B)P(C)$$

But $P(A \cap B \cap C) = 0 \neq P(A)P(B)P(C)$

A, B & C are not independent

Ex. If cards are drawn successively from a well-shuffled pack of 52 cards without replacement until an ace appears, the probability that the fourth card is the first ace to appear can be determined.

Sol. The probability of selecting 3 non-Ace and 1 Ace out of 52 cards is equal to $\frac{{}^{48}C_3 \times {}^4C_1}{{}^{52}C_4}$. Since we desire the 4th card to be the first ace, we need to consider the arrangement. There are 4 cards in the sample space, and they can be arranged in 4! ways. Among these, the favorable arrangement is in 3! ways, as we want the 4th position to be occupied by an Ace. Therefore, the required probability is $\frac{{}^{48}C_3 \times {}^4C_1}{{}^{52}C_4} \times \frac{3!}{4!}$.

Ex. In the process of drawing two balls from a box containing 6 red and 4 white balls without replacement, which of the following pairs is considered independent?

(A) Red on first draw and red on second draw
(B) Red on first draw and white on second draw

Sol. Let E represent the event 'Red on the first draw,' F represent the event 'Red on the second draw,' and G represent the event 'White on the second draw.'

$$P(E) = \frac{6}{10}, P(F) = \frac{6}{10}, P(G) = \frac{4}{10}$$

(A)

$$P(E \cap F) = \frac{{}^6P_2}{{}^{10}P_2} = \frac{1}{3}$$

$$P(E) \cdot P(F) = \frac{3}{5} \times \frac{3}{5} = \frac{9}{25} \neq \frac{1}{3}$$

E and F are not independent

(B)

$$P(E) \cdot P(G) = \frac{6}{10} \times \frac{4}{10} = \frac{6}{25}$$

$$P(E \cap G) = \frac{{}^6P_1 \times {}^4P_1}{{}^{10}P_2} = \frac{4}{15}$$

$$P(E) \cdot P(G) \neq P(E \cap G)$$

E and G are not independent