COMPOUND AND CONDITIONAL PROBABILITY

Compound event

When two or more than two events occur simultaneously, the event is said to be a compound event. Symbolically $A \cap B$ or AB represent the occurrence of both A & B simultaneously.

"$A \cup B$" or A + B represent the occurrence of either A or B.

Conditional Probability

Let's imagine we have two events, A and B. When we say "P(B|A)," we mean the probability of event B happening given that event A has already occurred. When we know event A has happened, we consider it as our new focus, replacing the original possibilities. This leads us to define conditional probability.

$$P(B \mid A)=\frac{P(A \cap B)}{P(A)}$$

which is called conditional probability of B given A

Multiplication of probability

Let E and F be two events associated with a sample space S. The occurrence of both events E and F simultaneously is denoted by$E \cap F$..

From the concept of conditional probability of event E given that F has occurred, we know that

$$P(E \mid F)=\frac{P(E \cap F)}{P(F)}, P(F) \neq 0$$

$$P(E \cap F)=P(F) \cdot P(E \mid F) \quad \ldots(i)$$

Also, we know that

$$P(F \mid E)=\frac{P(F \cap E)}{P(E)}, P(E) \neq 0$$

$$P(E \cap F)=P(E) \cdot P(F \mid E) \quad \ldots(ii)$$

Combining (i) and (ii), we find that

$$P(E \cap F)=P(E) \cdot P(F \mid E)$$

$$P(F) \cdot P(E \mid F) \text{ provided } P(E) \neq 0 \& P(F) \neq 0$$

Multiplication rule of probability for more than two events

If E, F and G are three events of sample space, we can extend the multiplication rule as given below,

$P(E \cap F \cap G)=$ (Probability of occurrence of E) x (Probability of occurrence of F while E has occurred) × (Probability of occurrence of G while E and F both have occurred)

$$P(E \cap F \cap G)=P(E) \cdot P(F \mid E) \cdot P(G \mid (E \cap F))$$

Similarly, the multiplication rule of probability can be extended for four or more events.

Ex. If $P(A/B)=0.2$ and $P(B)=0.5$ and $P(A)=0.2$. Find $P(A \cap \overline{B})$.

Sol. Let A be the event of occurrence of 4 always on the second die

$=\{(1,4),(2,4),(3,4),(4,4),(5,4),(6,4)\}$; $\quad \therefore n(A)=6$

and B be the event of occurrence of such numbers on both dice whose sum is 8 = {(6,2), (5,3), (4,4), (3,5), (2,6)}.

Thus, $A \cap B=\{(4,4)\}$

$\therefore n(A \cap B)=1$

$$\therefore P\left(\frac{B}{A}\right)=\frac{n(A \cap B)}{n(A)}=\frac{1}{6} \text{ or } \frac{P(A \cap B)}{P(A)}=\frac{\frac{1}{36}}{\frac{6}{36}}=\frac{1}{6}$$

Complementation rule

When one of two separate events is certain to happen, they're called complementary events. Because one of them must occur, the combined probability of both complementary events equals 1, or 100% of all possible outcomes.

For instance, consider a spinner with only red and blue sections. The chance of the spinner landing on either red or blue is 1. The arrow will always land on red or blue, covering all possible outcomes.

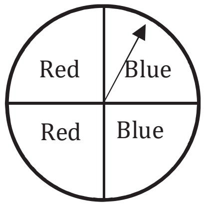

Fig: 1

$$P(\text{ red or blue }) = P(\text{ red }) + P(\text{ blue })$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

Here are some examples of situations involving complementary events:

- Tossing a coin to get heads or tossing a coin to get tails.
- Flicking a light switch to turn a light on or flicking a light switch to turn a light off.
- Securing a door by locking it or making it accessible by unlocking it.

While some complementary events, like coin flips, have equal chances (50-50), not all of them do. For example, for the spinner shown:

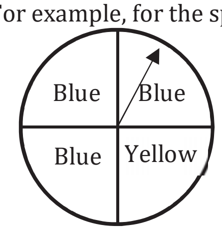

Fig: 2

$$P(\text{ blue or yellow }) = P(\text{ blue }) + P(\text{ yellow })$$

$$= \frac{3}{4} + \frac{1}{4}$$

$$= 1$$

The events B(blue) and Y(yellow) are complementary because their probabilities add up to 1. But the two complements are not equal in size.

Note that some disjoint events are NOT complementary events. Here, R(red) and B(blue) are disjoint events. However, their probabilities do NOT add up to 1 or 100 percent:

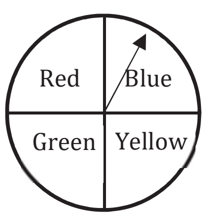

Fig: 3

$$P(\text{ red or blue }) = P(\text{ red }) + P(\text{ blue })$$

$$= \frac{1}{4} + \frac{1}{4}$$

$$= \frac{1}{2}$$

Because the total probability of two complementary events adds up to 1, if you know the probability of one complement, you can determine the probability of the other.
For events A and B, suppose the probability of B is 0.4. That means:

$$P(A) + P(B) = 1$$
$$P(A) + 0.4 = 1$$

Therefore, the probability of P(A) is 0.6, because:

$$P(A) + P(B) = 1$$
$$0.6 + 0.4 = 1$$

The Complement Rule states that for any two complements, A and B, the value of $P(A) = 1 - P(B)$. In other words, subtract the complement you know from 1 to find the unknown complement.
A and B are complements. $P(B) = 0.3$. Find $P(A)$.
To figure this out, subtract the complement you know, 0.3, from 1 to find P(B)

$$\begin{aligned} P(B) &= 1 - P(A) \\ &= 1 - 0.3 \\ &= 0.7 \end{aligned}$$

What is the probability that the arrow will land on red, green, or yellow?

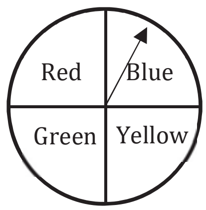

Fig: 4

The events are disjoint so the probability of one of them occurring is the sum of their individual probabilities.

$$\begin{aligned} P(\text{ red or blue or green }) &= P(\text{ red }) + P(\text{ blue }) + P(\text{ green }) \\ &= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \\ &= \frac{3}{4} \end{aligned}$$

The probability of the complementary event of the spinner landing on yellow is:

$$\begin{aligned} P(\text{ yellow }) &= 1 - P(\text{ red or blue or green }) \\ &= 1 - \frac{3}{4} \\ &= \frac{1}{4} \end{aligned}$$

The chance of the Mets winning tonight's game is 0.6. Let's figure out how probable it is for the Mets to lose tonight's game.

When we talk about winning and losing, they're like two sides of the same coin—they cover all the possibilities. So, if we know the probability of winning, we can use that to find out the probability of losing using a special formula for complementary events.

$$\begin{aligned} P(\text{ lose }) &= 1 - P(\text{ win }) \\ &= 1 - 0.6 \end{aligned}$$

Next, subtract the known probability from 1.

$$\begin{aligned} P(\text{ lose }) &= 1 - P(\text{ win }) \\ &= 1 - 0.6 \\ &= 0.4 \end{aligned}$$

Then, state the answer as the probability for the complementary event.
The answer is the probability that the Mets will lose tonight's game is 0.4. In other words, there is a 40% chance that the Mets will lose the game tonight.

Ex. Y and Z are complements. If the probability of Y occurring is 14%, what is the probability of Z occurring?

Sol. First, substitute the value for the known probability into the formula for complementary events. In this case, since, the known probability is reported in percentage, we will subtract from 100 instead of 1:

$$\begin{aligned} P(Z) &= 100 \text{ percent } - P(Y) \\ &= 100 \text{ percent } - 14 \text{ percent} \end{aligned}$$

Next, subtract the known probability from 100 percent:

$$\begin{aligned} P(Z) &= 100 \text{ percent } - P(Y) \\ &= 100 \text{ percent } - 14 \text{ percent} \\ &= 86 \text{ percent} \end{aligned}$$

Then, state the answer as the probability for the complementary event:
The answer is the probability that Z will occur is 86%.