Chapter 15

Limits

INTRODUCTION

The nation of limit is the basis of calculus and the notion of derivative of a function builds on the notion of limit.

In the study of limits, we enable a variable to undergo continuous change and observe how the functional values change accordingly. Subsequently, we will delve into fundamental concepts of derivatives.

LIMITS

Consider the real-valued function y = f(x), defined in the vicinity of the point x = a. A real number i is denoted as the limit of the function f as x approaches a when.

$$X \to \text{ A I.E. } \lim_{X\to A} F(X) = I$$

The concept of continuity is prevalent in various applications of calculus. One may conceptually perceive continuous functions as those for which the graphs can be drawn without the necessity of lifting the pencil off the paper.

Existence of Limits

$$\lim_{x\to a} f(x) = l \text{ exists if}$$

$$\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = \text{ finite}$$

$$\lim_{h\to 0} f(a+h) = \lim_{h\to 0} f(a-h) = \text{ finite}$$

Where $\lim\limits_{x\to a^+} f(x)$ is called right hand limit and $\lim\limits_{x\to a^-} f(x)$ is called left hand limit.

Conclusion:

$\lim\limits_{x\to a} f(x)$ Existence is attributed to a limit at x = a, if both its left-hand limit (LHL) and right-hand limit (RHL) at x = a, are equivalent and finite.

$\lim\limits_{x\to a^+} f(x) = \lim\limits_{x\to a^-} f(x) =$ finite, other wise limits dose not exists.

Ex. Evalute $\lim\limits_{x\to 0} f(x)$ if $f(x) = \frac{|x|}{x}$

Sol. L.H.L. $= \lim\limits_{x\to 0^-} f(x) = \lim\limits_{h\to 0} f(0-h)$

$\lim\limits_{h\to0}\frac{|0-h|}{0-h} = \lim\limits_{h\to0}\frac{|-h|}{-h} = \lim\limits_{h\to0}\frac{h}{-h} = -1$

$\text{R.H.L} = \lim\limits_{x\to0^+} f(x) = \lim\limits_{h\to0} f(0+h)$

$\lim\limits_{h\to0}\frac{|0+h|}{0+h} = \lim\limits_{h\to0}\frac{|h|}{h} = 1$

Here left and limit ≠ right hand limit, so limit dose not exist.

Ex. Find $\lim\limits_{x\to2} f(x)$ if $f(x) = \frac{[x]}{x^2+1}$ Where [x] represents the greatest integer that is less than or equal to x.

Sol. R. H. L. $\lim\limits_{x\to2^+} f(x) = \lim\limits_{h\to0} f(2+h)$

$\lim\limits_{h\to0}\frac{[2+h]}{(2+h)^2+1}$

$\lim\limits_{h\to0}\frac{2}{(2+h)^2+1} = \frac{2}{3}$

L.H.L. $\lim\limits_{x\to2^-} f(x) = \lim\limits_{h\to0} f(2-h)$

$\lim\limits_{h\to0}\frac{[2-h]}{(2-h)^2+1}$

$\lim\limits_{h\to0}\frac{1}{(2-h)^2+1} = \frac{1}{3}$

LHL ≠ RHL, so $\lim\limits_{x\to2} f(x)$ does not exist

In general, limits for the greatest integer function do not exist when x → integer

Ex. Evaluate the value of $\lim\limits_{x\to2^-}[\frac{x}{2}]^3 + \frac{x^3}{2}$ where [.] denote the step function.

Sol.

$\lim\limits_{x\to2^-}\left([\frac{x}{2}]^3 + \frac{x^3}{2}\right)$

$\lim\limits_{h\to0}\left([\frac{2-h}{2}]^3 + \frac{(2-h)^3}{2}\right)$

$2 - h < 2$

$\frac{2-h}{2} < 1$

$[\frac{2-h}{2}] = 0$

$\lim\limits_{h\to0}\frac{(2-h)^3}{2} = 4$

Indeterminate Form

The function f(x) adopts the expression $f(x) = \frac{0}{0}$, at $x = a$ Then we assert that is indeterminate at, $x = a$, and some indeterminate forms include: