

LIMITS OF EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Limit Laws for exponential functions

$$\lim_{x \rightarrow \infty} ex = \infty;$$

$$\lim_{x \rightarrow -\infty} ex = 0;$$

$$\lim_{x \rightarrow \infty} e^{-x} = 0;$$

$$\lim_{x \rightarrow -\infty} e^{-x} = \infty;$$

These laws were created to prove that if an exponential exponent goes to infinity in the limit, the exponential function will also go to infinity. In the same way, if the exponent flows to minus infinity in the limit, the exponential will flow to zero.

Limit Laws for Logarithmic functions

The right-handed limit was operated for $\lim_{x \rightarrow 0^+} \ln x = -\infty$. Because we can't use negative x's in a logarithm function, the right-handed limit was used. As x from both the right and left sides of the point in issue should be assessed, whereas x's to the left of zero are negative, the standard limit cannot exist. We can see that if a log's argument goes to zero from the right (i.e., it is always positive), the record will go to negative infinity in the limit. In contrast, if the argument goes to infinity, the log will also go to infinity in the limit. Because we can't plug negative values into the logarithm, we can't look at a logarithm's limit as x approaches minus infinity.

Rules of Exponential and Logarithmic Functions.**Exponential Rules**

- $a^x a^y = a^{x+y}$
- $\frac{a^x}{a^y} = a^{x-y}$
- $(a^x)^y = a^{xy}$
- $a^x b^x = (ab)^x$
- $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$
- $a^0 = 1$
- $a^{-x} = \frac{1}{a^x}$

Logarithmic Rules

- $\log_b(xy) = \log_b x + \log_b y$
- $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$
- $\log_b x^m = m \log_b x$
- $\log_b p^2 = \log_b p + \log_b p = 2 \log_b p$
- $\log_a p = \frac{(\log_b p)}{(\log_b a)}$
- $\log_b 1 = 0$
- $\log_b b = 1$
- $\log_b b^x = x$

Derivatives of Exponential and Logarithmic Functions

The formulas for the derivatives of exponential and logarithmic functions are listed below.

With respect to x, ex derivative is stated as:

$$\frac{d}{dx}(ex) = ex$$

The log x derivative about x is written as follows:

$$\frac{d}{dx}(\log x) = \frac{1}{x}$$

We can obtain different formulas by applying exponential and logarithmic laws to these two formulas.

$$\frac{d}{dx}(e^{ax}) = ae^{ax} \text{ is the derivative of } e^{ax} \text{ about } x.$$

$$\text{The } n\text{th derivative of } e^{ax} \text{ for } x \text{ is } \frac{d^n}{dx^n}(e^{ax}) = a^n e^{ax}$$

Similarly, several formulas for the derivative of exponential functions can be found.

Ex. Evaluate the derivative $y = e^{x^4}$

Sol.: According to the given function, i.e, $y = e^{x^4}$

The derivative would be for x

$$\frac{d}{dx}(e^{x^4}) = e^{x^4} \frac{d}{dx}(x^4) = e^{x^4} 4 \times x^3 = 4x^3 e^{x^4}$$