EVALUATION OF TRIGONOMETRIC LIMITS

STANDARD LIMIT

(A)

1. $\ell\lim\limits_{x\to 0}\frac{\sin x}{x}=\lim\limits_{x\to 0}\frac{\tan x}{x}=1$

2. $\ell\lim\limits_{x\to 0}\frac{\tan^{-1}x}{x}=\lim\limits_{x\to 0}\frac{\sin^{-1}x}{x}=1$

3. $\ell\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}}=e;\lim\limits_{x\to 0}(1+ax)^{\frac{1}{x}}=e^{a}$

4. $\ell\lim\limits_{x\to\infty}(1+\frac{1}{x})^{x}=e;\lim\limits_{x\to\infty}(1+\frac{a}{x})^{x}=e^{a}$

5. $\ell\lim\limits_{x\to 0}\frac{e^{x}-1}{x}=1;\lim\limits_{x\to 0}\frac{a^{x}-1}{x}=\log_{e}a=\ell na, a>0$

6. $\ell\lim\limits_{x\to 0}\frac{\ln(1+x)}{x}=1$

7. $\ell\lim\limits_{x\to a}\frac{x^{n}-a^{n}}{x-a}=na^{n-1}$

(B) f f(x) → 0, when x → a, then

1. $\ell\lim\limits_{x\to a}\frac{\sin f(x)}{f(x)}=1$

2. $\ell\lim\limits_{x\to a}\cos f(x)=1$

3. $\ell\lim\limits_{x\to a}\frac{\tan f(x)}{f(x)}=1$

4. $\ell\lim\limits_{x\to a}\frac{e^{f(x)}-1}{f(x)}=1$

5. $\ell\lim\limits_{x\to a}\frac{b^{f(x)}-1}{f(x)}=\ell nb,(b>0)$

6. $\ell\lim\limits_{x\to a}\frac{\ln(1+f(x))}{f(x)}=1$

7. $\ell\lim\limits_{x\to a}(1+f(x))^{\frac{1}{f(x)}}=e$

(C) $\lim\limits_{x\to a}f(x)=A>0$ and $\lim\limits_{x\to a}\phi(x)=B$(a finite quantity), then $\ell_{x\to a}[f(x)]^{\phi(x)}=A^{B}$.

Ex. Evaluate: $\lim\limits_{x\to 0}\frac{\tan x-\sin x}{x^{3}}$

Sol. $\lim\limits_{x\to 0}\frac{\tan x-\sin x}{x^{3}}=\lim\limits_{x\to 0}\frac{\tan x(1-\text{co}\ \ x)}{x^{3}}=\lim\limits_{x\to 0}\frac{\tan x\cdot 2\sin^{2}\frac{x}{2}}{x^{3}}$

$$\lim_{x\to 0}\frac{1}{2}\cdot\frac{\tan x}{x}\cdot\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^{2}=\frac{1}{2}$$

Ex. Evaluate: $\lim\limits_{n\to\infty}\frac{\sin\frac{a}{n}}{\tan\frac{b}{n+1}}$

Sol. As $n\to\infty,\frac{1}{n}\to 0$ and $\frac{a}{n}$ also tends to zero

$\sin\frac{a}{n}$ should be written as $\frac{\sin\frac{a}{n}}{\frac{a}{n}}$ so that it looks like $\lim\limits_{\theta\to 0}\frac{\sin\theta}{\theta}$

The given limit $=\lim\limits_{n\to\infty}\left(\frac{\sin\frac{a}{n}}{\frac{a}{n}}\right)\left(\frac{\frac{b}{n+1}}{\tan\frac{b}{n+1}}\right)\cdot\frac{a(n+1)}{n\cdot b}$

$$\lim_{n\to\infty}\left(\frac{\sin\frac{a}{n}}{\frac{a}{n}}\right)\left(\frac{\frac{b}{n+1}}{\tan\frac{b}{n+1}}\right)\cdot\frac{a}{b}\left(1+\frac{1}{n}\right)=1\times 1\times\frac{a}{b}\times 1=\frac{a}{b}$$

Ex. Evaluate $\lim\limits_{x\to 1}\frac{\sqrt{x}+\sqrt{\sqrt{x}}+\sqrt{\sqrt{\sqrt{x}}}+\sqrt{\sqrt{\sqrt{\sqrt{x}}}}-4}{x-1}$

Sol. $\lim\limits_{x\to 1}\frac{\sqrt{x}+\sqrt{\sqrt{x}}+\sqrt{\sqrt{\sqrt{x}}}+\sqrt{\sqrt{\sqrt{\sqrt{x}}}}-4}{x-1}$

$$\lim_{x\to 1}\frac{x^{1/2}+x^{1/4}+x^{1/8}+x^{1/16}-4}{x-1}$$

$$\lim_{x\to 1}\left(\frac{x^{1/2}-1}{x-1}+\frac{x^{1/4}-1}{x-1}+\frac{x^{1/8}-1}{x-1}+\frac{x^{1/16}-1}{x-1}\right)$$

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}$$

$$\frac{8+4+2+1}{16}=\frac{15}{16}$$