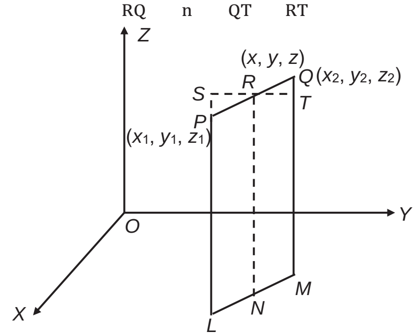
SECTION FORMULA

To determine the coordinates of point R, which divides the line segment PQ internally in the ratio of m : n, initiate the process by projecting perpendiculars from points P, R, and Q onto the xy-plane. Denote the points where these perpendiculars meet the xy-plane as L, N, and M, respectively.

It is noted that triangles ΔPRS and ΔRQT are similar. Consequently, this similarity implies that the corresponding sides of these triangles are in proportion.

$$\frac{PR}{RQ}=\frac{m}{n}=\frac{SP}{QT}=\frac{SR}{RT}$$



$$SP=(SL-PL)=(RN-PL)=(z-z_1)$$
$$QT=(QM-TM)=(QM-RN)=(z_2-z)$$
$$\frac{m}{n}=\frac{(z-z_1)}{(z_2-z)}$$
$$(mz_2-mz)=nz-nz_1$$
$$z(m+n)=mz_2+nz_1$$

Similarly $y=\frac{(my_2+ny_1)}{(m+n)}, x=\frac{(mx_2+nx_1)}{(m+n)}$

The coordinates of point R are

$$\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n},\frac{mz_2+nz_1}{m+n}\right)$$

Likewise, if point R divides the line segment PQ externally in the ratio of m : n, then the coordinates of R are determined as follows:

$$\left(\frac{mx_2-nx_1}{m-n},\frac{my_2-ny_1}{m-n},\frac{mz_2-nz_1}{m-n}\right)$$

Ex. Determine the distance between points $A(2,-1,3)$ and $B(-2,-1,3)$.

Sol. Distance $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

$AB=\sqrt{(2+2)^2+(-1+1)^2+(3-3)^2}=4$ units

Ex.2 Demonstrate the collinearity of the points $(2,-3,4)$, $(-1,2,1)$, and $\left(0,\frac{1}{3},2\right)$ determine the ratio at which the third point divides the line connecting the first two points.

Sol. Designate point A as (2, -3, 4) and point B as (-1, 2, 1).

Let point C divide the line connecting A and B in the ratio of λ : 1.

Therefore, the coordinates of point C are determined as follows:

$$\left(\frac{2-\lambda}{\lambda+1},\frac{-3+2\lambda}{\lambda+1},\frac{4+\lambda}{\lambda+1}\right)\equiv\left(0,\frac{1}{3},2\right)$$
$$\frac{2-\lambda}{\lambda+1}=0$$
$$\lambda=2$$
$$\frac{-3+2\lambda}{\lambda+1}=\frac{1}{3}$$
$$-9+6\lambda=\lambda+1$$
$$5\lambda=10$$
$$\lambda=2$$
$$\frac{4+\lambda}{\lambda+1}=2$$
$$4+\lambda=2\lambda+2$$
$$\lambda=2$$

Solving each equation for λ yields a value of 2.
Consequently, points A, C, and B are collinear.
Hence, point C divides the line segment between A and B internally in a 2 : 1 ratio.

Important formulas/ points

- Distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
- Section formula $x = \frac{mx_2+nx_1}{m+n}, y = \frac{my_2+ny_1}{m+n}, z = \frac{mz_2+nz_1}{m+n}$
- For internal division m: n is positive.
- For external division m: n is negative.