PROJECTION OF THE LINE SEGMENT JOINING TWO POINTS ON A GIVEN LINE

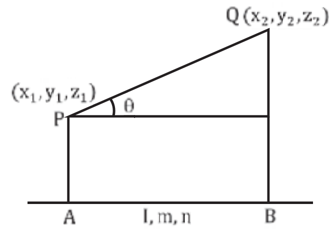
The projection of the line segment connecting two specified points (x_1, y_1, z_1) and (x_2, y_2, z_2) onto the line with direction cosines l, m, n is expressed as:

$$I(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)$$

This is evident from the vector.

Clearly, $\vec{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

and the line $\vec{AB} = l\hat{i} + m\hat{j} + n\hat{k}$



The projection of $\vec{PQ}$ on $\vec{AB}$

$$\frac{\vec{PQ}\cdot\vec{AB}}{|\vec{AB}|} = \frac{l(x_2-x_1)+m(y_2-y_1)+n(z_2-z_1)}{\sqrt{l^2+m^2+n^2}}$$

$$l(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)$$

Ex. Consider two given points A (-1, 2, 1) and B (4, 3, 5). Determine the projection of the line segment AB onto a line that forms angles of 120° and 135° with the y and z-axes, respectively, and forms an acute angle with the x-axis.

Sol. Consider an acute angle 'α' that the provided line forms with the x-axis. Then.

$$\cos^2\alpha + \cos^2 120^\circ + \cos^2 135^\circ = 1$$

$$\cos^2\alpha = 1 - \frac{1}{4} - \frac{1}{2} = \frac{4-2-1}{4} = \frac{1}{4}$$

$$\cos\alpha = \pm\frac{1}{2} \text{ but } \alpha \text{ is acute}$$

$$\cos\alpha = +\text{ve}$$

$$\cos\alpha = \frac{1}{2} = \cos 60^\circ$$

$$\alpha = 60^\circ$$

Therefore, the direction cosines of the given straight line are $\cos 60^\circ, \cos 120^\circ, \cos 135^\circ$

$$\frac{1}{2}, -\frac{1}{2}, -\frac{1}{\sqrt{2}}$$

Hence the projection of AB on the line

$$= \frac{1}{2}(4+1) - \frac{1}{2}(3-2) - \frac{1}{\sqrt{2}}(5-1)$$

$$= \frac{5}{2} - \frac{1}{2} - 2\sqrt{2}$$

$$= (2 - 2\sqrt{2}) \text{ units}$$