DIRECTION RATIOS:

Consider three numbers, a, b, and c, which are proportional to the direction cosines l, m, and n of a line. These three numbers, a, b, and c, are referred to as the direction ratios of the line.

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \pm\frac{\sqrt{l^2+m^2+n^2}}{\sqrt{a^2+b^2+c^2}} = \pm\frac{1}{\sqrt{a^2+b^2+c^2}}$$

$$I = \pm\frac{a}{\sqrt{a^2+b^2+c^2}} = \pm\frac{a}{\sqrt{\Sigma a^2}}$$

$$m = \pm\frac{b}{\sqrt{a^2+b^2+c^2}} = \pm\frac{b}{\sqrt{\Sigma a^2}}$$

$$n = \pm\frac{c}{\sqrt{a^2+b^2+c^2}} = \pm\frac{c}{\sqrt{\Sigma a^2}}$$

Only two scenarios are possible in this context

$$\left(\frac{a}{\sqrt{\Sigma a^2}}, \frac{b}{\sqrt{\Sigma b^2}}, \frac{c}{\sqrt{\Sigma c^2}}\right) \text{ or } \left(-\frac{a}{\sqrt{\Sigma a^2}}, -\frac{b}{\sqrt{\Sigma b^2}}, -\frac{c}{\sqrt{\Sigma c^2}}\right)$$

In accordance with the given condition.

The direction ratios of the line segment connecting points (x_1, y_1, z_1) and (x_2, y_2, z_2) are given by $x_2 - x_1, y_2 - y_1$ and $z_2 - z_1$ and that of vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ are a_1, a_2, a_3.

Ex. Determine the direction cosines of a line passing through the points P (1, 4, 6) and Q (5, 1, 11), oriented in such a way that it forms an acute angle with the positive direction of the y-axis.

Sol. The direction ratios of the line segment connecting points P (1, 4, 6) and Q (5, 1, 11) are

$$5 - 1, 1 - 4, 11 - 6$$

$$4, -3, 5$$

Therefore, the direction cosines are.

$$\pm\frac{4}{\sqrt{16+9+25}}, \pm\frac{-3}{\sqrt{16+9+25}}, \pm\frac{5}{\sqrt{16+9+25}}$$

$$\pm\frac{4}{5\sqrt{2}}, \pm\frac{-3}{5\sqrt{2}}, \pm\frac{5}{5\sqrt{2}}$$

As the line forms an acute angle with the y-axis, the direction cosines of the line segment can be expressed as follows.

$$-\frac{2\sqrt{2}}{5}, \frac{3\sqrt{2}}{10}, -\frac{\sqrt{2}}{2}$$