DIRECTION COSINES OF A LINE

Consider a line OP that forms angles α, β, and γ with the positive direction of the X-axis, Y-axis, and Z-axis, respectively. The cosines of these angles, denoted as cos α, cos β, and cos γ, are referred to as the direction cosines of the line OP and are typically denoted by l, m, and n, respectively.

$$\cos\alpha = I$$
$$\cos\beta = m$$
$$\cos\gamma = n$$

If (x, y, z) be the coordinates of P and OP = r then

$$x = r\cos\alpha, y = r\cos\beta, z = r\cos\gamma.$$

The direction cosines of OP are

$$\frac{x}{r}, \frac{y}{r}, \frac{z}{r} (\text{ or equivalently } \frac{x}{OP}, \frac{y}{OP}, \frac{z}{OP})$$

The distance of P(x, y, z) from origin = r

$$\sqrt{x^2 + y^2 + z^2} = r$$
$$x^2 + y^2 + z^2 = r^2$$
$$r^2\cos^2\alpha + r^2\cos^2\beta + r^2\cos^2\gamma = r^2$$
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$
$$I^2 + m^2 + n^2 = 1$$
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$
$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$$

Given two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ the distance between these points is.

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} = r$$
$$I = \cos\alpha = \frac{x_2 - x_1}{r}, m = \cos\beta = \frac{y_2 - y_1}{r}, n = \cos\gamma = \frac{z_2 - z_1}{r}$$

The cosines of the angles formed by a vector with the coordinate axes are known as the direction cosines of the vector.

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k} \text{ are } \frac{a_1}{\sqrt{a_1^2 + a_2^2 + a_3^2}}, \frac{a_2}{\sqrt{a_1^2 + a_2^2 + a_3^2}}, \frac{a_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}}$$

Thus the direction cosines of x-axis, y-axis and z-axis are (1,0,0)(0,1,0) (0,0,1) Respectively.

Ex. **1.** A line OP passing through the origin O makes angles of 60° and 30° with the OY and OZ axes, respectively. Determine the angle at which it is inclined to the OX axis.

2. Determine the direction cosines of a line that is equally inclined to all axes.

Sol. **1.** Consider the line OP making an angle α with the x-axis.

$$\cos^2\alpha + \cos^2 60^\circ + \cos^2 30^\circ = 1$$
$$\cos^2\alpha + \frac{1}{4} + \frac{3}{4} = 1$$
$$\cos\alpha = 0$$
$$\alpha = 90^\circ$$

2. Consider angles α, β, and γ formed by a line with the coordinate axes, then...

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$
$$\alpha = \beta = \gamma$$
$$3\cos^2\alpha = 1$$
$$\cos\alpha = \pm\frac{1}{\sqrt{3}}$$

Thus the required direction cosines are $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$