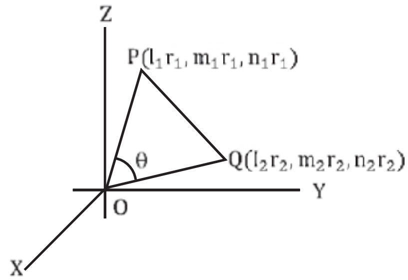
ANGEL BETWEEN TWO LINES

Let I_1, m_1, n_1 and I_2, m_2, n_2 be the direction of lines OP and OQ and $OP = r_1, OQ = r_2$ then the coordinate of P and Q are (l_1r_1, m_1r_1, n_1r_1) and (l_2r_2, m_2r_2, n_2r_2) respectively.



Let $\angle POQ = \theta$, then

$$\cos\theta = \frac{OP^2+OQ^2-PQ^2}{2.OP.OQ} = \frac{r_1^2+r_2^2-(l_1r_1-I_2r_2)^2-(m_1r_1-m_2r_2)^2-(n_1r_1-n_2r_2)^2}{2\cdot r_1r_2}$$

$$\cos\theta = I_1I_2 + m_1m_2 + n_1n_2$$

$$\theta = \cos^{-1}(I_1I_2 + m_1m_2 + n_1n_2)$$

$$\sin^2\theta = 1-\cos^2\theta = (l_1^2+m_1^2+n_1^2)(l_2^2+m_2^2+n_2^2) - (I_1I_2 + m_1m_2 + n_1n_2)^2$$

$$\sin^2\theta = (m_1n_2 - m_2n_1)^2 + (n_1I_2 - n_2I_1)^2 + (I_1m_2 - I_2m_1)^2$$

$$\sin\theta = \pm\sqrt{(m_1n_2 - m_2n_1)^2 + (n_1I_2 - n_2I_1)^2 + (I_1m_2 - I_2m_1)^2}$$

1. The two line with direction cosines I_1, m_1, n_1 and I_2, m_2, n_2 are perpendicular to each other if $\theta = \frac{\pi}{2}$

$$\cos\theta = l_1I_2 + m_1m_2 + n_1n_2 = 0$$

2. The two line with direction cosines I_1, m_1, n_1 and I_2, m_2, n_2 are parallel to each other if $\theta = 0$ or π

$$\sin\theta = 0$$

$$(m_1n_2 - m_2n_1)^2 + (n_1I_2 - n_2I_1)^2 + (I_1m_2 - I_2m_1)^2 = 0$$

$$\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$$

3. The angel between two line having direction ratios a_1, b_1, c_1 and a_2, b_2, c_2 is given by.$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{(a_1^2+b_1^2+c_1^2)(a_2^2+b_2^2+c_2^2)}}$

Ex. 1. Determine the angle between the lines with direction ratios 1, 2, 3 and -3, 2, 1.

2. Determine the angle formed between two body diagonals of a cube.

Sol. 1. Let θ be the angle in question, then.

$$\cos\theta = \frac{1\times-3+2\times2+3\times1}{\sqrt{1+4+9}\sqrt{1+4+9}} = \frac{4}{14} = \frac{2}{7}$$

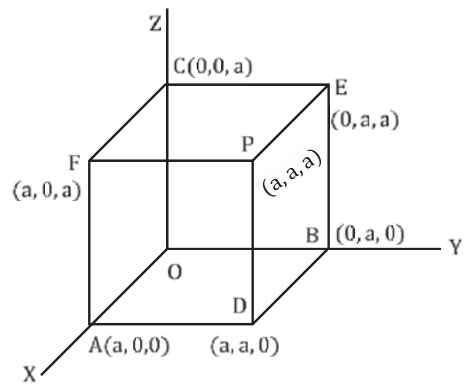
$$\theta = \cos^{-1}(\frac{2}{7})$$

2. From the adjoining figure, the direction ratios of the diagonal OP and CD of a given cube are given by $a-0, a-0, a-0$ and $a-0, a-0, 0-a$

And hence their respective direction cosines are.

$$\frac{a}{\sqrt{a^2+a^2+a^2}}, \frac{a}{\sqrt{a^2+a^2+a^2}}, \frac{a}{\sqrt{a^2+a^2+a^2}} \text{ i.e., } \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$$

$$\frac{a}{\sqrt{a^2+a^2+a^2}}, \frac{a}{\sqrt{a^2+a^2+a^2}}, \frac{-a}{\sqrt{a^2+a^2+a^2}} \text{ i.e. } \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}$$



Let θ be the angle between these diagonals, then

$$\cos\theta = \frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}\frac{-1}{\sqrt{3}} = \frac{1}{3} + \frac{1}{3} - \frac{1}{3} = \frac{1}{3}$$

$$\theta = \cos^{-1}\left(\frac{1}{3}\right)$$