RECTANGULAR HYPERBOLA

1. Asymptote:

A line that gradually approaches and becomes tangent at infinity is termed the asymptote of the curve. In the case of a hyperbola, the equation of the hyperbola and the combined equation of asymptotes vary only by a constant term, and the asymptotes pass through the center.

Let the hyperbola be $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Combined equation of asymptotes is

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=0 \Rightarrow y=\pm\frac{b}{a}x$$

2. Rectangular Hyperbola:

A hyperbola is termed a rectangular hyperbola if its asymptotes are perpendicular to each other.

$$\frac{b}{a}\times-\frac{b}{a}=-1 \Rightarrow b=a$$

$x^2-y^2=a^2$ is the equation of rectangular hyperbola whose asympotate are $y=\pm x$ eccentricity of a rectunglar hyperbola is always $\sqrt{2}$..

$$e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{1+1}(\because b=a)$$

$$e=\sqrt{2}$$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

All solutions of the form $x^2-y^2=a^2$are derived by substituting $b=a$ in $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

3. Rectangular hyperbola whose asymptotes are the co-ordinate axes :

The equations for the x-axis and y-axis are $y=0$ and $x=0$, respectively.

The combined equation of the asymptotes is $xy=0$.

The equation of a rectangular hyperbola with asymptotes along the coordinate axes is:

$$xy=c^2 \text{ or } xy=-c^2$$

$x^2-y^2=a^2$ Is a ractangular hyperbola whose asymptotes. $y=\pm x$

Let the axes be rotated about the origin through an angle of 45° in a clockwise direction.

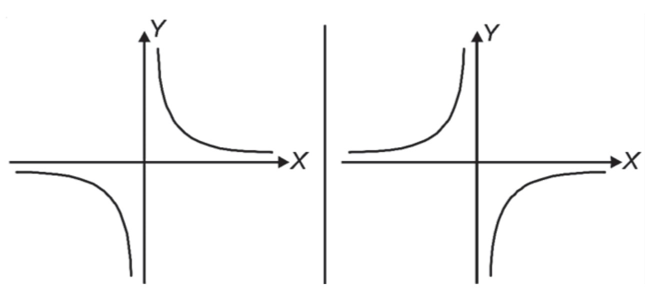
Putting $x=\frac{x+y}{\sqrt{2}}, y=\frac{-x+y}{\sqrt{2}}$, we get

$$\left(\frac{x+y}{\sqrt{2}}\right)^2-\left(\frac{-x+y}{\sqrt{2}}\right)^2=a^2$$

$$4xy=2a^2$$

$$xy=\frac{a^2}{2}$$

$$xy=c^2 \text{ whose } c^2=\frac{a^2}{2}$$



4. Paramatric Form

The paramatric equation of the hyperbola $xy=c^2$ are $x=ct$ and $y=\frac{c}{t}$.

A general point of rectangular hyperbola $xy=c^2$ is $(ct,\frac{c}{t})$

5. Equation Of Tangnet

(a) The equation of tangent at (x_1, y_1) is $xy_1 + x_1y = 2c^2$ or $\frac{x}{x_1} + \frac{y}{y_1} = 2$

(b) The equation of tangent at $(ct, \frac{c}{t})$ is $\frac{x}{t} + ty = 2c$

6. Equation Of The Normal

(a) The equation of the normal at (x_1, y_1) is $xx_1 - yy_1 = x_1^2 - y_1^2$

(b) The equation of the normal at $(ct, \frac{c}{t})$ is $t^3x - ty - ct^4 + c = 0$

Ex. Demonstrate that the equation $3x^2 - 3v^2 - 18x + 12v + 2 = 0$ represents a rectangular hyperbola. Determine its center, foci, and eccentricity.

Sol. The given equation is

$$3x^2 - 3y^2 - 18x + 12y + 2 = 0$$
$$3(x^2 - 6x) - 3(y^2 - 4y) + 2 = 0$$
$$3(x^2 - 6x + 9) - 3(y^2 - 4y + 4) = 13$$
$$\frac{(x-3)^2}{\frac{13}{3}} - \frac{(y-2)^2}{\frac{13}{3}} = 1$$

This represents a hyperbola where the length of its transverse axis is equal to the length of the conjugate axis. $= \sqrt{\frac{13}{3}}$

Therefore, the provided hyperbola is a rectangular hyperbola.

The center of the hyperbola is located at (3, 2), and its eccentricity is. $= \sqrt{2}$

The foci are $(3 \pm \sqrt{\frac{26}{3}}, 2)$.

Ex. Demonstrate that two concentric hyperbolas, with axes meeting at an angle of 45°, intersect orthogonally.

Sol. Consider the equation for the rectangular hyperbola as follows:

$$x^2 - y^2 = a^2 \quad \text{... (1)}$$

Considering that the asymptotes of one hyperbola are the axes of the other, and vice versa, the equation of the other hyperbola can be expressed as:

$$xy = c^2 \quad \text{... (2)}$$

Suppose the intersection point of the two hyperbolas (1) and (2) is denoted as P(a secθ, a tanθ).

The equations for the tangent at point P to hyperbolas (1) and (2) are given, respectively, by:

$$x - y\sin\alpha = a\cos\alpha$$
$$y + x\sin\alpha = \frac{2c^2}{a}\cos\alpha$$

In this observation, we note that the product of the slopes of the tangents to the hyperbolas is...

$$= \frac{1}{\sin\alpha} \times -\sin\alpha = -1$$

The tangents are perpendicular, implying that the two hyperbolas intersect orthogonally.