LOCUS PROBLEMS

To determine the locus of a moving point P, follow the procedure:

1. Represent the coordinates of the moving point as P (α, β).

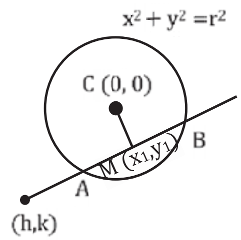
2. Based on the given geometrical conditions, establish the relationship in terms of α and β. This relationship should involve only α, β, and known quantities.

3. Simplify the obtained result in terms of α and β and substitute α with x and β with y. The resulting equation is the equation of the desired locus.

Ex. Secants are drawn to the circle $x^2 + y^2 = r^2$ through a fixed point (h, k). Demonstrate that the locus of the midpoints of the portions of the secants intercepted by the circle is $x^2 + y^2 = hx + ky$.

Sol. 1st Method

Let $M(x, y_1)$ denote the midpoint of the portion intercepted by $x^2 + y^2 =$ on an arbitrary secant passing through the fixed point (h, k).



Then CM perpendicular AB

$$\frac{y_1}{x_1} \cdot \frac{y_1 - k}{x_1 - h} = -1$$

$$x_1^2 + y_1^2 - hx_1 - ky_1 = 0$$

Hence the locus of $M(x_1, y_1)$ has the equation

$$x^2 + y^2 - hx - ky = 0$$

2nd Method

The equation of the chord AB in terms of the mid-point

$$M(x_1, y_1) \text{ is } S_1 = T$$

$$x_1^2 + y_1^2 - r^2 = xx_1 + yy_1 - r^2$$

$$x_1^2 + y_1^2 = xx_1 + yy_1$$

This chord passes through (h, k)

$$x_1^2 + y_1^2 = hx_1 + ky_1$$

Hence the locus of $M(x_1, y_1)$ is $x^2 + y^2 = hx + ky$