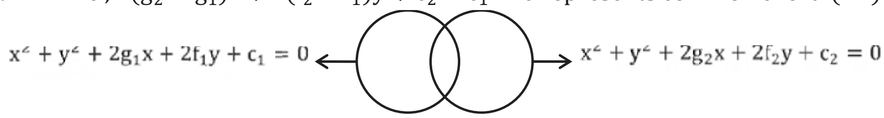
FAMILY OF CIRCLES

1. Family of circles passing through an intersection point $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ is

$$x^2 + y^2 + 2g_1x + 2f_1y + c_1 + \lambda(x^2 + y^2 + 2g_2x + 2f_2y + c_2) = 0, \lambda \in R - \{-1\}.$$

If $\lambda = -1$ i.e., $2(g_2 - g_1)x + 2(f_2 - f_1)y + c_2 - c_1 = 0$ represents common chord (AB)

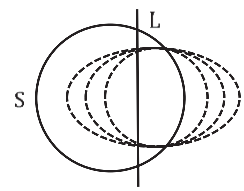


2. The family of circles passing through the point of intersection of the circle.

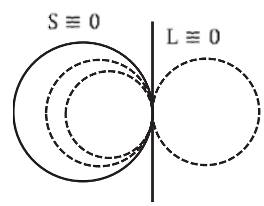
$$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0 \text{ and the line}$$

$L \equiv ax + by + k = 0$ meeting the circle in two distinct points,is

$$S + \lambda L = 0, \lambda \in R.$$



If, on the other hand, the line L touches the circle $S = 0$ at point P, then the equation $S + \lambda L = 0$ represents a family of circles, each of which touches the line $L = 0$ at point P.



As a corollary, the equation for a family of circles touching a constant line is: $y - y_1 = m(x - x_1)$ at the fixed point(x_1, y_1) is

$(x - x_1)^2 + (y - y_1)^2 + \lambda[y - y_1 - m(x - x_1)] = 0$, where λ is a parameter.

4. Family of circles passing through a point $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + \lambda\left(y - y_1 - \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)\right) = 0$$

5. The equation for a family of circles passing through two specified points (x_1, y_1) and (x_2, y_2)can be expressed as the form $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + \lambda\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$, λ being parameter.

6. Equation of the circle with a tangent $T = 0$ at (x_1, y_1) is given by $(x - x_1)^2 + (y - y_1)^2 + \lambda T = 0$.

Ex. Determine the equation of the circle passing through (1,1), (3,1) and (2,2).

Sol. Family of circle passing through (1,1) and (3,1) is

$$(x - 1)(x - 3) + (y - 1)(y - 1) + \lambda(y - 1) = 0$$
$$x^2 - 4x + 3 + y^2 - 2y + 1 + \lambda(y - 1) = 0$$

Equation (i) passes through (2,2),

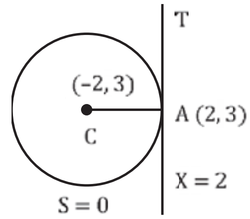
$$4 - 8 + 3 + 4 - 4 + 1 + \lambda(2 - 1) = 0$$

$\lambda = 0$ hence equation of required circle is

$$x^2 - 4x + 3 + y^2 - 2y + 1 = 0$$
$$x^2 + y^2 - 4x - 2y + 4 = 0$$

Ex. Find the equation of the circle which passes through the point (1,1) and which touches the circle $x^2 + y^2 + 4x - 6y - 3 = 0$ at the point (2,3) on it.

Sol. Let $S \equiv x^2 + y^2 + 4x - 6y - 3 = 0$ be the given circle.



Its centre is $C(-2,3)$.

Note that the line joining C to $A(2,3)$ is parallel to x-axis

The equation of tangent at A to $S \equiv 0$ is $x = 2$

Now the equation of a family of circles touching the line $x = 2$ at $(2,3)$ is

$$(x^2 + y^2 + 4x - 6y - 3) + \lambda(x - 2) = 0$$

As (1,1) lies on it

$$(1 + 1 + 4 - 6 - 3) + \lambda(1 - 2) = 0$$
$$-3 - \lambda = 0$$
$$\lambda = -3$$

Thus the equation of circle is

$$x^2 + y^2 + 4x - 6y - 3 - 3(x - 2) = 0$$
$$x^2 + y^2 + x - 6y + 3 = 0$$