Chapter 10

Circle, Conic Sections

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INTRODUCTION OF CONIC SECTIONS -

When a plane intersects a cone at a point other than the vertex, several situations arise:

(a) If $\beta = 90°$, the section forms a circle.

(b) When $\alpha < \beta < 90°$, the section takes the shape of an ellipse.

(c) In the case of $\alpha = \beta$, the section becomes a parabola.

(d) When $0 \leq \beta < \alpha$, the section turns into a hyperbola. Here, β represents the angle formed by the plane with the vertical axis of the cone.

Circle:

A circle is a set of points in a plane that are all at an equal distance from a fixed point. The equation of a circle with a radius of r and a center at (h, k) is expressed as

$$(x - h)^2 + (y - k)^2 = r^2$$

Parabola:

A parabola consists of points in a plane that are equidistant from a fixed line and a fixed point. For a parabola with $a > 0$, a focus at (a, 0), and a directory of $x = -a$, the equation is represented as $y^2 = 4ax$. In the parabola equation $y^2 = 4ax$, the length of the latus rectum is given by 4a.

Ellipse:

An ellipse is defined as the set of points in a plane for which the sum of distances from two fixed points is constant. An ellipse with its foci on the x-axis can be represented as:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

In ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

The length of the latus rectum is determined by $\frac{2b^2}{a}$

Hyperbola:

The hyperbola is defined as the set of points in a plane for which the difference of distances from two fixed points remains constant. A hyperbola with its foci on the x-axis can be represented as

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

In a Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

The length of the latus rectum is determined by $\frac{2b^2}{a}$

Ex. Determine the equation for a circle that has its center located at (0, 0) and a radius of r.

Sol. Given, Centre = $(h, k) = (0,0)$

Radius = r

Therefore, the equation of the circle is $x^2 + y^2 = r^2$

Ex. Determine the equation for a circle that is centered at (-3, 2) and has a radius of 4.

Sol. Given, Centre = (h, k) = (-3, 2)

Radius = r = 4

Therefore, the equation of the required circle is

$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x + 3)^2 + (y - 2)^2 = 4^2$$
$$(x + 3)^2 + (y - 2)^2 = 16$$

Ex.. Determine the center and radius of the circle represented by the equation

$$x^2 + y^2 + 8x + 10y - 8 = 0$$

Sol. The provided equation is

$$x^2 + y^2 + 8x + 10y - 8 = 0$$
$$(x^2 + 8x) + (y^2 + 10y) = 8$$

Rearranging terms to complete the squares within the parentheses, we get:

$$(x^2 + 8x + 16) + (y^2 + 10y + 25) = 8 + 16 + 25$$
$$(x + 4)^2 + (y + 5)^2 = 49$$
$$[x - (-4)]_w^2 + [y - (-5)]^2 = 7^2$$

Comparing with the standard form, h = -4, k = -5 and r = 7

Therefore, the given circle has center at (– 4, –5) and radius 7.

Ex. Determine the equation of a circle that passes through the points (2, -2) and (3, 4) and whose center lies on the line x + y = 2.

Sol. Let's assume the equation of the circle be $(x – h)^2 + (y – k)^2 = r^2$.

Given that the circle passes through the points (2, –2) and (3, 4). We can write two equations.

$$(2 - h)^2 + (-2 - k)^2 = r^2 \quad \text{... (1)}$$
$$(3 - h)^2 + (4 - k)^2 = r^2 \quad \text{... (2)}$$

Also, it is given that the center of the circle lies on the line x + y = 2, which can be expressed as:.

$$h + k = 2 \quad \text{... (3)}$$

By solving the equations (1), (2), and (3), we can find the values of h, k, and r^2, which are as follows:

$$h = 0.7, k = 1.3 \text{ and } r^2 = 12.58$$

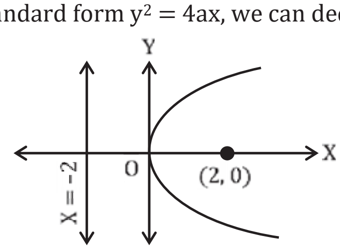
Hence, the equation of the required circle is

$$(x - 0.7)^2 + (y - 1.3)^2 = 12.58$$

Ex. Determine the focus's coordinates, axis, the equation for the directrix, and the latus rectum of the parabola given by the equation $y^2 = 8x$.

Sol. The presence of y^2 in the provided equation indicates that the axis of symmetry lies along the x-axis.

Since the coefficient of x is positive, it signifies that the parabola opens to the right. By comparing the given equation to the standard form $y^2 = 4ax$, we can deduce that the value of 'a' is 2.



Consequently, the parabola's focus is located at (2, 0), and the equation representing its directrix is x = -2 (as depicted in the figure).

Length of the latus rectum = 4a = 4 × 2 = 8

Ex.. Determine the equation of the parabola that has its vertex at (0, 0) and its focus at (0, 2).

Sol. With the information provided, where the vertex is at (0,0) and the focus is at (0,2), it's evident that the focus is situated on the y-axis.

Hence, the axis of the parabola coincides with the y-axis.

Consequently, the equation for this parabola takes the form of $x^2 = 4ay$.

$$x^2 = 4(2)y$$
$$x^2 = 8y$$

This equation represents the desired parabola.

Ex. Determine the coordinates of the foci, the vertices, the lengths of the major and minor axes, and the eccentricity of the ellipse described by the equation

$$9x^2 + 4y^2 = 36$$

Sol. Given the equation $9x^2 + 4y^2 = 36$, it can be expressed in standard form as follows:

In this case, 9 is greater than 4, indicating that the major axis aligns with the y-axis.

By comparing with $$(\frac{x^2}{b^2}) + (\frac{y^2}{a^2}) = 1,$$

$$a^2 = 9, b^2 = 4$$
$$a = 3, \quad b = 2$$
$$c = \sqrt{(a^2 - b^2)} = \sqrt{(9-4)} = \sqrt{5}$$
$$e = \frac{c}{a} = \frac{\sqrt{5}}{3}$$

As a result, the foci are located at $(0, \sqrt{5})$ and $(0, -\sqrt{5})$, the vertices are positioned at (0, 3) and (0, -3), the length of the major axis is 6 units, the length of the minor axis is 4 units, and the eccentricity of the ellipse is $\frac{\sqrt{5}}{3}$.

Ex.. Determine the equation of the hyperbola with foci at $(0, \pm 12)$ and a latus rectum of length 36.

Sol. Given the foci as $(0, \pm 12)$, we can establish that c = 12.

Length of the latus rectum $= \frac{2b^2}{a} = 36$

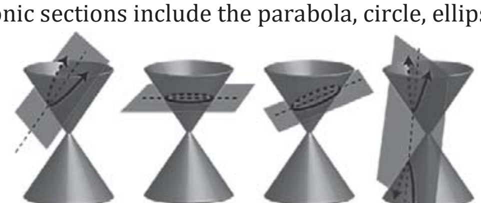
$$b^2 = 18a$$
$$c^2 = a^2 + b^2$$
$$144 = a^2 + 18a$$
$$a^2 + 18a - 144 = 0$$
$$a = -24, 6$$

The value of a cannot be negative. Therefore, a = 6 and so, $b^2 = 108$.

Therefore, the equation of the hyperbola can be expressed as $(\frac{y^2}{36}) - (\frac{x^2}{108}) = 1$ or $3y^2 - x^2 = 108$

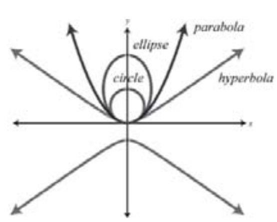
Section of a Right Circular Cone by Different Planes

A conic section is a geometric curve formed by the intersection of a right circular cone and a flat plane. The primary conic sections include the parabola, circle, ellipse, and hyperbola.



Parabola Circle Hyperbola Ellipse

Our objective is to create graphical representations of these conic sections on a rectangular coordinate plane.



The Distance and Midpoint Formulas

Let's commence by revisiting the **distance formula**. When you have two points represented as (x_1, y_1) and (x_2, y_2) in a rectangular coordinate system, you can determine the distance'd' between these points using the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Moreover, the point that precisely divides the line segment created by these two points is referred to as the **midpoint** and can be calculated using the following formula.

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

The midpoint represents a coordinated pair formed by calculating the average of the x-values and the average of the y-values.

Ex. For the given points $(-2, -5)$ and $(-4, -3)$compute both the distance and midpoint between them.

Sol. In this instance, we'll apply the formulas using the given points:

$$(x_1, y_1)(x_2, y_2) = (-2, -5)(-4, -3)$$

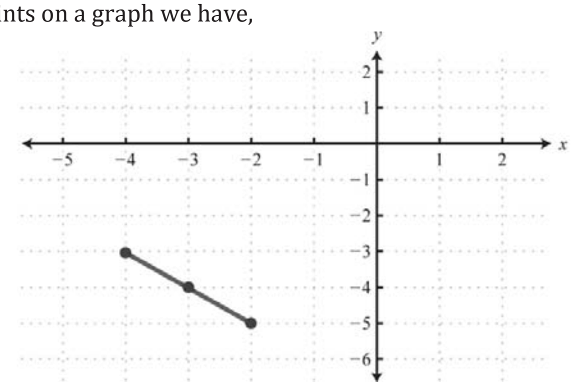
It's advisable to present the formula in its general form prior to substituting specific values for the variables. This approach enhances clarity and minimizes the likelihood of errors.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{[-4 - (-2)]^2 + [-3(-5)]^2}$$
$$d = \sqrt{(-4 + 2)^2 + (-3 + 5)^2}$$
$$d = \sqrt{(-2)^2 + (2)^2}$$
$$d = \sqrt{4 + 4}$$
$$d = \sqrt{8} = 2\sqrt{2}$$

Next determine the midpoint.

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-2+(-4)}{2}, \frac{-5+(-3)}{2}\right)$$
$$= \left(\frac{-6}{2}, \frac{-8}{2}\right)$$
$$= (-3, -4)$$

Plotting these points on a graph we have,



Answer: Distance= $2\sqrt{2}$ units; midpoint = $(-3,-4)$

Ex. The circle's diameter is determined by the two points (-1, 2) and (1, -2). Calculate the circle's radius and use it to compute the circle's area.

Sol. To find the diameter, employ the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{[1 - (-1)]^2 + [-2 - 2]^2}$$
$$d = \sqrt{(2)^2 + (-4)^2}$$
$$d = \sqrt{4 + 16}$$
$$d = \sqrt{20} = 2\sqrt{5}$$

Remember that the radius of a circle is half of its diameter.

Therefore, if d=$2\sqrt{5}$ units,

Then $r = \frac{d}{2} = \frac{2\sqrt{5}}{2} = \sqrt{5}$

The area of a circle is determined by the formula $A=\pi r^2$ and

We have $A = \pi(\sqrt{5})^2$

$A = \pi \cdot 5 = 5\pi$

Area is measured in square units.

Answer: Radius=$\sqrt{5}$ units; and area= 5π square units

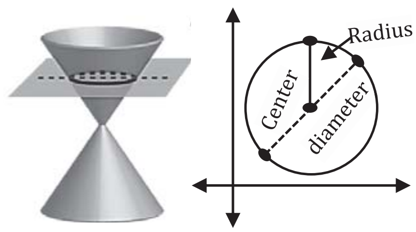
CIRCLE

A circle is defined as the path traced by a point moving in a plane, such that its distance from a specific fixed point (located in the same plane) remains constant. This fixed point is known as the center of the circle, and the constant distance is referred to as the radius of the circle.

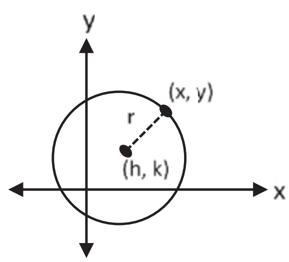
Equation of A Circle in Various Forms

The Circle in Standard Form

A **circle** represents a collection of points within a plane, each at a consistent distance from a central point known as the center, and this fixed distance is called the **radius**. The **diameter** is a line segment's length that passes through the center and has its endpoints on the circle. Additionally, a circle can be created through the intersection of a cone and a plane that is perpendicular to the cone's axis.



In a rectangular coordinate plane, where the center of a circle with radius r is (h, k), we have



Determine the distance between the points (h, k) and (x, y) by applying the distance formula.

$$\sqrt{(x-h)^2+(y-k)^2}=r$$

When both sides are squared, it results in the equation for a **circle in standard form.**

$$(x-h)^2+(y-k)^2=r^2$$

This form clearly reveals the center and radius of the circle.

For example, given the equation $(x-2)^2+(y+5)^2=16$

$$(x-h)^2+(x-k)^2=r^2$$

$$(x-2)^2+[y-(-5)]^2=4^2$$

In this case, the center is $(2,-5)(2,-5)$ and $r=4$

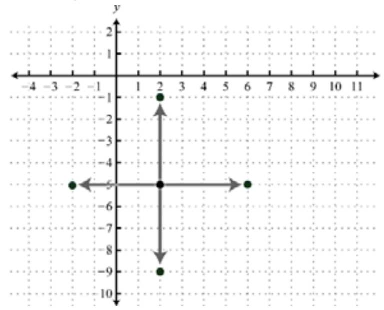
More examples follow:

Equation	Center	Radius
$(x-3)^2+(y-4)^2=25$	(3,4)	$r = 5$
$(x-1)^2+(y+2)^2=7$	$(1,-2)$	$r = \sqrt{7}$
$(x+4)^2+(y-3)^2=1$	$(-4,3)$	$r=1$
$x^2+(y+6)^2=8$	$(0,-6)$	$r=2\sqrt{2}$

The characteristics of a circle's graph are entirely defined by its center and radius.

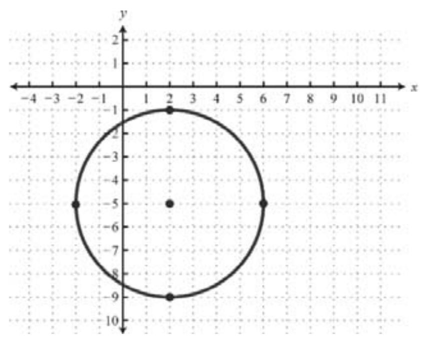
Ex. Graph: : $(x-2)^2+(y+5)^2=16$

Sol. Expressed in this format, it becomes evident that the center is located at (2, -5), and the radius is equal to 4 units. To construct the circle, start from the center and mark points 4 units in all directions: up, down, left, and right.



Afterward, connect these four points to outline the circle.

Answer:



As with any graph, we are interested in finding the x- and y-intercepts.

Ex. Find the intercepts $(x-2)^2+(y+5)^2=16$

Sol. To find the y-intercepts set $x=0$:

$$(x-2)^2+(y+5)^2=16$$
$$(0-2)^2+(y+5)^2=16$$
$$4+(y+5)^2=16$$

To solve this equation, we can apply the method of square root extraction.

$$(y+5)^2=12$$
$$y+5=\pm\sqrt{12}$$
$$y+5=\pm 2\sqrt{3}$$
$$y=-5\pm 2\sqrt{3}$$

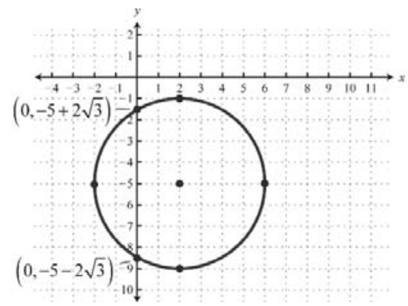
Hence, the points where the graph intersects the y-axis are $(0,-5-2\sqrt{3})$and

$$\left(0,-5+2\sqrt{3}\right)$$

To determine the x-intercepts, set $y=0$

$$(x-2)^2+(y+5)^2=16$$
$$(x-2)^2+(0+5)^2=16$$
$$(x-2)^2+25=16$$
$$(x-2)^2=-9$$
$$x-2=\pm\sqrt{-9}$$
$$x=2\pm 3i$$

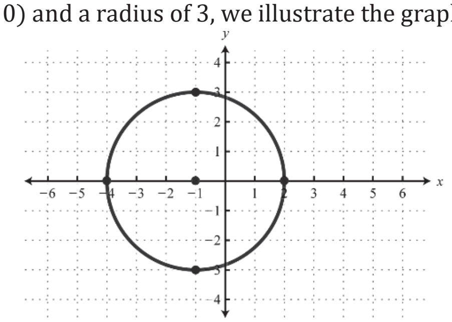
Since the solutions are complex, we can infer that there are no real x-intercepts. This observation aligns with the graph's characteristics.



Answer: x-intercepts = none; y-intercepts = $(0, -5-2\sqrt{3})$ and $(0, -5+2\sqrt{3})$
Given the center and radius of a circle, we can find its equation.

Ex. Plot the circle centered at (-1, 0) with a radius of 3 units. Write its equation in standard form and identify its intercepts.

Sol. With the center at (-1, 0) and a radius of 3, we illustrate the graph as follows:



Replace the variables h, k, and r with their values to obtain the equation in standard form.
Given (h, k) = (-1, 0) and r = 3, this leads to:

$$(x-h)^2 + (y-k)^2 = r^2$$
$$[x-(-1)]^2 + (y-0)^2 = 3^2$$
$$(x+1)^2 + y^2 = 9$$

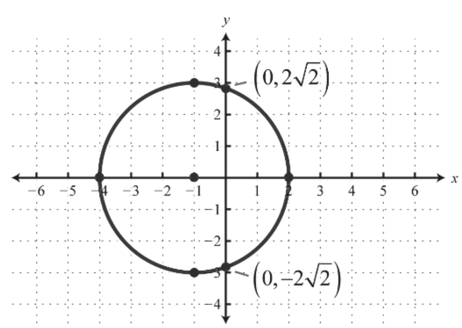
Utilize the equation of the circle $(x+1)^2 + y^2 = 9$, to calculate the y-intercepts.

$$(x+1)^2 + y^2 = 9$$

Set x = 0 to and solve for y.

$$(0+1)^2 + y^2 = 9$$
$$1 + y^2 = 9$$
$$y^2 = 8$$
$$y = \pm\sqrt{8}$$
$$y = \pm 2\sqrt{2}$$

Hence, the y-intercepts are located at $(0, -2\sqrt{2})$ and $(0, 2\sqrt{2})$. To determine the x-intercepts, set y = 0 and solve for x.



Answer: Equation = $(x+1)^2 + y^2 = 9$; y-intercepts $= (0, -2\sqrt{2})$ and $(0, 2\sqrt{2})$ x − intercepts $= (-4, 0)$ and $(2, 0)$

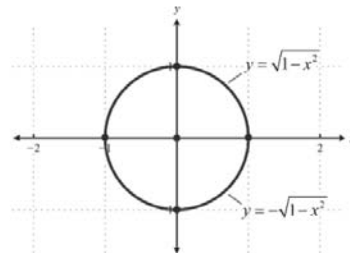
A notable circle is the **unit circle**, defined by the equation

$$x^2 + y^2 = 1$$
$$(x - 0)^2 + (y - 0)^2 = 1^2$$

In this form, it becomes evident that the center is at (0, 0), and the radius is 1 unit. Additionally, solving for y yields two functions:

$$x^2 + y^2 = 1$$
$$y^2 = 1 - x^2$$
$$y = \pm\sqrt{1 - x^2}$$

The function $y = \sqrt{1 - x^2}$ represents the upper half of the circle, while the function $y = -\sqrt{1 - x^2}$ Represents the lower half of the unit circle.



The Circle in General Form

We've observed that the position and size of a circle can be deduced from its center and radius, typically found in its standard form equation. Nevertheless, equations for circles are not always presented in standard form. The **general form of a circle's** equation is as follows:

$$x^2 + y^2 + cx + dy + e = 0$$

In this form, c, d, and e represent real numbers. The procedure for graphing a circle when provided with its equation in general form is as follows.

Ex. Graph: $x^2 + y^2 + 6x - 8y + 13 = 0$

Sol. To start, convert the equation into standard form.

Step 1: Combine the terms with the same variables and transfer the constant to the right side. In this instance, subtract 13 from both sides and group the terms related to x and those related to y in the following manner.

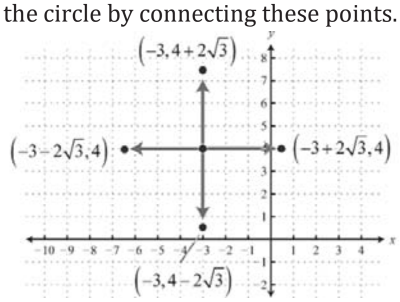
$$x^2 + y^2 + 6x - 8y + 13 = 0$$
$$(x^2 + 6x + \_\_) + (y^2 - 8y + \_\_) = -13$$

Step 2: Perform the process of completing the square for each group. The concept here is to add the appropriate value, which makes each group a perfect square, $(\frac{b}{2})^2$, to both sides for both groupings, Then, factor the expressions. Use $(\frac{6}{2})^2 = 3^2 = 9$ for the terms involving x and use $(\frac{-8}{2})^2 = (-4)^2 = 16$ for the terms involving y.

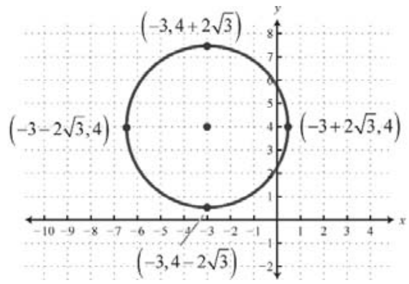
$$(x^2 + 6x + 9) + (y^2 - 8y + 16) = -13 + 9 + 16$$
$$(x + 3)^2 + (y - 4)^2 = 12$$

Step 3: Find the center and radius by examining the equation in standard form. In this instance, the center is located at (-3, 4), and the radius $r = \sqrt{12} = 2\sqrt{3}$.

Step 4: Starting from the center, extend the radius both vertically and horizontally, and then proceed to draw the circle by connecting these points.



Answer:



Ex. Find the center and radius: $4x^2 + 4y^2 - 8x + 12y - 3 = 0$.

Sol. To get the general form, begin by dividing both sides by 4.

$$\frac{4x^2+4y^2-8x+12y-3}{4} = \frac{0}{4}$$

$$x^2 + y^2 - 2x + 3y - \frac{3}{4} = 0$$

Now that we've established the general form for a circle, where both quadratic terms have a leading coefficient of 1, we can follow the steps to transform it into standard form. Start by adding $\frac{3}{4}$ to both sides and regrouping like variables.

$$(x^2 - 2x + \ \_) + (y^2 + 3y\ \_) = \frac{3}{4}$$

Then, proceed to perform the process of completing the square for both sets of terms. Use $(\frac{-2}{2})^2 = (-1)^2 = 1$ for the first group and $(\frac{3}{2})^2 = \frac{9}{4}$ for the second group.

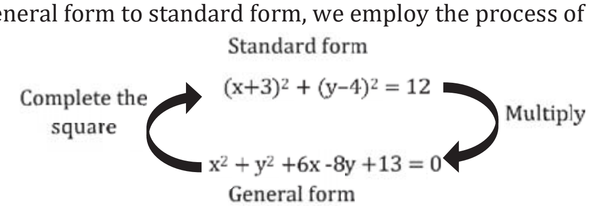
$$(x^2 - 2x + 1) + (y^2 + 3y + \frac{9}{4}) = \frac{3}{4} + 1 + \frac{9}{4}$$

$$(x-1)^2 + (y + \frac{3}{2})^2 = \frac{16}{4}$$

$$(x-1)^2 + (y + \frac{3}{2})^2 = 4$$

Answer: Center = $= \left(1, -\frac{3}{2}\right)$ radius $= r = 2$

In summary, when transitioning from standard form to general form, we multiply, and when moving from general form to standard form, we employ the process of completing the square.



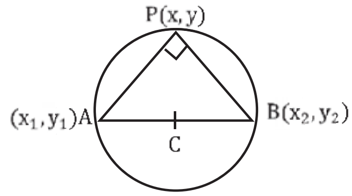
> The visual representation of a circle is solely determined by its center and radius.
> The standard form for a circle's equation is, $(x-h)^2 + (y-k)^2 = r^2$, where (h, k) represents the center, and r is the radius.
> When graphing a circle, place points at a distance of r units in the upward, downward, leftward, and rightward directions from the center. Connect these points to form the circle.
> If presented with a circle's equation in general form, $x^2 + y^2 + cx + dy + e = 0$, group terms with similar variables and complete the square for each grouping. This process transforms it into standard form, making it easy to determine the circle's center and radius.
> An equation can be identified as representing a circle when it is a quadratic equation in both x and y, with the coefficients of the squared terms being equal.

The Circle in Diameter form

If $A(x_1, y_1)$ and $B(x_2, y_2)$ represent the endpoints of the diameter of a circle, and $P(x, y)$ is a point on the circle other than A and B, then geometrically, it is established that $\angle APB$ is a right angle $(90°)$.

$$(\text{Slope of PA}) \times (\text{Slope of PB}) = -1$$
$$\left(\frac{y-y_1}{x-x_1}\right)\left(\frac{y-y_2}{x-x_2}\right) = -1$$
$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

This will be the circle of least radius passing through (x_1, y_1) and (x_2, y_2).



The Circle in Parametric Form

1. The Parametric equation of the circle $x^2 + y^2 = r^2$ are $x = r\cos\theta, y = r\sin\theta; \theta \in [0,2\pi)$ and $(r\cos\theta, r\sin\theta)$ are called the parametric co-ordinate.
2. The Parametric equation of the circle $(x-h)^2 + (y-k)^2 = r^2$ is $x = h + r\cos\theta, y = k + r\sin\theta$ the parametric equation of the circle where θ is parameter
3. The Parametric equation of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ are $x = -g + \sqrt{g^2 + f^2 - c}\cos\theta, y = -f + \sqrt{g^2+f^2-c}\sin\theta$ Where θ is parameter.

Ex. Determine the equation of the circle with a center at (1, -2) and a radius of 4.

Sol. The equation representing the circle is $(x-1)^2 + (y+2)^2 = 4^2$.
$$(x-1)^2 + (y+2)^2 = 16$$
$$x^2 + y^2 - 2x + 4y - 11 = 0$$

Ex. Determine the equation of the circle, given that the coordinates of the endpoints of its diameter are (–1, 2) and (4, –3).

Sol. We are aware that the equation of the circle formed with the line segment connecting (x_1, y_1) and (x_2, y_2) as a diameter can be expressed as $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$.
Here, $x_1 = -1, x_2 = 4, y_1 = 2$ and $y_2 = -3$.
So, the equation of the required circle is.
$$(x+1)(x-4) + (y-2)(y+3) = 0 \Rightarrow x^2 + y^2 - 3x + y - 10 = 0$$

Ex. If $y = 2x + m$ represents a diameter of the circle given by $x^2 + y^2 + 3x + 4y - 1 = 0$, determine the value of m.

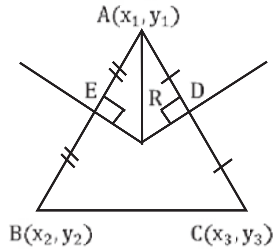
Sol. Centre of circle $= \left(-\frac{3}{2}, -2\right)$. This lies on diameter $y = 2x + m$
$$-2 = \left(-\frac{3}{2}\right) \times 2 + m$$
$$m = 1$$

Equation of a Circle Passing Through Non-Collinear Points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$

In this case, the equation of the circle can be determined using the following method.

1. Consider the circle is $x^2 + y^2 + 2gx + 2fy + c = 0$
$$x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c = 0 \quad \text{... (1)}$$
$$x_2^2 + y_2^2 + 2gx_2 + 2fy_2 + c = 0 \quad \text{... (2)}$$
$$x_3^2 + y_3^2 + 2gx_3 + 2fy_3 + c = 0 \quad \text{... (3)}$$
By (1), (2), (3) we can find the value of g.f. and c. and hence the equation of cirrcle.

2. To find the equation of circumcircle, find the point of intersection of perpendicular bisector of any two sides, let AB and AC. Let the point of intersection is O, which will be the centre of required circle.
$$OA = OB = OC \text{ will be the radius.}$$



3. Apart from these we sometimes also use the equation of a circle through three non-collinear points $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ given in the determinate from.

$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ x_1^2+y_1^2 & x_1 & y_1 & 1 \\ x_2^2+y_2^2 & x_2 & y_2 & 1 \\ x_3^2+y_3^2 & x_3 & y_3 & 1 \end{vmatrix} = 0$$

Ex. Prove that the point A (4, 0), B (6, 1), C (4, 3) and D (3, 2) are con cyclic.

Sol. Equation of family of circle passing through A, B is.

$$(x-4)(x-6)+(y-0)(y-1)+\lambda\begin{vmatrix} x & y & 1 \\ 4 & 0 & 1 \\ 6 & 1 & 1 \end{vmatrix} = 0$$

$$x^2+y^2-10x-y+24+\lambda(-x+2y+4)=0$$

Point C (4, 3) lies on the circle.

$$\lambda = -1$$

Equation of circle is $x^2+y^2-9x-3y+20=0$

Point D (3, 2) satisfies this circle.

Thus A, B, C, D are con cyclic.

Ex. The circle is characterized by two diameters with equations $x+y=6$ and $x+2y=4$, and the radius is 10. Determine the equation of the circle.

Sol. Note that the intersection point of the two diameters of a circle is the center of the circle. Thus, the center of the circle is the point where $x+y=6$ and $x+2y=4$ intersect, resulting in (8, -2). The equation of the required circle with centre (8, -2) and radius 10 is.

$$(x-8)^2+(y+2)^2=10^2$$

$$x^2+v^2-16x+4v-32=0$$