Chapter 1

Set Theory

- Introduction
- Sets and their Representations
- The Empty Set
- Finite and Infinite Sets
- Analysis of Two Sets
  - ➢ Equal Sets
  - ➢ Subsets
  - ➢ Intervals as subsets of R
  - ➢ Power Set
  - ➢ Universal Set
  - ➢ Venn Diagrams
- Operations on Sets
  - ➢ Union of sets
  - ➢ Intersection of sets
  - ➢ Difference of sets
  - ➢ Complement of a Set
  - ➢ Algebra on Sets
- Practical Problems on Union and Intersection of Two Sets
- Wavy Curve Method

INTRODUCTION

In mathematics, a well-defined collection of objects is called a SET .The study of sets is the basis of all the mathematical avenues. This chapter includes types of sets, relations of sets, operations on sets and even practical applications of sets. It also includes solving basic inequalities using wavy curve method.

SETS AND THEIR REPRESENTATIONS

The foundation of modern mathematics is built upon the concept of sets, a term first introduced by the German mathematician George cantor. He initially defined a set as "any collection into a whole of definite and distinct objects of our intuition or thought." this definition underwent discussions and modifications, eventually evolving into the widely accepted form: "a set is any collection of distinct and distinguishable objects of our intuition or thought"

In this chapter, the focus is on fostering a graphical approach among students from the outset, emphasizing the use of Venn diagrams to simplify problem-solving. The concept of relation proves invaluable in understanding functions; clarity in the concept of relation is essential to discern whether something functions as a function.

In the context of relations, 'a r b' signifies that 'a' is r-related to 'b,' where r represents any given relation between 'a' and 'b.' understanding the concept of function is crucial, as it lays the groundwork for delving into calculus, a paramount branch of mathematics. The term 'function,' derived from a Latin word meaning 'operation,' is also synonymous with mapping.

SET: Set is a well-defined collection of distinct objects"

The elements within a set share a common property, where an object possessing this property is considered a part of the set, and an object lacking this property is not included in the set.

For instance, the set of books authored by Shakespeare constitutes a set, whereas the set of interesting books written by Shakespeare is not a set. This is because a book deemed interesting by one person may not be similarly appreciated by another.

Examples of sets include the set of known planets in the solar system, the set of days in a week, the set of all whole numbers, and the set of consonants in the English alphabet.

Ex. Identify the sets from the following collections of objects:

1. The collection of all students of Aakash Institute.
2. The collection of most talented Artists of India.

3. The collection of bright students at IIT Kanpur.
4. The collection of all prime-ministers of India.
5. The collection of all lucky numbers.
6. The collection of Indian states.
7. The collection of all tasty dishes.
8. The collection of all secular nations.

Sol. Sets such as 1, 4, 6, and 8 are well-defined because there is no ambiguity regarding their members. However, 2 and 3, 6 and 7 do not accurately represent sets, as there is no clear criterion for being the most talented, bright, lucky, or tasty. These terms can be interpreted differently by various individuals.

Set-Notations

A set is typically represented by capital letters such as A, B, C, etc., while its individual members, elements, or objects are indicated by lowercase letters like a, b, c, etc. The Greek symbol is utilized to signify the phrase 'belongs to,' and this symbol is referred to as the membership relation.

For Instance:

$x \in A$ implies 'x belongs to A' or 'x is an element of A' or 'x is a member of A' or 'x is an object of A.'
A implies 'x does not belong to A.'
Examples include a set of the English alphabet and a set not containing the number 6 from the English alphabet.

Representation Of A Set

A set can be expressed using two methods

1. Roster form or tabular form 2. Set builder form or rule method.

Roster Or Tabular Form

In this representation, the elements of the set are enumerated, separated by commas and enclosed within braces or curly brackets {}. The order of listing is inconsequential, and no element is repeated.

For instance, the set A of all single-digit natural numbers can be expressed as:

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ or $A = \{1, 3, 5, 2, 6, 4, 9, 8, 7\}$ (order is immaterial)

Set-Builder Form:

In this approach, a variable (denoted as x) is selected to represent each element of the set that satisfies a specific property. Inside the braces, x is accompanied by a symbol (such as ":" or ";" or a vertical line "|" or an oblique line "/"), followed by the property or properties possessed by each element of the set.

For instance, the set A, comprising all even integers less than 10, can be denoted as:

A = {x: x is an even integer less than 10}
= {x | x is an even integer less than 10}
= {x; x is an even integer less than 10}
= {x / x is an even integer less than 10}

The symbol following x is interpreted as 'such that.' The roster form of set A is expressed as $A = \{0, 2, 4, 6, 8\}$

> '0' is a member of the set of even integers. Set builder form is also referred to as the rule method, property method, or symbolic method.

Ex. Express each of the following in an alternative form of set notation.

(a) A = {x / x N and x 6} (b) $B = \left\{\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}\right\}$

(c) C = {1, – 1, i, – i} (d) D = {2, 4, 8, 16, 32}

(e) E = {x: $x^2 - 5x + 6 = 0$} (f) F = {x | x is a letter of word IITJEE}

(g) G = {$n^3 - n^2$: n N and 2n 4} (h) H = {1, 8, 27, 64... 10)

Sol. (a) A = {1, 2, 3, 4, 5, 6}

(b) Each element is of the form $\frac{2n-1}{2n}$ hence

$B = \{x: x = \frac{2n-1}{2n}, nN, n4\}$

(c) 'C' is a set of fourth roots of unity

Hence C = {x: $x^4 = 1$}

(d) 2, 4, 8, 16, 32 are clearly of the form 2^n, where n is a natural number less than 6. So

$D = \{x: x = 2^n; nN, n < 6\}$

(e) The roots of given equation must form the solution, hence E = {2, 3}

(f) No element has to be repeated, hence F = {I, T, J, E}

(g) G = {4, 18, 48}

(h) All the listed numbers are cube of natural numbers. So

$H = \{x: x = n^3, nN, n10\}$

Standard Notations For Sets Of Numbers

	Set of all	Symbol	i.e.
1.	Natural number	N	N = {1, 2, 3.......}
2.	Integers	Z or I Z or I	= {.......–3, –2, –1, 0, 1, 2, 3 ...}
3.	(a)	Positive integers Z^+	Z^+ = {1, 2, 3 ...}
	(b)	Negative integers Z^-	Z^- = {.....–3, –2, –1}
4.	Integers excluding 0	I_0	I_0 = {±1, ±2, ±3........}
5.	Even integers	E	E = {0, ±2, ± 4........}
6.	Odd integers	O	O = {±1,±3,±5........}
7.	Rational numbers	Q	Q = {x : x =$\frac{p}{q}$, p and q are integers, q≠0}
8.	Non-zero rational numbers	Q_0	Q_0= {x : x ∈Q, x≠0}
9.	Positive rational numbers	Q^+	Q^+ = {x : x ∈Q, x > 0}
10.	Real numbers	R	Here all rational and irrational numbers are included
11.	Non-zero real numbers	R_0	R_0 = {x : x ∈ R, x≠ 0}
12.	Positive real number	R^+	R^+ = {x : x ∈ R, x > 0}
13.	Complex numbers	C	C = {a + ib; a, b∈ R and i =$\sqrt{-1}$ }
14.	Non-zero complex number	C_0	C_0 = {x : x ∈ C, x≠ 0}
15.	Natural numbers less than or	N_k	N_k = {1, 2, 3, 4 ...k} equal to K, where K is positive integer
16.	Whole numbers	W	W = {0, 1, 2, 3 ...}

R is a subset of C(R ⊂ C). Numbers that cannot be expressed in the form of p/q are Irrational. These include non-repeating and non-terminating decimals, which are termed Irrational. For example, $\sqrt{2}, \sqrt[5]{3}, \pi, e, \log_2 10$