FINITE AND INFINITE SETS

Finite Set:

A set characterized by having a limited number of elements is referred to as a finite set. It can be understood that these sets possess a definite or countable number of elements.

Example Of A Finite Set:

The set P = {4, 8, 12, 16, 20} is considered a finite set because it contains a limited number of elements.

Infinite Set:

This is precisely the opposite of a finite set. It signifies that an infinite set will encompass an unbounded number of elements/components.

Example Of An Infinite Set:

The collection of all prime numbers, the collection of all even numbers, and the collection of all odd numbers exemplify infinite sets.

Venn Diagram Of Finite And Infinite Sets

In mathematics, finite sets and infinite sets are entirely distinct from each other, differing in their definitions. To illustrate these concepts, we can employ Venn diagrams, which consist of overlapping closed circles. Each circle within the diagram represents a set, depicting the relationships between various sets.

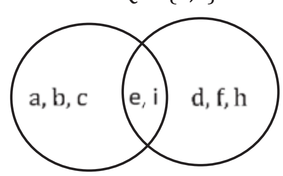
Consider the above Venn diagram. It consists of:

Set P = (a, b, c, e, i}

Set Q = {e, i, d, f, h}

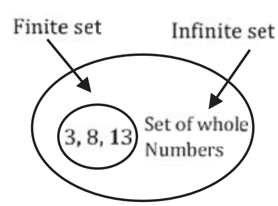
$P \cup Q = \{a, b, c, e, i, d, f, h\}$

$P \cap Q = \{e, i\}$



Given that sets P and Q are finite, with cardinalities n (P) = 5 and n (Q) = 5, it follows that the union

(P ∪ Q) and intersection (P ∩ Q) of these sets are also finite. Now, let's examine the following Venn diagram:



The diagram illustrates two sets, with the inner set containing the elements {3, 8, and 13}, representing a finite set. In contrast, the outer set, which encompasses all whole numbers, is an infinite set due to its limitless number of elements.

Properties Of Finite And Infinite Sets

Having established an understanding of finite and infinite sets through the Venn diagram, let's now delve into the associated properties.

- For a finite set, the following properties are applicable:
- The subsets of a finite set are invariably finite.

- Both the union and the intersection of the provided finite set yield a finite set.
- The power set of a finite set is also well-defined.
- In the case of finite sets, the Cartesian product is likewise finite.
- The cardinal number of a finite set is a clearly determined numerical value.

If a provided set is infinite, the following properties are applicable:
- The union of infinite sets always yields an infinite set.
- The power set of the given infinite sets is also infinite.
- The resulting superset of an infinite set is likewise infinite.
- Whether a subset of an infinite set is infinite or not depends on the number of elements in that particular subset.

Difference Between Finite And Infinite Sets

Finite and infinite sets form the fundamental classification of sets, indicating that a set of elements can be either finite or infinite. Now, let's explore the distinctions and comparisons between them:

FINITE SETS	INFINITE SETS
Elements: The number of elements is Countable for such sets.	**Elements:** The number of elements is uncountable for these types of sets.
Subset: A subset for these sets contains countable elements i.e. the subset is finite.	**Subset:** For these sets, a subset may or may not be countable.
Union of sets: The components in the union of such sets are countable/ finite.	**Union of sets:** The union result for such sets is also infinite.
Power set: The power set holds a finite number of elements.	**Power set:** The power set has an infinite number of elements.
Cardinality: The cardinality for these sets is equal to the number of elements in the set. That is, the cardinality of a set with n elements is equal to n.	**Cardinality:** The cardinality for these sets is equal to infinity (∞). This is due to the fact That the number of elements is Uncountable.
Example: Number of days in a week / months, months in a year, seasons in the year, set of prime numbers less than 50,etc.	**Example:** A set of all whole numbers, set of all-natural numbers, set of all real numbers, etc.

Ex. Determine whether an empty set is finite.

Sol. An empty set, denoted by {} or Ø, contains no elements. Since the empty set has zero elements, and zero is a definite quantity, we can conclude that an empty set is indeed a finite set.

Ex. Regarding the sets X = {3, 6, 9, 12} and Y = {6, 12, 18, 24}, provide observations on the nature of their union.

Sol. Given: X = {3, 6, 9, 12) and Y = {6, 12, 18, 24}

The union of X and Y is:

$$X \cup Y = \{3, 6, 9, 12, 18, 24\}$$

It's evident that both sets X and Y have a specific number of elements, and the union of X and Y follows suit.

Ex. A set containing elements as {..., -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5...} is finite or infinite, justify you're answer

Sol. Designating the set as Q, expressed as Q= {...,-5,-4,-3, -2, -1, 0, 1, 2, 3, 4, 5...}.

It can be affirmed that the set lacks both a starting and ending point. Consequently, the number of elements in set Q is uncountable, affirming that set Q is infinite.

Ex. Provide observations or remarks regarding the nature of the given sets below:

A= {p: p N, 2<p<13}

B= {q: q ∈ W, q<21}

C= {x: X∈ R and x+6>15}

Sol. Given that: A= {p: p N, 2<p<13}

B= {q: q ∈ W, q<21}

C= {x: x ∈ R and x+6>15}

We will examine each of these sets individually:

A= {p: p N, 2<p<13}

Set A comprises natural numbers ranging from 2 to 13, all of which are countable. Therefore, set A is finite.

B= {q: q∈W, q<21}

Set B contains whole numbers that are less than 21. The elements are countable, indicating that set B is finite.

C= {x: x ∈ R and x+6>15}

The set comprises real numbers greater than 9, as demonstrated by the inequality
$x + 6 > 15$ (simplified to $x > 9$). Notably, this set has no endpoint, indicating that set C is infinite.

Ex. If the cardinality of set A, denoted as n(A), is 10, determine the cardinality of the power set of A.

Sol. Concept:

1. Cardinality of a set A is defined as the no. of elements present in the set A, denoted by n (A).
2. Power set: Let A be a non-empty set. Then, P (A) = {B | B C A} is called as the power set of A.
3. For any non-empty set, A with n (A) = x.
   The total number of subsets of set A is given by: 2^x.
   In other words. $n (P (A)) = 2^x$.

Calculation:

Given: n (A) = 10, here x = 10.

$$n (P(A)) = 2^{10}$$

Hence, the cardinality of power set of A is 2^{10}