

**VALUES OF TRIGONOMETRIC RATIOS OF TYPICAL ANGLES**

In academic literature, the angles  $0^\circ$ ,  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ , and  $90^\circ$  are commonly referred to as standard angles. The trigonometric ratios for these standard angles can be found in the table below.

	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\operatorname{cosec} \theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec \theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\cot \theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

**Value for  $0^\circ$  and  $90^\circ$** 

Let's examine an angle of  $90^\circ$  and another angle close to  $0^\circ$ , as shown in the following image.



Let the smallest angle be  $A$  then

$\sin A = \text{perpendicular/hypotenuse}$ ,

$\cos A = \text{base/hypotenuse}$

Since angle  $A$  is very small, the corresponding perpendicular is also very small, resulting in a value of  $\sin A$  that is close to zero.

Moreover, the length of the triangle's base will be approximately equal to the length of the hypotenuse. Therefore, the value of  $\cos A$  will be approximately close to one.

After all this, we can say that.

$\sin A = 0$  and  $\cos A = 1$

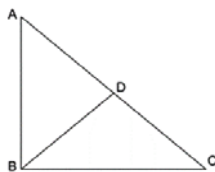
Now we know that

$$\tan A = \frac{\sin A}{\cos A} = \frac{0}{1} = 0$$

From this, we have shown that  $\sin 0^\circ = 0$ ,  $\cos 0^\circ = 1$ , and  $\tan 0^\circ = 0$ . Consequently, we can also deduce that  $\sin 90^\circ = 1$ ,  $\cos 90^\circ = 0$ , and  $\tan 90^\circ$  is undefined.

**Value for  $30^\circ$  and  $60^\circ$** 

Let's examine the triangle below.



Given that  $AC = 2BA$  and from the image above where  $AD = DC = BD$  according to plane geometry, we establish that angle  $C = 60^\circ$  and angle  $A = 30^\circ$ .

Now, as we understand

$$\sin C = \frac{BA}{AC} = \frac{BA}{2BA} = \frac{1}{2} = \sin 30^\circ$$

$$\cos C = \frac{BA}{AC} = \frac{BA}{2BA} = \frac{1}{2} = \cos 60^\circ,$$

$$\text{Now } \cos A = \frac{BC}{AC} = \sqrt{1 - \left(\frac{BA}{AC}\right)^2} = \sqrt{1 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2} = \cos 30^\circ$$

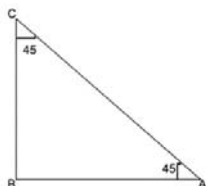
$$\cos C = \frac{BC}{AC} = \sqrt{1 - \left(\frac{BA}{AC}\right)^2} = \sqrt{1 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2} = \sin 60$$

Here we can summarize the trigonometric ratios for  $30^\circ$  and  $60^\circ$  as follows:

	$30^\circ$	$60^\circ$
Sin	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
Cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$

#### Value for $45^\circ$

Let's examine the triangle depicted below.



This is a right triangle where the other two angles are  $45^\circ$  each. Such a triangle is of the isosceles type.

This means  $AB = BC = \frac{AC}{\sqrt{2}}$ ,

$$\sin 45 = \frac{BC}{AC} = \frac{1}{\sqrt{2}}$$

$$\cos 45 = \frac{AB}{AC} = \frac{1}{\sqrt{2}}$$

Hence proved

	$45^\circ$
Sin	$\frac{1}{\sqrt{2}}$
Cos	$\frac{1}{\sqrt{2}}$

We've demonstrated how to derive the values of standard angles. To gain a deeper understanding of the topic, let's explore some examples or problems that can be solved using these trigonometric ratios for standard angles.

**Ex.** What is the value of  $\cos 45 \sin 30 + \sin 45 \cos 30$  ?

**Sol.** Using the table of trigonometric ratios for standard angles, we can derive the following values:

$$\cos 45 = \sin 45 = \frac{1}{\sqrt{2}}$$

$$\sin 30 = \frac{1}{2}$$

$$\cos 30 = \frac{\sqrt{3}}{2}$$

Let's put these values in the expression.

$$\cos 45 \sin 30 + \sin 45 \cos 30 = \frac{1}{\sqrt{2}} \times \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} = \frac{1 + \sqrt{3}}{2}$$

Here the above value is our answer.

**Ex.** If,  $\sin 60 + \cos 30 + \cos 60 + \sin 30 = \sin \theta$  then, what is the value of  $\theta$ ?

**Sol.** Using the table of trigonometric ratios for standard angles, we can find the values and substitute them into the equation as follows:

$$\sin 60 + \cos 30 + \cos 60 + \sin 30 = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{1}{2} = \sin \theta$$

$$\frac{\sqrt{3} + 1}{1} = \sin \theta$$

Now, we can say that.  $\theta = 60^\circ$