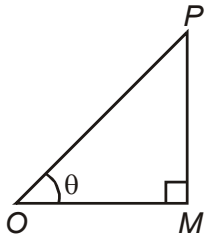
TRIGONOMETRIC RATIOS/FUNCTIONS OF ACUTE ANGLES

Trigonometric ratios of standard angles

Trigonometric Functions



1. $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{MP}{OP}$
2. $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{OM}{OP}$
3. $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{MP}{OM}$
4. $\cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{OM}{MP}$
5. $\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{OP}{OM}$
6. $\text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{OP}{MP}$

Trigonometric identities

Fundamental Identities

1. $\sin^2\theta + \cos^2\theta = 1$

   $\sin^2\theta = 1 - \cos^2\theta$

   $\cos^2\theta = 1 - \sin^2\theta$
2. $1 + \tan^2\theta = \sec^2\theta$

   $\sec^2\theta - \tan^2\theta = 1$
3. $1 + \cot^2\theta = \text{cosec}^2\theta$

   $\text{cosec}^2\theta - \cot^2\theta = 1$

Also note the range within which different trigonometric function lie

1. $-1 \le \sin\theta \le 1; |\sin\theta| \le 1$
2. $-1 \le \cos\theta \le 1; |\cos\theta| \le 1$
3. $0 \le \sin^2\theta \le 1; 0 \le \cos^2\theta \le 1$
4. $\text{cosec}\,\theta \le -1$ or $\text{cosec}\,\theta \ge 1$
5. $\sec\theta \le -1$ or $\sec\theta \ge 1$
6. $0 < \cos A < \frac{\sin A}{A} < \frac{1}{\cos A}; 0 < A < \frac{\pi}{2}$
7. If $\theta \ll$ then $\sin\theta \simeq \theta$

Every trigonometric ratio can be represented in relation to all other trigonometric ratios,

For example,

$$\sin\theta = \frac{\pm 1}{\sqrt{1+\cot^2\theta}}; \cos\theta - \frac{\pm\cot\theta}{\sqrt{1+\cot^2\theta}}$$

$$\tan\theta = \frac{1}{\cot\theta}; \sec\theta = \frac{\pm\sqrt{1+\cot^2\theta}}{\cot\theta}$$

$$\text{cosec}\,\theta = \pm\sqrt{1+\cot^2\theta}$$

Addition and Subtraction Formulae (Compound Angle)

1. $\sin(A+B) = \sin A\cos B + \cos A\sin B$
2. $\cos(A+B) = \cos A\cos B - \sin A\sin B$
3. $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$
4. $\sin(A-B) = \sin A\cos B - \cos A\sin B$
5. $\cos(A-B) = \cos A\cos B + \sin A\sin B$
6. $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$
7. $\cot(A+B) = \frac{\cot A\cot B - 1}{\cot A + \cot B}$

8. $\cot(A-B)=\frac{\cot A\cot B+1}{\cot B-\cot A}$
9. $\sin(A+B)\sin(A-B)=\sin^2 A-\sin^2 B$
10. $\cos(A+B)\cos(A-B)=\cos^2 A-\sin^2 B$
11. $\sin(A+B+C)=\sin A\cos B\cos C+\cos A\sin B\cos C+\cos A\cos B\sin C-\sin A\sin B\sin C$

    Or

    $\sin(A+B+C)=\cos A\cos B\cos C(\tan A+\tan B+\tan C-\tan A\tan B\tan C)$
12. $\cos(A+B+C)=\cos A\cos B\cos C-\sin A\sin B\cos C-\sin A\cos B\sin C-\cos A\sin B\sin C$

    Or

    $\cos(A+B+C)=\cos A\cos B\cos C(1-\tan A\tan B-\tan B\tan C-\tan C\tan A)$
13. $\tan(A+B+C)=\frac{\tan A+\tan B+\tan C-\tan A\tan B\tan C}{1-\tan A\tan B-\tan B\tan C-\tan C\tan A}$
14. $\tan(\frac{\pi}{4}+A)=\frac{1+\tan A}{1-\tan A}$
15. $\tan(\frac{\pi}{4}-A)=\frac{1-\tan A}{1+\tan A}$

Two special series:

1. $\sin(a)+\sin(a+b)+\sin(a+2b)+\cdots\ldots\sin(a+(n-1)b)$

   $=\frac{\sin[\alpha+(n-1)(\frac{\beta}{2})]\cdot\sin(\frac{n\beta}{2})}{\sin(\frac{\beta}{2})}$.
2. $\cos a+\cos(a+b)+\cos(a+2b)+\cdots\ldots.+\cos(a+(n-1)b)$

   $=\frac{\cos[\alpha+(n-1)(\frac{\beta}{2})]\cdot\sin(\frac{n\beta}{2})}{\sin(\frac{\beta}{2})}$