TRIGONOMERTIC RATIOS OF MULTIPLE AND SUB-MULTIPLE ANGLES

T-Ratios of multiple angles: (An angle of the form nq, n $\in$ I)

1. $\sin 2A = 2\sin A\cos A = \frac{2\tan A}{1+\tan^2 A}$
2. $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \frac{1-\tan^2 A}{1+\tan^2 A}$

   Thus, $1 + \cos 2A = 2\cos^2 A$

   $1 - \cos 2A = 2\sin^2 A$
3. $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$

   $\text{Cot}2\theta = \frac{(\text{Cot}^2\theta - 1)}{2\text{Cot}\theta}$
4. (i) $\sin 3A = 3\sin A - 4\sin^3 A$

   (ii) $\cos 3A = 4\cos^3 A - 3\cos A$

   (iii) $\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}, \cot 3A = \frac{\cot^3 A - 3\cot A}{3\cot^2 A - 1}$
5. $\cos A\cos 2A\cos^2 A$

   $\cos 2^{n-1}A = \frac{\sin 2^n A}{2^n \sin A}$

T-Ratios of submultiple angle (An angle of the from $\frac{\theta}{n}$, n $\in$ I)

1. $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = \frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$
2. $\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} = 2\cos^2\frac{\theta}{2} - 1 = 1 - 2\sin^2\frac{\theta}{2} = \frac{1-\frac{\tan^2\theta}{2}}{1+\frac{\tan^2\theta}{2}}$
3. $\tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$
4. $\cot\theta = \frac{\cot^2\frac{\theta}{2}-1}{2\cot\frac{\theta}{2}}$
5. $\cos^2\frac{\theta}{2} = \frac{1+\cos\theta}{2}$
6. $\sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2}$
7. $\tan^2\frac{\theta}{2} = \frac{1-\cos\theta}{1+\cos\theta}$
8. $\cot^2\frac{\theta}{2} = \frac{1+\cos\theta}{1-\cos\theta}$
9. $\frac{1-\cos\theta}{\sin\theta} = \tan\frac{\theta}{2}$
10. $\frac{1+\cos\theta}{\sin\theta} = \cot\frac{\theta}{2}$

T-Ratio of some special angles

(i) $\sin 15^\circ = \cos 75^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$

(ii) $\cos 15^\circ = \sin 75^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$

(iii) $\tan 15^\circ = \cot 75^\circ = 2 - \sqrt{3}$

(iv) $\cot 15^\circ = \tan 75^\circ = 2 + \sqrt{3}$

(v) $\sin 18^\circ = \cos 72^\circ = \frac{\sqrt{5}-1}{4}$

(vi) $\cos 18^\circ = \sin 72^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4}$

(vii) $\sin 36^\circ = \cos 54^\circ = \frac{\sqrt{10-2\sqrt{5}}}{4}$

(viii) $\cos 36^\circ = \sin 54^\circ = \frac{\sqrt{5}+1}{4}$

(ix) $\tan 22\frac{1}{2}^\circ = \sqrt{2} - 1$

(x) $\cot 22\frac{1}{2}^\circ = \sqrt{2} + 1$

Ex. Prove that $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = 4$

Sol. $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = \frac{\cos 10^\circ - \sqrt{3}\sin 10^\circ}{\sin 10^\circ \cos 10^\circ}$

$= \frac{4(\frac{1}{2}\cos 10^\circ - \frac{\sqrt{3}}{2}\sin 10^\circ)}{\sin 20^\circ} = \frac{4(\sin 30^\circ \cos 10^\circ - \cos 30^\circ \sin 10^\circ)}{\sin 20^\circ} = 4$

Remember

(i) $\left|\sin\frac{A}{2} + \cos\frac{A}{2}\right| = \sqrt{1 + \sin A}$

Or $\sin\frac{A}{2} + \cos\frac{A}{2} = \pm\sqrt{1 + \sin A}$

i.e. $\begin{cases} +, \text{ if } 2n\pi - \frac{\pi}{4} \le \frac{A}{2} \le 2n\pi + \frac{3\pi}{4} \\ -, \text{ otherwise} \end{cases}$

(ii) $\left|\sin\frac{A}{2} - \cos\frac{A}{2}\right| = \sqrt{1 - \sin A}$

Or $\left(\sin\frac{A}{2} - \cos\frac{A}{2}\right) = \pm\sqrt{1 - \sin A}$

i.e. $\begin{cases} +, \text{ if } 2n\pi - \frac{\pi}{4} \le \frac{A}{2} \le 2n\pi + \frac{5\pi}{4} \\ -, \text{ otherwise} \end{cases}$