TRANSFORMATION FORMULA

Formula to transform the sum or differences into product

We know that,

$\sin(A + B) = \sin A\cos B + \cos A\sin B$ ...(i)

$\sin(A - B) = \sin A\cos B - \cos A\sin B$ ...(ii)

$\cos(A + B) = \cos A\cos B - \sin A\sin B$ ...(iii)

$\cos(A - B) = \cos A\cos B + \sin A\sin B$ ...(iv)

Adding (i) and (ii), we get

$2\sin A\cos B = \sin(A + B) + \sin(A - B)$

Subtracting (ii) from (i), we get

$2\cos A\sin B = \sin(A + B) - \sin(A - B)$

Adding (iii) and (iv), we get

$2\cos A\cos B = \cos(A + B) + \cos(A - B)$

Subtracting (iv) from (iii), we get

$-2\sin A\sin B = \cos(A + B) - \cos(A - B)$

or $2\sin A\sin B = \cos(A - B) - \cos(A + B)$

Formula to transform the sum or differences into product

We know that,

$\sin(A + B) + \sin(A - B) = 2\sin A.\cos B$ ...(i)

Let $A + B = C$

$A - B = D$

Solving for A and B, we get

$A = \frac{C+D}{2}$ and $B = \frac{C-D}{2}$

Substituting the values of A and B in (i), we get

$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

Similarly, the following formulae can be deduced:

$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

$\cos C + \cos D = 2\cos\frac{C+D}{2}.\cos\frac{C-D}{2}$

$\cos C - \cos D = -2\sin\frac{C+D}{2}.\sin\frac{C+D}{2}$

Ex: Prove that $2\cos x.\cos 3x - 2\cos 5x.\cos 7x - \cos 4x + \cos 12x = 0$.

Sol.: LHS $= 2\cos x.\cos 3x - 2\cos 5x.\cos 7x - \cos 4x + \cos 12x$

We know that, $2\cos A.\cos B = \cos(A + B) + \cos(A - B)$

$\therefore$ $2\cos x.\cos 3x = \cos 4x + \cos 2x$

and, $2\cos 5x.\cos 7x = \cos 12x + \cos 2x$

Now, LHS $= \cos 4x + \cos 2x - \cos 12x - \cos 2x - \cos 4x + \cos 12x = 0 =$ RHS

Hence, proved.

Ex. Prove that $(\sin 20^\circ + \sin 40^\circ) + (\cos 20^\circ + \cos 40^\circ) = \left(\sqrt{3} + 1\right)\cos 10^\circ$

Sol.: L.H.S. $= (\sin 20^\circ + \sin 40^\circ) + (\cos 20^\circ + \cos 40^\circ)$

We know that,

$\sin C + \sin D = 2\sin(\frac{C+D}{2}) \cdot \cos(\frac{C-D}{2})$ and, $\cos C + \cos D = 2\cos(\frac{C+D}{2}) \cdot \cos(\frac{C-D}{2})$,

Now,

$$\begin{aligned}
\text{L.H.S.} &= 2\sin\frac{60^\circ}{2}\cdot\cos\frac{20^\circ}{2}+2\cos\frac{60^\circ}{2}\cdot\cos\frac{20^\circ}{2}\\
&= 2\times\sin 30^\circ\cdot\cos 10^\circ+2\times\cos 30^\circ\cdot\cos 10^\circ\\
&= 2\times\frac{1}{2}\cos 10^\circ+2\times\frac{\sqrt{3}}{2}\cdot\cos 10^\circ\\
&= \cos 10^\circ+\sqrt{3}\cos 10^\circ\\
&= \left(\sqrt{3}+1\right)\cos 10^\circ = \text{ R.H.S.}
\end{aligned}$$