Chapter 3

Trigonometry

INTRODUCTION

Trigonometric Equations

An equation that includes one or more trigonometric ratios of an unknown angle is referred to as a trigonometric equation.

Solution of Trigonometric Equation

A solution of trigonometric equation is the value of the unknown angle that makes the equation true.

For instance, If $\sin\theta = \frac{1}{\sqrt{2}}$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, \dots\dots$$

Therefore, due to their periodic nature, a trigonometric equation may have an infinite number of solutions and can be categorized as:

(a) Principal solution

(b) General solution.

Principle solution of a trigonometric equation

The solutions of a trigonometric equation that fall within the interval $[0, 2\pi)$ are termed Principal solutions. For example, determine the Principal solutions of the equation

$$\sin x = \frac{1}{2}$$

there exists two values

i.e. $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ which lie in $[0,2\pi)$ and whose sine is $\frac{1}{2}$

Principal solutions of the equation $\sin x = \frac{1}{2}$ are $\frac{\pi}{6}, \frac{5\pi}{6}$

General solution of a trigonometric equation

The mathematical expression incorporating an integer 'n' that yields all solutions to a trigonometric equation is referred to as the general solution. Below are the general solutions for some standard trigonometric equations.

- Trigonometric functions of any angle
- Trigonometric functions defined as circular function
  - Sign of trigonometric functions in different quadrants
  - Variations in values of trigonometry functions in different quadrants
  - Graph of sine function with its varying values on unit circle
- Graphs and other useful data of trigonometric functions
  - Transformation of the graphs of trigonometric functions
- Trigonometric ratios of allied angles
- Trigonometric ratios for compound angles
  - Cosine of the difference and sum of two angles
  - Sine of the difference and sum of two angles
  - Tangent of the difference and sum of two angles
  - Important conditional identities
  - range of $f(\theta) = a\cos\theta + b\sin\theta$
- Transformation formula
  - Formula to transform the product into sum or differences
  - Formula to transform the sum or differences into product
- Trigonometric ratios of multiple and sub-multiple angles
  - Formulas for multiple angles
- Values of trigonometric ratios of typical angles
- Sum of sine or cosine of n angles in A.P.
- Conditional identities
  - Standard identities in triangle
- Finding range or expressions using trigonometric substitution

GENERAL SOLUTION OF SOME STANDARD EQUATIONS

If $\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)n\alpha$

where $\alpha \in [-\frac{\pi}{2}, \frac{\pi}{2}], n \in I$.

If $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha$

Where $\alpha \in [0, \pi], n \in I$.

If $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha$

where $\alpha \in (-\frac{\pi}{2}, \frac{\pi}{2}), n \in I$.

If $\sin^2\theta = \sin^2\alpha \Rightarrow \theta = n\pi \pm \alpha, n \in I$

If $\cos^2\theta = \cos^2\alpha \Rightarrow \theta = n\pi \pm \alpha, n \in I$.

If $\tan^2\theta = \tan^2\alpha \Rightarrow \theta = n\pi \pm \alpha, n \in I$

Note: α is called the principal angle