CONDITIONAL IDENTITIES

When three angles A, B, and C adhere to a given relationship, various identities can be derived that establish connections between the trigonometric ratios of these angles in a triangle.

ABC, $A + B + C = \pi$

$\sin(A + B) = \sin(\pi - C) = \sin C$

And $\cos(A + B) = \cos(\pi - C) = -\cos C$

Also, $\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \frac{\pi}{2}$

If $A + B + C = \pi$, then $\tan(A + B) = \tan(\pi - C) = -\tan C$

Similarly, $\tan(B + C) = \tan(\pi - A) = -\tan A$ and $\tan(C + A) = \tan(\pi - B) = -\tan B$

Hence $\sin\left(\frac{A+B}{2}\right) - \sin\left(\frac{\pi}{2} - \frac{C}{2}\right) - \cos\frac{C}{2}$

$\cos\left(\frac{A+B}{2}\right) - \cos\left(\frac{\pi}{2} - \frac{C}{2}\right) - \sin\frac{C}{2}$

$\tan\left(\frac{A+B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right) = \cot\left(\frac{C}{2}\right)$

Remember:

If $A + B + C = \pi$, then

(i) $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$

(ii) $\cos 2A + \cos 2B + \cos 2C = -1 - 4 \cos A \cos B \cos C$

(iii) $\cos A + \cos B + \cos C = 1 + 4 \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$

(iv) $\sin A + \sin B + \sin C = 4 \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$

(v) $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

(vi) $\cot A \cot B + \cot B \cot C + \cot C \cot A = 1$

(vii) $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2} \cot\frac{B}{2} \cot\frac{C}{2}$

(viii) $\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1$

Ex. If $A = \cos^2\theta + \sin^4\theta$, then this expression holds true for all values of θ.

(1) $1 \le A \le 2$ (2) $\frac{13}{16} \le A \le 1$ (3) $\frac{3}{4} \le A \le \frac{13}{16}$ (4) $\frac{3}{4} \le A \le 1$

Sol. $A = \cos^2\theta + \sin^2\theta \sin^2\theta$

$A \le \cos^2\theta + \sin^2\theta.1$ $(\because \sin^2\theta \le 1)$

$A \le 1$

Again, $A = (1 - \sin^2\theta) + \sin^4\theta$

$A = (\sin^2\theta - \frac{1}{2})^2 + (1 - \frac{1}{4})$

$A \ge \frac{3}{4}$

Hence, $\frac{3}{4} \le A \le 1$

Ex. Solve $\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}$

(1) 0 (2) $\frac{1}{2}$ (3) $\frac{1}{3}$ (4) $-\frac{1}{8}$

Sol. $\frac{1}{2\sin\frac{\pi}{7}} \times (2\sin\frac{\pi}{7}\cos\frac{\pi}{7})\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}$

$\frac{1}{2\sin\frac{\pi}{7}} \times \frac{1}{2}(2\sin\frac{2\pi}{7}\cos\frac{2\pi}{7})\cos\frac{4\pi}{7}$

$\frac{1}{4\sin\frac{\pi}{7}} \times \frac{1}{2}(2\sin\frac{4\pi}{7}\cos\frac{4\pi}{7})$

$$\frac{1}{8}\times\frac{\sin\frac{8\pi}{7}}{\sin\frac{\pi}{7}}=\frac{\sin(\pi+\frac{\pi}{7})}{8\sin\frac{\pi}{7}}$$

$$\frac{-\sin\frac{\pi}{7}}{8\sin\frac{\pi}{7}}=-\frac{1}{8}$$

Ex. The period of the function, f (x) = 3 sin (2x + 1) measured in radians is

(1) 2π (2) π (3) $\frac{\pi}{2}$ (4) $-\pi$

Sol. Period of sin x is 2π, the period of

$f(x) = 3\sin(2x+1)$ is $\frac{2\pi}{2}=\pi$

Alternatively we have

$f(x) = 3\sin(2x+1) = 3\sin(2\pi+2x+1)$

$3\sin\{2(\pi+x)+1\} = f(\pi+x)$

Period of f (x) is π