

**Algebraic operation on function**

If  $f$  and  $g$  are real-valued functions of  $x$  with domains  $A$  and  $B$ , respectively, then both  $f$  and  $g$  are defined within their respective domains.  $A \cap B$  Now we defined  $f + g$ ,  $f - g$ ,  $(f, g)$  and  $(f/g)$  as follows:

$$\left. \begin{aligned} (f \pm g)(x) &= f(x) \pm g(x) \\ (f, g)(x) &= f(x) \cdot g(x) \end{aligned} \right\} - \text{domain in each case is } A \cap B$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \text{ domain is } \{x \mid x \in A \cap B \text{ such that } g(x) \neq 0\}.$$

$$(kf)(x) = hf(x) \text{ where } k \text{ is a scalar.}$$

**Equal or identical Function**

Two functions,  $f$  and  $g$ , are considered equal if.

1. The domain of  $f$  equals the domain of  $g$ .
2. The range of  $f$  is equal to the range of  $g$ , and
3.  $f(x) = g(x)$ , for every  $x$  belonging to their common domain eg.  $f(x) = \frac{1}{x}$  &  $g(x) = \frac{x}{x^2}$  are identical function.

**Ex.** The mathematical function  $f(x) = \log\left(\frac{x-1}{x-2}\right)(x-1) - \log(x-2)$  and  $g(x) = \log$  are identical when  $x$  lies in the interval.

**Sol.** Since  $f(x) = \log(x-1) - \log(x-2)$ .

$$\text{Domain of } f(x) \text{ is } x > 2 \text{ or } x \in (2, \infty) \quad \dots (1)$$

$$g(x) = \log\left(\frac{x-1}{x-2}\right) \text{ is defined if } \frac{x-1}{x-2} > 0$$

$$x \in (-\infty, 1) \cup (2, \infty) \quad \dots (2)$$

$$\text{From (1) and (2)} \quad x \in (2, \infty)$$