EVEN AND ODD FUNCTIONS

1. Even function:

If $f(-x) = f(x)$ then $f(x)$ is said to be even function.

Example: $f(x) = \cos x$ is a even function

$$[f(-x) = \cos(-x) = \cos x = f(x)]$$

2. Odd function:

If $f(-x) = -f(x)$ then $f(x)$ is said to odd function.

Example: If $f(x) = x^3 + \tan^3 x$ is a odd function because

$$f(-x) = (-x)^3 + [\tan(-x)]^3$$
$$-x^3 - \tan^3 x$$
$$-[x^3 + \tan^3 x]$$
$$-f(x)$$
$$f(-x) = -f(x)$$

1. An even function exhibits symmetry about the y-axis, while an odd function is symmetrical about the origin (i.e., in opposite quadrants).

2. The addition and subtraction of two even functions always result in an even function.

3. The sum of an even and an odd function is neither an even nor an odd function.

4. Any function 'f' can be expressed as the sum of an even function and an odd function.

$$f(x) = \frac{1}{2}[f(x) + f(-x)] + \frac{1}{2}[f(n) - f(-x)]$$

Where, $\frac{1}{2}[f(x) + f(-x)]$ is an even and $\frac{1}{2}[f(n) - f(-x)]$ is an odd function.

5. $f(x) = 0$ is the only function which is both odd and even.

Extension of Domain

Let a function f(x) be defined on the interval [0, a], it can be extended on [-a, a] so that f(x) is either odd or even on the interval [-a, a].

Even extension

f(x) is given to be defined in [0, a]

$-a \le x \le 0$ i.e., $x \epsilon [-a, 0]$

We define f(x) in [−a, 0] such that f(x) = f(−x). If $f_e(x)$ be the even extension, then

$$f_e(x) = \begin{cases} f(x), & x \epsilon [0, a] \\ f(-x), & x \epsilon [-a, 0] \end{cases}$$

Odd extension

f(x) is defined in [0, a]

$-x \in [-a, 0]$

We define f(x) in [−a, 0] such that f(x) = f(−x). Let $f_o(x)$ be the odd extension, then

$$F_o(x) = \begin{cases} f(x), & x \epsilon [0, a] \\ -f(-x), & x \epsilon [-a, 0] \end{cases}$$

Ex. Is a function $f(x) = x \cdot \frac{e^x + e^{-x}}{e^x - e^{-x}}$ even?

Sol: Yes, f(x) is even, because

$$f(-x) = (-x)\frac{e^{-x} + e^x}{e^{-x} - e^x} = x\frac{e^x + e^x}{e^x - e^{-x}} = f(x)$$
$$f(-x) = f(x).$$