KINETIC THEORY OF GASES

The kinetic theory of gases provides a theoretical framework for understanding the experimental gas laws. This model is based on several assumptions:

(a) Gas particles are extremely small, and their volume is considered negligible compared to the volume of the container (although this assumption may be violated if the container volume is very small).

(b) There is no interaction between gas particles, although certain conditions of temperature and pressure may affect this interaction.

(c) Gas molecules are in a constant state of motion unaffected by gravity, and this motion is described as random straight-line motion or Brownian motion.

(d) Continuous motion leads to collisions between gas molecules and the walls of the container. These collisions are responsible for the pressure exerted by the gas on the container walls.

(e) Gas molecules move at different speeds, and their speeds change with each collision.

(f) All collisions are considered perfectly elastic, meaning there is no loss of energy.

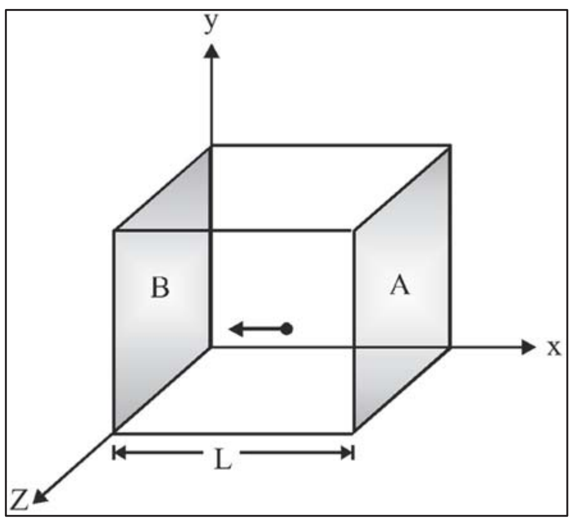
(g) The average kinetic energy of the gas depends solely on the absolute temperature.

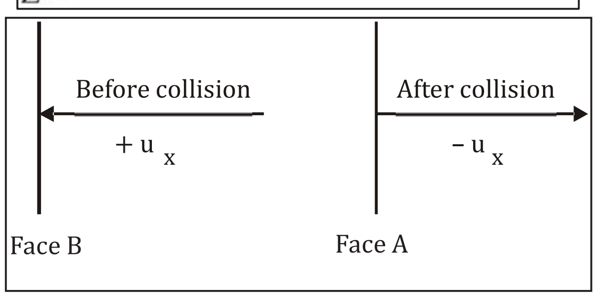
The Kinetic Gas Equation

Derivation of Equation for Kinetic Molecular

Let's examine a cube with a side length of L, containing N0 molecules, each with a mass of m, moving in various directions with a velocity of u. Consequently, these molecules collide both with each other and the sides of the container.

Velocity u can be resolved into three components u_x, u_y and u_z along their axes such that $u^2 = u_x^2 + u_y^2 + u_z^2$ (assume velocity in ms^{-1} and distance in meter)





In the simplest scenario, we focus on the motion of a molecule solely along the x-axis. The molecule moves towards face B with a velocity of ux. Upon colliding with face B, it then moves towards face A with a velocity of (-ux) due to the elastic nature of collisions, which results in a change in direction without altering the velocity.

∴ Momentum before collision on face B = mu_x

Momentum after collision on face B = $-mu_x$

Change in momentum due to one collision on face B

$$= mu_x - (-mu_x) = 2mu_x$$

To strike face B again distance travelled = 2L

Time taken to strike face B again = $\frac{2L}{u_x}$seconds

∴ Number of collisions per second on face B along x-axis = $\frac{u_x}{2L}$

∴ Change in momentum due to $\frac{u_x}{2L}$ collisions per second on face B along x-axis

$$= 2mu_x \cdot \frac{u_x}{2L} =$$

Change in momentum per second due to collisions of one molecule on opposite faces A and B along x-axis = $\frac{mu_x^2}{L}$

Similarly for y-axis change in momentum per second = $\frac{2mu_y^2}{L}$ and for z-axis = $\frac{2mu_z^2}{L}$

Total change in momentum per second due to collisions of a single molecule on six faces along three axes

$$= \frac{2m\ \ _x^2}{L} + \frac{2mu_y^2}{L} + \frac{2mu_z^2}{L}$$

$$= \frac{2m}{L}(u_x^2 + u_y^2 + u_z^2) = \frac{2mu^2}{L}$$

But rate of change in momentum per second = Force

∴ Force = $\frac{2mu^2}{L}$

∴ Pressure = $\frac{\text{Force}}{\text{Area of six faces}} = \frac{\frac{2mu^2}{L}}{6\ L^2} = \frac{mu^2}{3\ L^3} = \frac{mu^2}{3\ V}$ [L^3 = volume V]

∴ Pressure due to collisions of N_0 molecules on six faces of a cube = $\frac{1}{3} mN_0u^2$

$$PV = \frac{1}{3}\ mN_0u^2 = \frac{1}{3}\ Mu^2$$

mN_0 = M (molar mass)

N_0 = Avogadro's number

u = root mean square velocity (U_{rms})

Translational Kinetic Energy of n Moles

$$\frac{1}{2} Mu^2 = \frac{3}{2} PV = \frac{3}{2}\ nRT$$

Average Translational Kinetic Energy Per Molecule

$$= \frac{3}{2}\frac{RT}{N_0} = \frac{3}{2} KT$$

WhereK(= $\frac{R}{N_0}$) is called Boltzmann's constant.

Its numerical value is 1.38×10^{-16} erg K^{-1} molecule^{-1}

Thus average K.E. is proportional to absolute temperature.

If T = 0 K (i.e., – 273.15ºC), then average KE = 0

Thus, absolute zero (0 K) is the temperature at which molecular motion ceases.

Molecular Speeds in Gases

Different Types of Molecular Velocities

$$PV = \frac{1}{3}MU_{rms}^2$$

(i) **Root means square velocity (U_{rms}):**

Root Mean Square Speed

It is characterized as the square root of the average of the squared velocities of all the molecules within a given gas sample.

$$U_{rms} = \sqrt{\frac{U_1^2+U_2^2+\cdots U_n^2}{N}}$$

∴ U_{rms} (root mean square velocity) $= \sqrt{\frac{3P}{M}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3P}{d}}$ Where d is the density.

➢ If N_1 molecules have velocity u_1 and N_2 molecules have velocity u_2, then

$$U_{rms} = \sqrt{\frac{N_1u_1^2+N_2u_2^2}{N_1+N_2}}$$

(ii) **Average velocity (U_{av}):**

Average Speed

It is determined by calculating the average of the squared velocities of gas molecules at a specific temperature, using the arithmetic mean.

$$U_{av} = \frac{U_1^2+U_2^2+\cdots U_n^2}{n}$$

➢ U_{av} (average velocity) $= \sqrt{\frac{8RT}{\pi M}}$

(iii) **Most probable velocity (U_{mp}):**

Most Probable Speed

It is characterized as the velocity held by the majority of gas molecules at a specified temperature.

➢ U_{mp} (most probable velocity) $= \sqrt{\frac{2RT}{M}}$

➢ If P and T both are given, use equation in terms of temperature i.e., use

$$U_{rms} = \sqrt{\frac{3RT}{M}} \text{ and not } \sqrt{\frac{3pv}{M}}$$

➢ To have velocity in ms^{-1} (MKS) takes R = 8.314 J $mol^{-1}K^{-1}$, M in kg.

➢ If density is in kg m^{-3} and P in N m^{-2}, velocity will be in ms^{-1}.

Relation between rms velocity, average velocity and most probable velocity.

➢ $U_{rms}: U_{av}: U_{mp} = \sqrt{\frac{3RT}{M}} : \sqrt{\frac{8RT}{\pi M}} : \sqrt{\frac{2RT}{M}} = \sqrt{3} : \sqrt{\frac{8}{\pi}} : \sqrt{2} =$ 1.2248: 1.1284: 1

$$U_{rms} = 1.2248\ U_{mp}$$

$$U_{av} = 1.1284\ U_{mp}$$

$$U_{mp} = 1.0854\ U_{av}$$