HEISENBERG'S UNCERTAINTY PRINCIPLE

Bohr's theory conceptualizes an electron as a material particle, enabling the accurate determination of its position and momentum. However, when viewed as a wave, as proposed by de Broglie, it becomes challenging to simultaneously pinpoint the exact position and velocity of the electron with high precision at a specific moment, given that the wave extends across a spatial region.

In 1927, Werner Heisenberg introduced a principle known as the Heisenberg uncertainty principle, asserting that "It is impossible to measure simultaneously the exact position and exact momentum of a body as small as an electron."

The uncertainty of measurement of position, Δx, and the uncertainty of momentum Δp or $m\Delta v$, are related by Heisenberg's relationship as: ($p = mv$, $\Delta p = m\Delta v$)

$$\Delta x \,.\, \Delta p \geq \frac{h}{4\pi}$$

or $$\Delta x.\ m\Delta v \geq \frac{h}{4\pi}$$

or $$\Delta x.\ \Delta v \geq \frac{h}{4\pi m}$$

where h is Planck's constant.

$\Delta x\ \Delta v$ = uncertainty product

For an electron of mass m (9.10×10^{-28} g), the product of uncertainty is quite large.

$$\Delta x \,.\, \Delta v \geq \frac{6.624 \times 10^{-27}}{4\pi m}$$

$$\geq \frac{6.624 \times 10^{-2}}{4 \times 3.14 \times 9.10 \times 10^{-2}}$$

$$= 0.57 \text{ erg sec per gram approximately}$$

When, $\Delta x = 0$, $\Delta v = \infty$ and vice-versa.

For larger particles with significant mass, the uncertainty product has a negligible value. When the position is known with high accuracy (i.e., Δx is very small), Δv becomes large, and vice versa.

- Expressed in terms of uncertainties in energy (ΔE) and time (Δt), this principle is formulated as Heisenberg replaced the notion of definite orbits with the concept of probability.
- According to Heisenberg, $\Delta E \cdot \Delta t \geq \frac{h}{4\pi}$, allowing us only to define the probability of finding electrons around the nucleus.

Ex. Why electron cannot exist inside the nucleus according to Heisenberg's uncertainty principle?

Sol. Diameter of the atomic nucleus is of the order of 10^{-15} m

The maximum uncertainty in the position of electron is 10^{-15} m.

Mass of electron = 9.1×10^{-31} kg.

$$\Delta x.\ \Delta p = \frac{h}{4\pi}$$

$$\Delta x \times (m.\Delta v) = \frac{h}{4}\pi$$

$$\Delta v = \frac{h}{4\pi} \times \frac{1}{\Delta x \cdot m} = \frac{6.63 \times 10^{-34}}{4 \times \frac{22}{7}} \times \frac{1}{10^{-15} \times 9.1 \times 10^{-31}}$$

$$\Delta v = 5.80 \times 10^{10}\ ms^{-1}$$

This value is much higher than the velocity of light and hence not possible.

De Broglie Relationship & Heisenberg's Uncertainty Principle

Ex. The mass of a particle is 1 mg and its velocity is 4.5×10^{5} cm per second. What should be the wavelength of this particle if $h = 6.652 \times 10^{-27}$ erg second.

(1) 1.4722×10^{-24} cm (2) 1.4722×10^{-29} cm

(3) 1.4722×10^{-32} cm (4) 1.4722×10^{-34} cm

Sol. Given that

$$m = 1 \text{ mg} = 1 \times 10^{-3} \text{ g}$$

$$c = 4.5 \times 10^{5} \text{ cm/sec.}$$

$$h = 6.652 \times 10^{-27} \text{ erg sec.}$$

$$\therefore \quad \lambda = \frac{h}{mc} = \frac{6.652 \times 10^{-27}}{1 \times 10^{-3} \times 4.5 \times 10^{5}} = \frac{6.652 \times 10^{-29}}{4.5} = 1.4722 \text{ cm} \times 10^{-29} \text{cm}$$