BOHR'S MODEL

Introduction of Bohr's Model

In 1915, Neil Bohr introduced the Bohr model of the atom, a conceptual framework that emerged as a modification to Ernest Rutherford's previous atomic model. Rutherford's model, known as the nuclear model, postulated the existence of a central nucleus with a positive charge, surrounded by orbiting electrons carrying a negative charge. Bohr's innovative model built upon Rutherford's groundwork, refining the understanding of atomic structure by incorporating distinct electron orbits and proposing quantized energy levels for electrons within these orbits. This model played a pivotal role in advancing our comprehension of atomic behavior during the early 20th century.

Bohr's Theory for Hydrogen and Hydrogen Like Atoms

The Bohr model of the hydrogen atom, pioneered by Niels Bohr in 1913, marked a significant breakthrough as it became the initial atomic model capable of effectively elucidating the radiation spectra emitted by atomic hydrogen. This model was conceived with the purpose of addressing specific shortcomings identified in Ernest Rutherford's atomic model. Bohr's hydrogen atom model holds historical significance by not only bridging the gaps perceived in Rutherford's model but also serving as a catalyst for the development of quantum mechanics through the introduction of quantum theory. This innovative approach laid the foundation for a more comprehensive understanding of the behavior of atomic particles, shaping the course of scientific exploration in the early 20th century.

Postulates Of Bohr's Model

(i.) An atom comprises a centrally located, small, dense, and positively charged nucleus, with electrons orbiting around the nucleus in circular paths known as orbits. The Coulombic force of attraction between the nucleus and electrons is balanced by the centrifugal force of the revolving electrons.

(ii.) Among the infinite circular orbits, only those orbits are permissible in which the angular momentum of the electron is an integral multiple of $\frac{h}{2\pi}$. In other words, the angular momentum of an electron can have fixed values such as $\frac{h}{2\pi}, \frac{2h}{2\pi}, \frac{3h}{2\pi}$, and so on, indicating that the angular momentum of the electron is quantized.

$$mvr = \frac{nh}{2\pi}$$

m and v represent the mass and velocity of the electron, respectively, while r denotes the radius of the orbit. The integer n is subsequently associated with the orbit number and shell number, and h stands for Planck's constant.

(iii.) The energy values of these circular orbits are predetermined, resulting in the electron in an atom having only specific energy values. The energy is intrinsic to an orbit and cannot possess any arbitrary value, thus making the energy of an electron quantized.

(iv.) While electrons occupy these fixed orbits, they do not dissipate energy; in other words, the energy of an electron remains constant (unchanging with time). These stable orbits are referred to as allowed energy levels or stationary states, elucidating the stability of the atom.

(v.) The energy levels are designated as K, L, M, N, and are numbered sequentially as 1, 2, 3, 4, etc., moving outward from the nucleus. Additionally, as the distance of the energy level or shell from the nucleus increases, the energy of the energy level also increases.

$$E_N > E_M > E_L > E_k$$

(vi.) The release or absorption of energy in the form of radiations only takes place when an electron transitions from one stationary state to another.

$$\Delta E = E_{higher} - E_{lower}$$

$$\Delta E = h\nu$$

In this equation, hν represents the energy of the absorbed or emitted photon, corresponding to the difference in energy levels. Energy is absorbed when an electron transitions from a lower energy level (ground state) to a higher energy level (excited, unstable state), and energy is emitted when electrons transition from a higher energy level to a lower energy level.

- Bohr's model is applicable for a one electron species only, like H, He^{+}, Li^{+2}, Be^{+3} etc.
- Derivation of Radius of different orbits in one electron species (using Bohr's model):

$$mvr = \frac{nh}{2\pi} \quad \text{........ (1)}$$

$$q_1 = e, \quad q_2 = Ze$$

$$\frac{mv^2}{r} = \frac{kq_1q_2}{r^2} = \frac{kze^2}{r^2} \quad \text{........ (2)}$$

$$\Rightarrow \frac{mn^2h^2}{4\pi^2m^2r^2\cdot r} = \frac{kZe^2}{r^2}$$

$$\Rightarrow r = \frac{mn^2h^2}{4\pi^2m^2kZe^2}$$

$$\Rightarrow r = \frac{n^2h^2}{4\pi^2mkZe^2}$$

$$\Rightarrow \frac{(6.625\times10^{-34})^2}{4\pi^2\times9.1\times10^{-31}\times9\times10^9\times(1.6\times10^{-1}\ )^2} \times \frac{n^2}{Z}$$

$$r = 0.529 \times \frac{n^2}{z} \text{Å}$$

$$r \propto \frac{n^2}{z}$$

for a particular atom $r \propto n^2$

Radius of 1^{st} orbit of H atom $r = 0.529$ Å

Ex. Calculate ratio of radius of 1^{st} orbit of H atom to Li^{+2} ion:

Sol. $\frac{\text{Radius of } 2^{nd} \text{ orbit of H atom}}{\text{Radius of } 3^{rd} \text{ orbit of } Li^{+2} \text{ atom}} = \frac{n^2}{z} \times \frac{z_1}{n_1^2} = \frac{4}{3} =$

Derivation of Velocity of electron in Bohr's orbit

$$v = \frac{nh}{2\pi mr}, \text{ putting value of r.}$$

$$v = \frac{nh\times4\pi^2mkZe^2}{2\pi m^2h^2}$$

$$v = \frac{2\pi kZe^2}{h.n.} = \frac{2\pi ke^2}{h} \times \frac{Z}{n}$$

$$v = 2.18 \times 10^6 \frac{Z}{n} \text{ m/s}$$

$$v \propto \frac{Z}{n}$$

Derivation of total energy of electron / system

T.E. of system = K.E. of e^- + P.E. of system (nucleus and e^-)

kinetic energy of electron = $\frac{1}{2}$ mv^2 $= \frac{1}{2}\frac{kze^2}{r}$

$$PE = -\frac{kze^2}{r}$$

$$TE = \frac{-KZe^2}{2r}$$

$$T.E. = -13.6 \times \frac{z^2}{n^2} \text{ eV/atom}$$

$$T.E. = -2.18 \times 10^{-18} \frac{z^2}{n^2} \text{ J/atom}$$

As the shell number or distance increases, the values of total energy (T.E.) and potential energy (P.E.) increase (although their magnitudes decrease) and reach their maximum at infinity, which is zero. The negative sign indicates that the electron is subject to the attractive forces of the nucleus.

$$K.E. = -\frac{PE}{2}$$

$$T.E. = \frac{P.E}{2}$$

$$T.E = -K.E.$$

Calculation of energy of energy level in H atom

(i) When n = 1 (ground level)

$$K.E. = 13.6 \text{ eV (atom)}$$

$$P.E. = -27.2 \text{ eV / atom}$$

$$T.E. = -13.6 \text{ eV/atom}$$

(ii) When n = 2/2nd energy level / 1st excited state

$$K.E. = \frac{13.6}{4} \text{ eV/atom} = 3.4 \text{ eV/ atom}$$

$$P.E. = -6.8 \text{ eV/atom}$$

$$T.E. = -3.4 \text{ eV/atom}$$

$$E_2 - E_1 = -3.4 + 13.6 \text{ eV/atom} = 10.2 \text{ eV/atom}$$

(iii) When n = 3 / 3rd energy level/2nd excited state.

$$K.E. = \frac{13.6}{9} \text{ eV/atom} = 1.51 \text{ eV/atom}$$

$$P.E. = -3.02 \text{ eV/atom}$$

$$T.E. = -1.51 \text{ eV/atom}$$

$$-E_3 - E_2 = -1.51 \text{ eV/atom} + 3.4 \text{ eV/atom}$$

$$= 1.89 \text{ eV atom.}$$

(iv) when n = 4 / 3rd excited state

$$K.E. = \frac{13.6}{4^2}$$

$$\Rightarrow \quad = 0.85 \ \frac{13.6}{16\times10} \text{ eV/atom}$$

$$P.E. = -1.70 \text{ eV/atom}$$

$$T.E. = -0.85 \text{ eV/atom}$$

$$E_4 - E_3 = 0.66$$

As the distance increases (with an increase in n), the energy of the energy level also increases. However, the energy difference between consecutive energy levels continues to grow, reaching its maximum between levels 2 and 1 (consecutive levels).

If reference value (P.E at ∞) is assigned value other than zero. –

(i) All K.E. data remains same.

(ii) P.E./T.E. of each shell will be.

changed however difference in P.E./T.E. between 2 shells will remain unchanged.

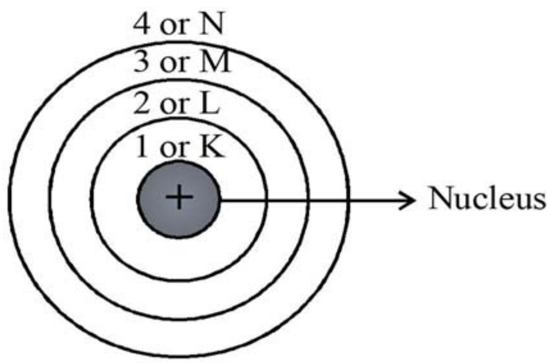
Bohr's Atomic Model

This was the inaugural model founded on Planck's quantum theory, elucidating both the stability of the atom and the line spectrum of hydrogen. Grounded in the quantum theory of light, it is exclusively applicable to single-electron species.

Ex. H, He^{+}, Li^{2+}, etc.

Assumptions of Bohr's Model

The electron within the hydrogen atom follows a circular path of a constant radius and energy while orbiting the nucleus. These paths are referred to as orbits, stationary states, energy shells, or allowed energy states. The stationary states for electrons are denoted by numbers such as n = 1, 2, 3, ..., or designated as K, L, M, N, ..., etc. shells (as shown in the figure). These integral numbers are identified as principal quantum numbers, and the orbits are concentrically arranged around the nucleus.

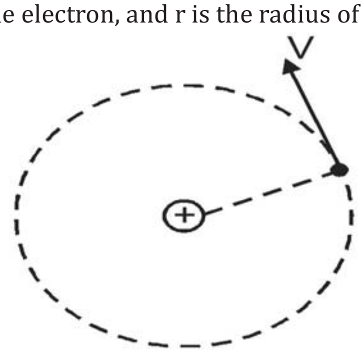


Bohr's orbit

- Electrons orbit exclusively in those paths where the angular momentum of the electron is quantized. Consequently, an electron is restricted to move solely in orbits where its angular momentum is an integral multiple of $\frac{h}{2\pi}$.

$$mvr = n\frac{h}{2\pi}$$

Here, n takes values from 1 to infinity, h represents Planck's constant, m is the mass of the electron, v denotes the velocity of the electron, and r is the radius of the orbit.



- The energy of an electron within an orbit remains constant over time, indicating that the electron's energy in a specific orbit remains unchanged; there is no loss or gain of energy.
- An electron transitions from a lower stationary state to a higher one only when it absorbs the necessary amount of energy. Upon returning to the lower energy level, the electron emits the same amount of energy. The energy change occurs discretely rather than continuously.
- The frequency of radiation absorbed or emitted when transition occurs between two stationary states that differ in energy by ΔE is given by

$$\nu = \frac{\Delta E}{h} = \frac{E_2 - E_1}{h}$$

where E_1 and E_2 are the energies of the lower and higher allowed energy states, respectively. This expression is called Bohr's frequency rule.

Mathematical forms of Bohr's Postulates

Calculation of The Radius of The Bohr's Orbit:

Imagine an electron with mass 'm' and charge 'e' orbiting around a nucleus with a charge 'Ze' (where 'Z' is the atomic number and 'e' represents the charge). The electron has a tangential or linear velocity 'v,' and the orbit's radius is denoted by 'r.'

According to Coulomb's law, the electrostatic force of attraction (F) between the moving electron and nucleus is:

$$F = \frac{KZe^2}{r^2}$$

where $K = \text{constant} = \frac{1}{4\pi\varepsilon_0} = 9 \text{ x } 10^9 \text{ Nm}^2/C^2$

And the centrifugal force F $\frac{mv^2}{r} =$

For the stable orbit of an electron both the forces are balanced.

i.e. $$\frac{mv^2}{r} = \frac{KZe^2}{r^2}$$

then $$v^2 = \frac{KK\ ^2}{mr} \quad \text{......... (i)}$$

From the postulate of Bohr,

$$mvr\ \frac{nh}{2\pi} = \Rightarrow \qquad v = \frac{nh}{2\pi mr}$$

On squaring $$v^2 = \frac{n^2h^2}{4\pi^2m^2r^2} \quad \text{........ (ii)}$$

From equation (i) and (ii)

$$\frac{KZe^2}{mr} = \frac{n^2\ h^2}{4\pi^2\ m^2r^2}$$

On solving, we will get

$$r = \frac{n^2\ h^2}{4\pi^2 mKZe^2}$$

On putting the value of e, h, m, the radius of n^{th} Bohr orbit is given by:

$$r_n = \frac{n^2}{Z} 0.529 \text{ x Å} \quad \propto \frac{n^2}{Z} \quad \Rightarrow$$

$$r \Rightarrow \quad \frac{r_1}{r_2} = \frac{n_1^2}{n_2^2} \times \frac{Z_2}{Z_1}$$

Ex. Calculate radius ratio for 2^{nd} orbit of He^+ ion & 3^{rd} orbit of Be^{+++} ion.

Sol. r_1 (radius of 2^{nd} orbit of He^+ ion) = $(\frac{2^2}{2})$ 0.529 Å

r_2 (radius of 3^{rd} orbit of Be^{+++} ion) = $(\frac{3^2}{4})$ 0.529 Å

Therefore, $$\frac{r_1}{r_2} = \frac{0.529\times 2^2/2}{0.529\times 3^2/4} = \frac{8}{9}$$

Calculation of Velocity of an Electron in Bohr's Orbit

Angular momentum of the revolving electron in n^{th} orbit is given by

$$mvr = \frac{nh}{2\pi}$$

$$v = \frac{nh}{2\pi mr} \quad \text{......... (iii)}$$

put the value of 'r' in the equation

then, $$v = \frac{nh \times 4\pi^2 mZe^2\ K}{2\pi mn^2\ h^2}$$

$$v = \frac{2\pi Ze^2 K}{nh}$$

on putting the values of π, e^-, h and K

velocity of electron in n^{th} or bit $v_n = 2.18 \times 10^6 \times$ m/sec; $v \propto \frac{Z}{n}$ Z; $v \propto \frac{1}{n}$

$$v \propto \frac{Z}{n} \Rightarrow \frac{v_1}{v_2} = \frac{Z_1}{Z_2} \times \frac{n_2}{n_1}$$

T, Time period of revolution of an electron in its orbit = $\frac{2\pi r}{v}$ substituting the value of 'r' and 'v' we get

Time Period, $$T = 1.52 \times 10^{-16} \times \frac{n^3}{Z^2}$$

$$\Rightarrow \quad T \propto \frac{n^3}{Z^2}$$

$$\Rightarrow \quad \frac{T_1}{T_2} = \frac{n_1^3}{n_2^3} \times \frac{Z_2^2}{Z_1^2}$$

f, Frequency of revolution of an electron in its orbit = $\frac{v}{2\pi r} = \frac{1}{T}$

Calculation of Energy of an Electron

The total energy of an electron revolving in a particular orbit is

$$T.E. = K.E. + P.E.$$

where:

P.E. = Potential energy,

K.E. = Kinetic energy,

T.E. = Total energy

The K.E. of an electron = $\frac{1}{2}$ mv^2

and the P.E. of an electron = $-\frac{KZe^2}{r}$

Hence, $$T.E. = \frac{1}{2} mv^2 - \frac{KZe^2}{r}$$

we know that, $$\frac{mv^2}{r} = \frac{KZe^2}{r^2}$$

or $$mv^2 = \frac{KZe^2}{r}$$

$$\Rightarrow \quad K.E. = \frac{1}{2} \frac{KZe^2}{r}$$

substituting the value of mv^2 in the above equation:

$$T.E. = \frac{KZe^2}{2r} - \frac{KZe^2}{r} = -\frac{KZe^2}{2r}$$

So, $$T.E. = -\frac{KZe^2}{2r}$$

$$\Rightarrow \quad T.E. = -K.E. = \frac{P.E.}{2}$$

substituting the value of 'r' in the equation of T.E.

Then $$T.E. = -\frac{KZe^2}{2} \times \frac{4\pi^2 Ze^2\ m}{n^2\ h^2} = -\frac{2\pi^2 Z^2 e^4 mK^2}{n^2\ h^2}$$

Thus, the total energy of an electron in n^{th} orbit is given by

$$T.E. = \quad E_n = -\frac{2\pi^2 me^4 k^2}{h^2}\left(\frac{z^2}{n^2}\right) \qquad \text{........ (iv)}$$

Putting the value of m, e, h and π we get the expression of total energy

$E_n = -13.6\ \frac{Z^2}{n^2}$ eV / atom as the value of n increases, energy of an electron in the orbit increases.

Note: The P.E. at the infinite = 0
The K.E. at the infinite = 0