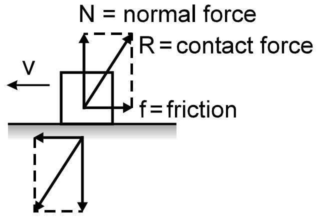
CHAPTER-7
FRICTION

1. FRICTION

When two bodies are kept in contact, electromagnetic forces act between the charged particles (molecules) at the surfaces of the bodies. Thus, each body exerts a contact force on the other. The magnitudes of the contact forces acting on the two bodies are equal but their directions are opposite and therefore the contact forces obey Newton's third law.



The direction of the contact force acting on a particular body is not necessarily perpendicular to the contact surface. We can resolve this contact force into two components, one perpendicular to the contact surface and the other parallel to it (figure. The perpendicular component is called the normal contact force or normal force (generally written as N) and the parallel component is called friction (generally written as f).

Therefore if R is contact force then R = $\sqrt{f^2 + N^2}$

2. REASONS FOR FRICTION

(i) Inter-locking of extended parts of one object into the extended parts of the other object.

(ii) Bonding between the molecules of the two surfaces or objects in contact.

3. FRICTION FORCE IS OF TWO TYPES.

a. Kinetic b. Static

(a) Kinetic Friction Force

Kinetic friction exists between two contact surfaces only when there is **relative motion** between the two contact surfaces. It stops acting when relative motion between two surfaces ceases.

DIRECTION OF KINECTIC FRICTION ON AN OBJECT

It is opposite to the relative velocity of the object with respect to the other object in contact considered.

Note that its direction is not opposite to the force applied it is opposite to the relative motion of the body considered which is in contact with the other surface.

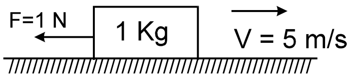
MAGNITUDE OF KINETIC FRICTION

The magnitude of the kinetic friction is proportional to the normal force acting between the two bodies. We can write

$$f_k = \mu_k N$$

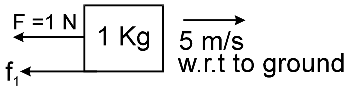
where N is the normal force. **The proportionality constant μ_k is called the coefficient of kinetic friction and its value depends on the nature of the two surfaces in contact.**

Example 1. Find the direction of kinetic friction force

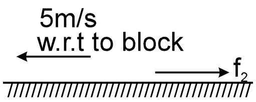


(a) on the block, exerted by the ground. (b) on the ground, exerted by the block.

Solution : (a)



(b)



where f_1 and f_2 are the friction forces on the block and ground respectively.

Example 2. In above example correct relation between magnitude of f_1 and f_2 is

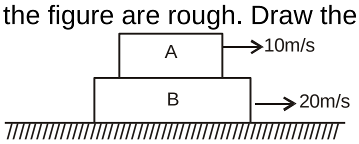
(A) $f_1 > f_2$ (B) $f_2 > f_1$

(C) $f_1 = f_2$ (D) not possible to decide due to insufficient data.

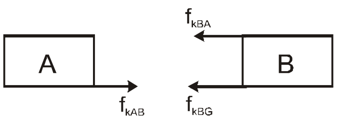
Solution : By Newton's third law the above friction forces are action-reaction pair and equal but opposite to each other in direction. Hence (C).

Also note that the direction of kinetic friction has nothing to do with applied force F.

Example 3. All surfaces as shown in the figure are rough. Draw the friction force on A & B

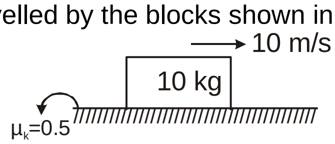


Solution :



Kinetic friction acts in such a way so as to reduce relative motion.

Example 4. Find out the distance travelled by the blocks shown in the figure before it stops.



Solution :

$N - 10\,g = 0$

$N = 100$ N

$f_x = \mu_k N$

$\mu = \mu_s = \mu_k$ when not mentioned

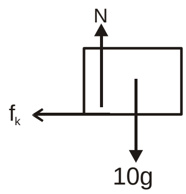
$f_x = 0.5 \times 100 = 50$ N

$F_x = ma$

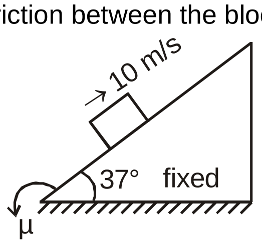
$50 = 10\,a \Rightarrow a = 5$

$\therefore\ v^2 = u^2 + 2as$

$0^2 = 10^2 + 2(-5)(S)$ $\quad\therefore\quad$ S = 10 m



Example 5. Find out the distance travelled by the block on incline before it stops. Initial velocity of the block is 10 m/s and coefficient of friction between the block and incline is $\mu = 0.5$.



Solution : N = mg cos37°

$\therefore$ mg sin 37° + µN = ma

a = 10 m/s^2 down the incline

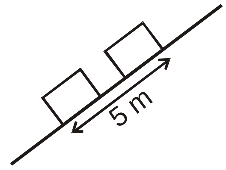
Now $v^2 = u^2 + 2as$

$0 = 10^2 + 2(-10)\,S$

$\therefore$ S = 5 m

Example 6. Find the time taken in the above example by the block to reach the initial position.

Solution : a = g sin 37° – µg cos 37°

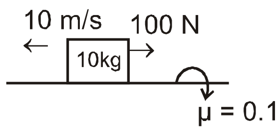


$\therefore$ a = 2 m/s^2 down the incline

$\therefore S = ut + \frac{1}{2}at^2 \Rightarrow S = \frac{1}{2} \times 2 \times t^2$

$\therefore$ t = sec.

Example 7. A block is given a velocity of 10 m/s and a force of 100 N in addition to friction force is also acting on the block. Find the retardation of the block?



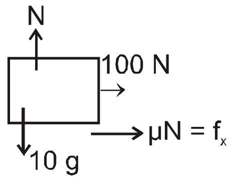
Solution : As there is relative motion

$\therefore$ Kinetic friction will act to reduce this relative motion.

$f_k = \mu N = 0.1 \times 10 \times 10 = 10$ N

100 + 10 = 10a

$a = \sqrt{5} = 11$ m/s^2

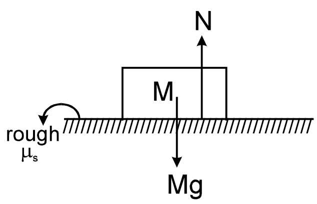


(b) STATIC FRICTION

It exists between the two surfaces when there is tendency of relative motion but no relative motion along the two contact surface.

For example consider a bed inside a room ; when we gently push the bed with a finger, the bed does not move. This means that the bed has a tendency to move in the direction of applied force but does not move as there exists static friction force acting in the opposite direction of the applied force.

Example 8. What is value of static friction force on the block?



Solution : In horizontal direction as acceleration is zero.

Therefore Σ F = 0.

$\therefore f = 0$

Direction of static friction force :

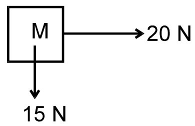
The static friction force on an object is opposite to its impending motion relative to the surface.

Following steps should be followed in determining the direction of static friction force on an object.

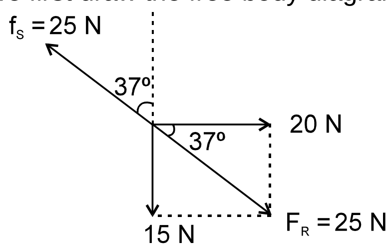
(i) Draw the free body diagram with respect to the other object on which it is kept.
(ii) Include pseudo force also if contact surface is accelerating.
(iii) Decide the resultant force and the component parallel to the surface of this resultant force.
(iv) The direction of static friction is opposite to the above component of resultant force.

Note : Here once again the static friction is involved when there is no relative motion between two surfaces.

Example 9. In the following figure an object of mass M is kept on a rough table as seen from above. Forces are applied on it as shown. Find the direction of static friction if the object does not move.



Solution : In the above problem we first draw the free body diagram of find the resultant force.



As the object doe not move this is not a case of kinetic friction. The direction of static friction is opposite to the direction of the resultant force F_R as shown in figure by f_s. Its magnitude is equal to 25 N.

4. MAGNITUDE OF KINETIC AND STATIC FRICTION

Kinetic friction :

The magnitude of the kinetic friction is proportional to the normal force acting between the two bodies. We can write

$$f_k = \mu_k N$$

where N is the normal force. **The proportionality constant μ_k is called the coefficient of kinetic friction and its value depends on the nature of the two surfaces in contact.** If the surfaces are smooth μ_k will be small, if the surfaces are rough μ_k will be large. It also depends on the materials of the two bodies in contact.

Static friction :

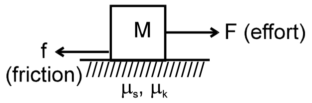
The magnitude of static friction is equal and opposite to the external force exerted, till the object at which force is exerted is at rest. **This means it is a variable and self adjusting force. However it has a maximum value called limiting friction.**

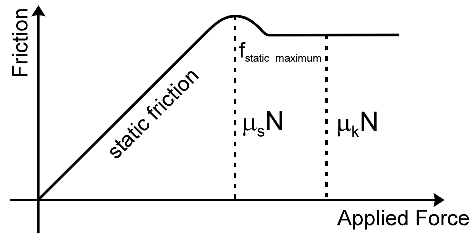
$$f_{max} = \mu_s N$$

The actual force of static friction may be smaller than $\mu_s N$ and its value depends on other forces acting on the body. The magnitude of frictional force is equal to that required to keep the body at relative rest.

$$0 \le f_s \le f_{smax}$$

Here μ_s and μ_k are proportionality constants. μ_s is called coefficient of static friction and μ_k is called coefficient of kinetic friction. They are dimensionless quantities independent of shape and area of contact. It is a property of the two contact surfaces. $\mu_s > \mu_k$ for a given pair of surfaces. If not mentioned then $\mu_s = \mu_k$ can be taken. Value of μ can be from 0 to ∞.

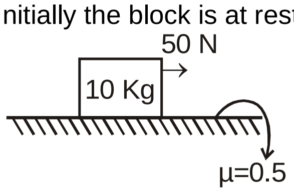




Following table gives a rough estimate of the values of coefficient of static friction between certain pairs of materials. The actual value depends on the degree of smoothness and other environmental factors. For example, wood may be prepared at various degrees of smoothness and the friction coefficient will vary.

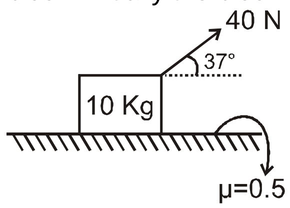
Material	μ_s	Material	μ_s
Steel and steel	0.58	Copper and copper	1.60
Steel and brass	0.35	Teflon and teflon	0.04
Glass and glass	1.00	Rubber tyre on dry concrete road	1.0
Wood and wood	0.35		
Wood and metal	0.40	Rubber tyre on wet concrete road	0.7

Example 10. Find acceleration of block. Initially the block is at rest.



Solution : zero

Example 11. Find out acceleration of the block. Initially the block is at rest.



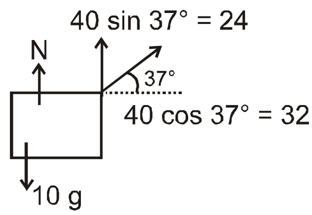
Solution : $N + 24 - 100 = 0$ for vertical direction

$\therefore\ N = 76$ N

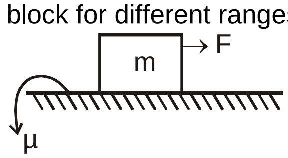
Now $0 \le f_s \le \mu_s N \Rightarrow 0 \le f_s \le 76 \times 0.5$

$0 \le f_s \le 38$ N $\quad \therefore\ 32 < 38$ Hence $f = 32$

$\therefore$ Acceleration of block is zero.



Example 12. Find out acceleration of the block for different ranges of F.

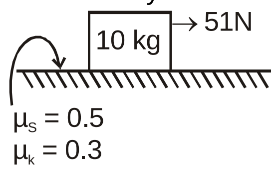


Solution : $0 \le f \le \mu_S N \quad \Rightarrow \quad 0 \le f \le \mu_S mg$

$a = 0$ if $F \le \mu_S mg$

$$a = \frac{F - \mu Mg}{M} \text{ if } F > \mu Mg$$

Example 13. Find out acceleration of the block. Initially the block is at rest.

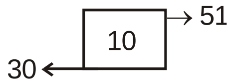


Solution : $0 \le f_s \le \mu_S N$

$0 \le f_s \le 50$

Now $51 > 50$

$\therefore$ Block will move but if the block starts moving then kinetic friction is involved.

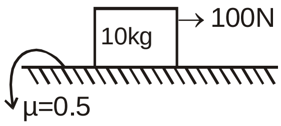


$K_F = \mu_k N = 0.3 \times 100 = 30$ N

$\therefore$ $51 - 30 = 10$ a

$\therefore$ a $= 2.1$ m/s^2

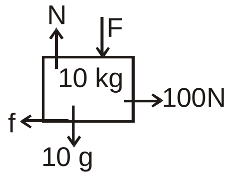
Example 14. Find out the minimum force that must be applied on the block vertically downwards so that the block doesn't move.



Solution : $100 - f_s = 0$

$\therefore$ $f_s = 100$ .....(1)

$F + 10\,g = N \Rightarrow N = 100 + F$ .....(2)



Now $0 \le f_S \le \mu N$

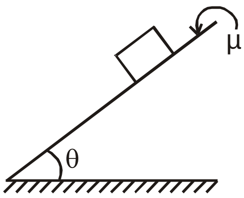
$100 \le 0.5$ N

$100 \le 0.5\ [100 + F]$

$200 \le 100 + F$; $F \ge 100$ N

$\therefore$ Minimum F = 100 N

Example 15. The angle of inclination is slowly increased. Find out the angle at which the block starts moving.



Solution : $0 \le f \le \mu_S N$

mg $\sin\theta > f_{smax}$

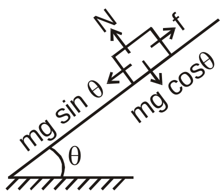
mg $\sin\theta > \mu N$

mg $\sin\theta > \mu$ mg $\cos\theta$

$\therefore \tan\theta > \mu$

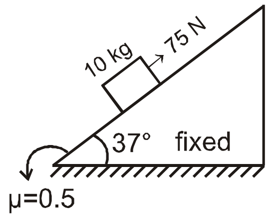
$\theta = \tan^{-1}\mu$

for $\tan\theta \le \mu$ no sliding on inclined plane.

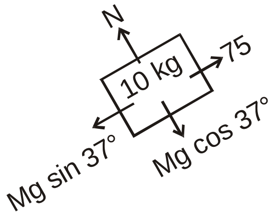


This method is used for finding out the value of μ practically.

Example 16. Find out the acceleration of the block. If the block is initially at rest.

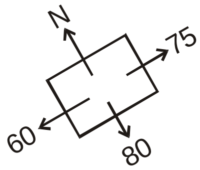


Solution : (FBD of the block excluding friction)



N = 10 g cos 37° = 80 N

Now $0 \le f_s \le \mu N$; $0 \le f_s \le 0.5 \times 80$ $\therefore$ $f_s \le 40$ N



We will put value of f in the last i.e. in the direction opposite to resultant of other forces. f acts down the incline and its value is of = 75 – 60 = 15 N

So acceleration is zero

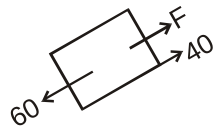
Example 17. In the above problem how much force should be added to 75 N force so that block starts to move up the incline.

Solution : $\therefore$ $60 + 40 = 75 + f_{extra}$ $\therefore$ $f_s = 25$ N

Example 18. In the above problem what is the minimum force by which 75 N force should be replaced with so that the block does not move.

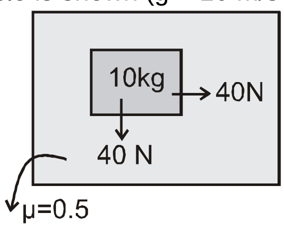
Solution : In this case the block has a tendency to move downwards.

Hence friction acts upwards.



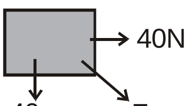
$\therefore$ F + 40 = 60 $\therefore$ F = 20 N

Example 19. Top view of a block on a table is shown (g = 10 m/s^2).



Find out the acceleration of the block.

Solution :

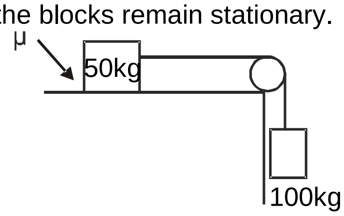


$F_R = 40\sqrt{2} \cong 56$N

Now $f_s \le \mu N$ $\therefore$ $f_s \le 50$; $F_R > f_{smax}$

Hence the block will move. $a = \dfrac{40\sqrt{2} - 50}{10} = \left(4\sqrt{2} - 5\right)$ m/s^2

Example 20. Find minimum μ so that the blocks remain stationary.



Solution : T = 100 g = 1000N

$\therefore$ f = 1000 to keep the block stationary

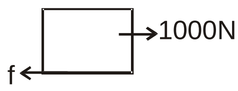
Now f_{max} = 1000N

$\mu N = 1000$

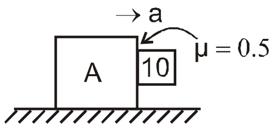
$\mu = 2$

Can μ be greater than 1 ?

Yes $0 < \mu \le \infty$



Example 21. Find out minimum acceleration of block A so that the 10 kg block doesn't fall.



Solution : Applying NL in horizontal direction

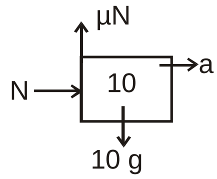
N = 10 a ......(1)

Applying NL in vertical direction

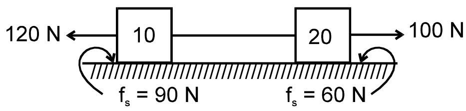
10 g = μ N ......(2)

10 g = μ 10 a from (1) & (2)

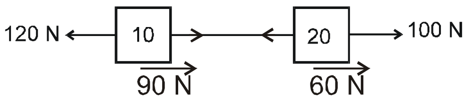
$\therefore\ a = \dfrac{g}{\mu} = 20\ m/s^2$



Example 22. Find the tension in the string in situation as shown in the figure below. Forces 120 N and 100 N start acting when the system is at rest and the maximum value of static friction on 10 kg is 90 N and that on 20 kg is 60N?

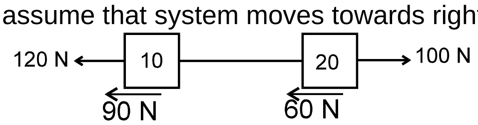


Solution : (i) Let us assume that system moves towards left then as it is clear from FBD, net force in horizontal direction is towards right. Therefore the assumption is not valid.



Above assumption is not possible as net force on system comes towards right. Hence system is not moving towards left.

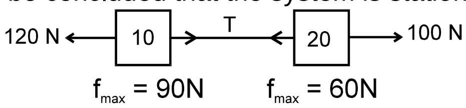
(ii) Similarly let us assume that system moves towards right.



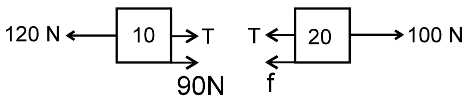
Above assumption is also not possible as net force on the system is towards left in this situation.

Hence assumption is again not valid.

Therefore it can be concluded that the system is stationary.



Assuming that the 10 kg block reaches limiting friction first then using FBD's.



120 = T + 90 $\Rightarrow$ T = 30 N

Also T + f = 100

$\therefore$ 30 + f = 100

f = 70 N which is not possible as the limiting value is 60 N for this surface of block.

$\therefore$ Our assumption is wrong and now taking the 20 kg surface to be limiting we have

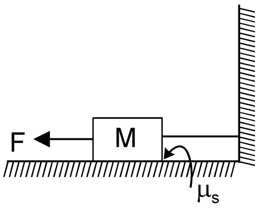
$T + 60 = 100$ N $\Rightarrow$ $T = 40$ N

Also $f + T = 120$ N $\Rightarrow$ $f = 80$ N

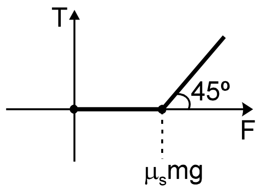
This is acceptable as static friction at this surface should be less than 90 N.

Hence the tension in the string is T = 40 N.

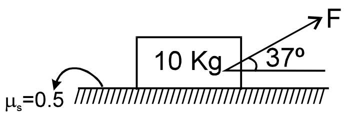
Example 23. In the following figure force F is gradually increased from zero. Draw the graph between applied force F and tension T in the string. The coefficient of static friction between the block and the ground is μ_s. {Initially string is horizontal & has zero tension.}



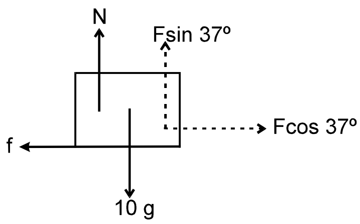
Solution : As the external force F is gradually increased from zero it is compensated by the friction and the string bears no tension. When limiting friction is achieved by increasing force F to a value till $\mu_s mg$, the further increase in F is transferred to the string.



Example 24. Force F is gradually increased from zero. Determine whether the block will first slide or lift up?



Solution : There are minimum magnitude of forces required both in horizontal and vertical direction either to slide on lift up the block. The block will first slide on lift up will depend upon which minimum magnitude of force is lesser.



For vertical direction to start lifting up

$F \sin 37^\circ + N - Mg \geq 0$.

N becomes zero just lifting condition.

$F_{lift} \geq \dfrac{10g}{3/5}$ $\quad\therefore\quad$ $F_{lift} \geq \dfrac{500}{3}$ N

For horizontal direction to start sliding $F \cos 37 \geq \mu_s N$

$F \cos 37^\circ > 0.5\,[10g - F \sin 37^\circ]$ $\;(\because\; N = 10\,g - F \sin 37^\circ)$

$$\text{Hence}F_{slide} > \frac{50}{\cos 37^\circ + 0.5 \sin 37^\circ}$$

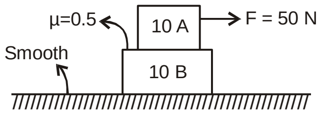
$$F_{slide} > \frac{500}{11}\,\text{N}$$

$F_{lift} > \dfrac{500}{3}$ N. $\quad\Rightarrow\quad F_{slide} < F_{lift}$

Therefore the block will begin to slide before lifting.

TWO BLOCK PROBLEMS

Example 25. Find the acceleration of the two blocks. The system is initially at rest and the friction coefficient are as shown in the figure?



Solution : **Method of solving**

Step 1 : Make force diagram.
Step 2 : Show static friction force by f because value of friction is not known.
Step 3 : Calculate separately for two cases.
Case 1 : Move together
Step 4 : Calculate acceleration.
Step 5 : Check value of friction for above case.
Step 6 : If required friction is less than available it means they will move together else move separately.
Step 7: (a) above acceleration will be common acceleration for both
Case 2 : Move separately
Step 7(b) If they move separately then kinetic friction is involved. Whose value is μN.
Step 8 : Calculate acceleration for above case.

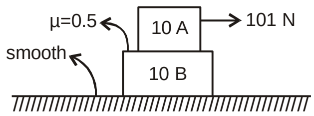
$f_{max} = \mu N$ $\therefore f \le 50$ N (available friction)

Move together | **Move separately**

(i) $a = \dfrac{50}{10+10} = 2.5$ m/s^2 No need to calculate

(ii) Check friction for B : f = 10 × 2.5 = 25
25 N is required which is less than available friction hence they will move together.
and $a_A = a_B = 2.5$ m/s^2

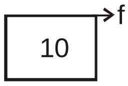
Example 26. Find the acceleration of the two blocks. The system is initially at rest and the friction coefficient are as shown in the figure?



Solution : $f_{max} = 50$ N $\therefore f \le 50$ N

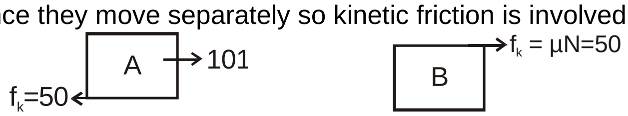
(i) If they move together $a = \dfrac{101}{20} = 5.05$ m/s^2

(ii) Check friction on B



f = 10 × 5.05 = 50.5 (required)
50.5 > 50 (therefore required > available). Hence they will not move together.

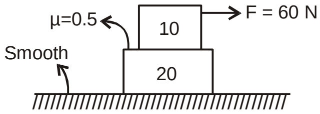
(iii) Hence they move separately so kinetic friction is involved.



$\therefore$ for $a_A = \dfrac{101-50}{10} = 5.1$ m/s^2 $\Rightarrow$ $a_B = \dfrac{50}{10} = 5$ m/s^2

Also $a_A > a_B$ as force is applied on A.

Example 27. Find the acceleration of the two blocks. The system is initially at rest and the friction coefficient are as shown in the figure?



Solution :

Move Together

$a = \dfrac{60}{30} = 2\ m/s^2$

Check friction on 20 kg.

f = 20 x 2

f = 40 (which is required)

40 < 50 (therefore required < available)

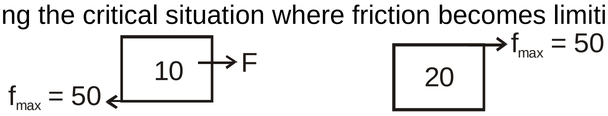
$\therefore$ will move together.

Move Separately

No need to calculate.

Example 28. In above example find maximum F for which two blocks will move together.

Solution : Observing the critical situation where friction becomes limiting.

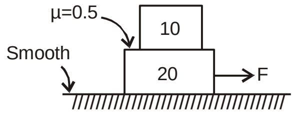


$\therefore\ F - f_{max} = 10\ a$ ......(1)

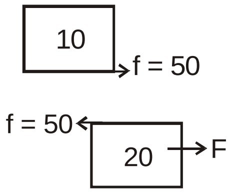
$f_{max} = 20\ a$ ......(2)

$\therefore\ F = 75\ N$

Example 29. Initially the system is at rest. find out minimum value of F for which sliding starts between the two blocks.



Solution : At just sliding condition limiting friction is acting.



$F - 50 = 20\ a$ ....(1)

$f = 10\ a$ ....(2)

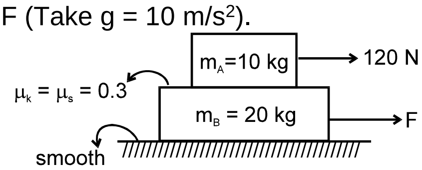
$50 = 10\ a$

$\therefore\ a = 5\ m/s^2$

hence $F = 50 + 20 \times 5 = 150\ N$

$\therefore\ F_{min} = 150\ N$

Example 30. In the figure given below force F applied horizontally on lower block, is gradually increased from zero. Discuss the direction and nature of friction force and the accelerations of the block for different values of F (Take g = 10 m/s^2).



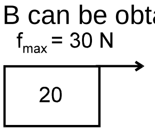
Solution : In the above situation we see that the maximum possible value of friction between the blocks is $\mu_s m_A g = 0.3 \times 10 \times 10 = 30\ N$.

Case (i) : When F = O.

Considering that there is no slipping between the blocks the acceleration of system will be

$a = \dfrac{120}{20+10} = 4\ m/s^2$

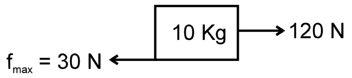
But the maximum acceleration of B can be obtained by the following force diagram.

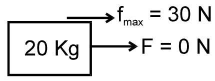


$a_B = \dfrac{30}{20} = 1.5\ m/s^2$ ($\because$ only friction force by block A is responsible for producing acceleration in block B)

Because 4 > 1.5 m/s^2 we can conclude that the blocks do not move together.

Now drawing the F.B.D. of each block, for finding out individual accelerations.

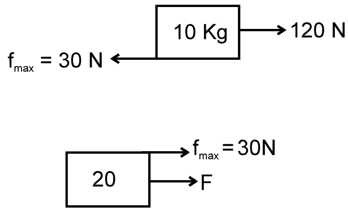




$a_A = \dfrac{120-30}{10} = 9$ m/s^2 towards right

$a_B = \dfrac{30}{20} = 1.5$ m/s^2 towards right.

Case (ii) F is increased from zero till the two blocks just start moving together.
As the two blocks move together the friction is static in nature and its value is limiting. FBD in this case will be

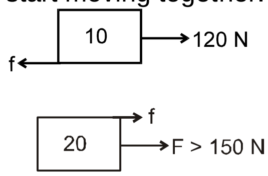


$a_A = \dfrac{120-30}{10} = 9$ m/s^2 $\Rightarrow$ $a_B = \dfrac{F+30}{20} = a_A$ $\Rightarrow$ $\dfrac{F+30}{20} = 9$

$\therefore$ F = 150 N

Hence when 0 < F < 150 N the blocks do not move together and the friction is kinetic. As F increases acceleration of block B increases from 1.5 m/s^2.
At F = 150 N limiting static friction start acting and the two blocks start moving together.

Case (iii) When F is increased above 150 N.
In this scenario the static friction adjusts itself so as to keep the blocks moving together. The value of static friction starts reducing but the direction still remains same. This happens continuously till the value of friction becomes zero. In this case the FBD is as follows



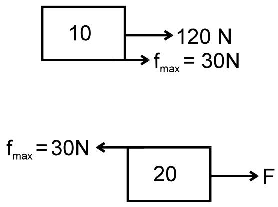
$a_A = a_B = \dfrac{120-f}{10} = \dfrac{F+f}{20}$

$\therefore$ when friction force f gets reduced to zero the above accelerations become

$a_A = \dfrac{120}{10} = 12$ m/s^2 $\Rightarrow$ $a_B = \dfrac{F}{20} = a_A = 12$ m/s^2 $\therefore$ F = 240 N

Hence when 150 ≤ F ≤ 240 N the static friction force continuously decreases from maximum to zero at F = 240 N. The accelerations of the blocks increase from 9 m/s^2 to 12 m/s^2 during the change of force F.

Case (iv) When F is increased again from 240 N the direction of friction force on the block reverses but it is still static. F can be increased till this reversed static friction reaches its limiting value. FBD at this juncture will be



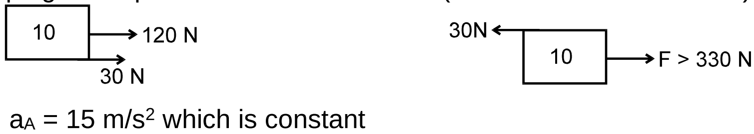
The blocks move together therefore.

$a_A = \dfrac{120+30}{10} = 15$ m/s^2

$\Rightarrow$ $a_B = \dfrac{F-30}{20} = a_A = 15$ m/s^2 $\therefore$ $\dfrac{F-30}{20} = 15$ m/s^2

Hence F = 330 N.

Case (v) When F is increased beyond 330 N. In this case the limiting friction is achieved and slipping takes place between the blocks (kinetic friction is involved).



$\therefore$ $a_A = 15$ m/s^2 which is constant

$a_B = \dfrac{F-30}{20}$ m/s^2 where F > 330 N.