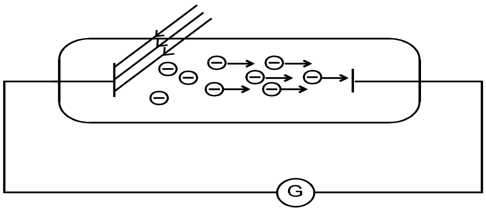
CHAPTER-31
MODERN PHYSICS -1

1 PHOTOELECTRIC EFFECT :

When electromagnetic radiations of suitable wavelength are incident on a metallic surface then electrons are emitted, this phenomenon is called photo electric effect.



1.1 Photoelectron : The electron emitted in photoelectric effect is called photoelectron.

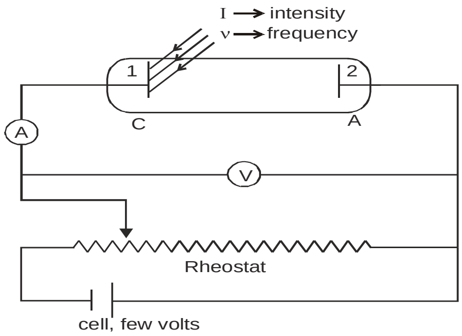
1.2 Photoelectric current : If current passes through the circuit in photoelectric effect then the current is called photoelectric current.

1.3 Work function : The minimum energy required to make an electron free from the metal is called work function. It is constant for a metal and denoted by ϕ or W. It is the minimum for Cesium. It is relatively less for alkali metals.

Work functions of some photosensitive metals

Metal	Work function (ev)	Metal	Work function (eV)
Cesium	1.9	Calcium	3.2
Potassium	2.2	Copper	4.5
Sodium	2.3	Silver	4.7
Lithium	2.5	Platinum	5.6

To produce photo electric effect only metal and light is necessary but for observing it, the circuit is completed. Figure shows an arrangement used to study the photoelectric effect.



Here the plate (1) is called emitter or cathode and other plate (2) is called collector or anode.

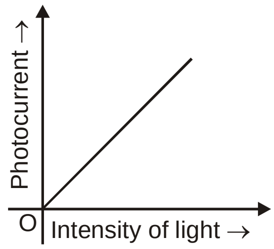
1.4 Saturation current : When all the photo electrons emitted by cathode reach the anode then current flowing in the circuit at that instant is known as saturation current, this is the maximum value of photoelectric current.

1.5 Stopping potential : Minimum magnitude of negative potential of anode with respect to cathode for which current is zero is called stopping potential. This is also known as cutoff voltage. This voltage is independent of intensity.

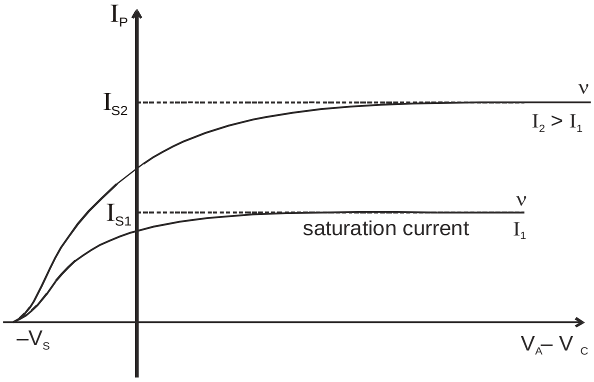
1.6 Retarding potential : Negative potential of anode with respect to cathode which is less than stopping potential is called retarding potential.

2. OBSERVATIONS : (MADE BY EINSTEIN)

2.1 A graph between intensity of light and photoelectric current is found to be a straight line as shown in figure. Photoelectric current is directly proportional to the intensity of incident radiation. In this experiment the frequency and retarding potential are kept constant.



2.2 A graph between photoelectric current and potential diffrence between cathode and anode is found as shown in figure.



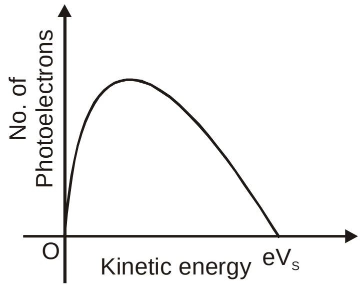
2.2 In case of saturation current,
rate of emission of photoelectrons = rate of flow of photoelectrons,
here, $v_s \rightarrow$ stopping potential and it is a positive quantity
Electrons emitted from surface of metal have different energies.
Maximum kinetic energy of photoelectron on the cathode = eV_s

$KE_{max} = eV_s$

Whenever photoelectric effect takes place, electrons are ejected out with kinetic energies ranging from

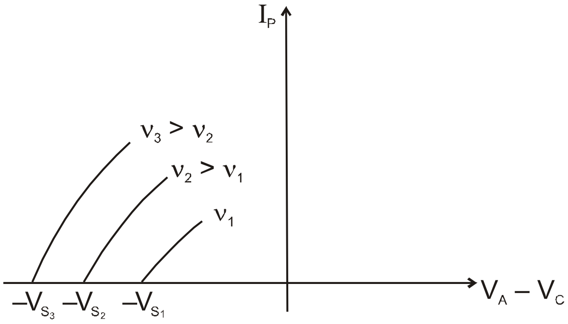
0 to $K.E_{max}$ i.e. $0 \leq KE_C \leq eV_s$

The energy distribution of photoelectron is shown in figure.



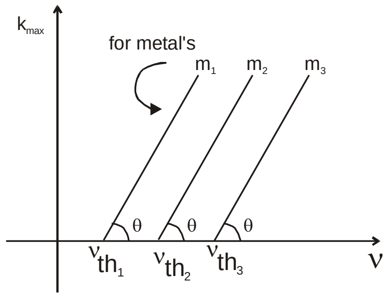
2.3 If intensity is increased (keeping the frequency constant) then saturation current is increased by same factor by which intensity increases. Stopping potential is same, so maximum value of kinetic energy is not effected.

2.4 If light of different frequencies is used then obtained plots are shown in figure.



It is clear from graph, as ν increases, stopping potential increases, it means maximum value of kinetic energy increases.

2.5 Graphs between maximum kinetic energy of electrons ejected from different metals and frequency of light used are found to be straight lines of same slope as shown in fiugre.



Graph between K_{max} and ν

m_1, m_2, m_3 : Three different metals.

It is clear from graph that there is a minimum frequency of electromagnetic radiation which can produce photoelectric effect, which is called **threshold frequency**.

ν_{th} = Threshold frequency

For photoelectric effect $\nu \geq \nu_{th}$

for no photoelectric effect $\nu < \nu_{th}$

Minimum frequency for photoelectric effect = ν_{th}

$\nu_{min} = \nu_{th}$

Threshold wavelength (λ_{th}) $\rightarrow$ The maximum wavelength of radiation which can produce photoelectric effect.

$\lambda \leq \lambda_{th}$ for photo electric effect

Maximum wavelength for photoelectric effect $\lambda_{max} = \lambda_{th}$.

Now writing equation of straight line from graph.

We have $K_{max} = A\nu + B$

When $\nu = \nu_{th}$, $K_{max} = 0$ and $B = -A\nu_{th}$

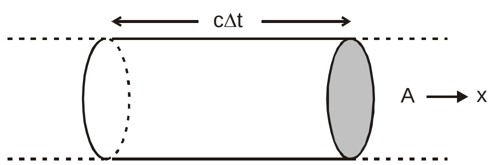
Hence **$[K_{max} = A(\nu - \nu_{th})]$**

and $A = \tan\theta = 6.63 \times 10^{-34}$ J-s (from experimental data)

later on 'A' was found to be 'h'.

2.6 It is also observed that photoelectric effect is an instantaneous process. When light falls on surface electrons start ejecting without taking any time.

3. THREE MAJOR FEATURES OF THE PHOTOELECTRIC EFFECT CANNOT BE EXPLAINED IN TERMS OF THE CLASSICAL WAVE THEORY OF LIGHT.



Intensity : The energy crossing per unit area per unit time perpendicular to the direction of propagation is called the intensity of a wave. Consider a cylindrical volume with area of crosssection A and length c Δt along the X-axis. The energy contained in this cylinder crosses the area A in time Δt as the wave propagates at speed c. The energy contained.

$U = u_{av}(c.\ \Delta t)A$

The intensity is $I = \dfrac{U}{A\Delta t} = u_{av}\,c$.

In the terms of maximum electric field, $I = \dfrac{1}{2}\varepsilon_0 E_0^2\, c$.

If we consider light as a wave then the intensity depends upon electric field.

If we take work function $W = I\ .\ A\ .\ t$, then $t = \dfrac{W}{IA}$

So for photoelectric effect there should be time lag because the metal has work function.

But it is observed that photoelectric effect is an instantaneous process.
Hence, light is not of wave nature.

3.1 **The intensity problem :** Wave theory requires that the oscillating electric field vector **E** of the light wave increases in amplitude as the intensity of the light beam is increased. Since the force applied to the electron is e**E**, this suggests that the kinetic energy of the photoelectrons should also increased as the light beam is made more intense. However observation shows that maximum kinetic energy is independent of the light intensity.

3.2 **The frequency problem :** According to the wave theory, the photoelectric effect should occur for any frequency of the light, provided only that the light is intense enough to supply the energy needed to eject the photoelectrons. However observations shows that there exists for each surface a characterstic cutoff frequency ν_{th}, for frequencies less than ν_{th}, the photoelectric effect does not occur, no matter how intense is light beam.

3.3 **The time delay problem :** If the energy acquired by a photoelectron is absorbed directly from the wave incident on the metal plate, the "effective target area" for an electron in the metal is limited and probably not much more than that of a circle of diameter roughly equal to that of an atom. In the classical theory, the light energy is uniformly distributed over the wavefront. Thus, if the light is feeble enough, there should be a measurable time lag, between the impinging of the light on the surface and the ejection of the photoelectron. During this interval the electron should be absorbing energy from the beam until it had accumulated enough to escape. However, no detectable time lag has ever been measured.

Now, quantum theory solves these problems in providing the correct interpretation of the photoelectric effect.

4 PLANCK'S QUANTUM THEORY :

The light energy from any source is always an integral multiple of a smaller energy value called quantum of light.hence energy $Q = NE$,
where $E = h\nu$ and N (number of photons) = 1,2,3,....
Here energy is quantized. $h\nu$ is the quantum of energy, it is a packet of energy called as **photon**.

$$E = h\nu = \frac{hc}{\lambda} \quad \text{and} \quad hc = 12400 \text{ eV Å}$$

5. EINSTEIN'S PHOTON THEORY

In 1905 Einstein made a remarkable assumption about the nature of light; namely, that, under some circumstances, it behaves as if its energy is concentrated into localized bundles, later called photons. The energy E of a single photon is given by

$$E = h\nu,$$

If we apply Einstein's photon concept to the photoelectric effect, we can write

$$h\nu = W + K_{max}, \qquad \text{(energy conservation)}$$

Equation says that a single photon carries an energy $h\nu$ into the surface where it is absorbed by a single electron. Part of this energy W (called the work function of the emitting surface) is used in causing the electron to escape from the metal surface. The excess energy ($h\nu - W$) becomes the electron's kinetic energy; if the electron does not lose energy by internal collisions as it escapes from the metal, it will still have this much kinetic energy after it emerges. Thus K_{max} represents the maximum kinetic energy that the photoelectron can have outside the surface. There is complete agreement of the photon theory with experiment.

Now $IA = Nh\nu \Rightarrow N = \frac{IA}{h\nu}$ = no. of photons incident per unit time on an area 'A' when light of intensity 'I' is incident normally.

If we double the light intensity, we double the number of photons and thus double the photoelectric current; we do not change the energy of the individual photons or the nature of the individual photoelectric processes.

The second objection (the frequency problem) is met if K_{max} equals zero, we have

$h\nu_{th} = W$,

Which asserts that the photon has just enough energy to eject the photoelectrons and none extra to appear as kinetic energy. If ν is reduced below ν_{th}, $h\nu$ will be smaller than W and the individual photons, no matter how many of them there are (that is, no matter how intense the illumination), will not have enough energy to eject photoelectrons.

The third objection (the time delay problem) follows from the photon theory because the required energy is supplied in a concentrated bundle. It is not spread uniformly over the beam cross section as in the wave theory.

Hence Einstein's equation for photoelectric effect is given by

$$h\nu = h\nu_{th} + K_{max} \qquad K_{max} = \frac{hc}{\lambda} - \frac{hc}{\lambda_{th}}$$

Example 1. In an experiment on photo electric emission, following observations were made;

(i) Wavelength of the incident light = 1.98×10^{-7} m;

(ii) Stopping potential = 2.5 volt.

Find : (a) Kinetic energy of photoelectrons with maximum speed.

(b) Work function and

(c) Threshold frequency;

Solution : (a) Since $v_s = 2.5$ V, $K_{max} = eV_s$ so, $K_{max} = 2.5$ eV

(b) Energy of incident photon

$$E = \frac{12400}{1980} \text{eV} = 6.26 \text{ eV} \qquad W = E - K_{max} = 3.76 \text{ eV}$$

(c) $h\nu_{th} = W = 3.76 \times 1.6 \times 10^{-19}$ J $\quad \therefore \quad \nu_{th} = \dfrac{3.76 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} \approx 9.1 \times 10^{14}$ Hz

Example 2. A beam of light consists of four wavelength 4000 Å, 4800 Å, 6000 Å and 7000 Å, each of intensity 1.5×10^{-3} Wm^{-2}. The beam falls normally on an area 10^{-4} m^2 of a clean metallic surface of work function 1.9 eV. Assuming no loss of light energy (i.e. each capable photon emits one electron) calculate the number of photoelectrons liberated per second.

Solution : $E_1 = \dfrac{12400}{4000} = 3.1$ eV, $\quad E_2 = \dfrac{12400}{4800} = 2.58$ eV $\qquad E_3 = \dfrac{12400}{6000} = 2.06$ eV

and $E_4 = \dfrac{12400}{7000} = 1.77$ eV

Therefore, light of wavelengths 4000 Å, 4800 Å and 6000 Å can only emit photoelectrons.

$\therefore$ Number of photoelectrons emitted per second = No. of photons incident per second)

$$= \frac{I_1A_1}{E_1} + \frac{I_2A_2}{E_2} + \frac{I_3A_3}{E_3} = IA\left(\frac{1}{E_1} + \frac{1}{E_2} + \frac{1}{E_3}\right)$$

$$= \frac{(1.5 \times 10^{-3})(10^{-4})}{1.6 \times 10^{-19}}\left(\frac{1}{3.1} + \frac{1}{2.58} + \frac{1}{2.06}\right) = 1.12 \times 10^{12} \quad \textbf{Ans.}$$

Example 3. A small potassium foil is placed (perpendicular to the direciton of incidence of light) a distance r (= 0.5 m) from a point light source whose output power P_0 is 1.0W. Assuming wave nature of light how long would it take for the foil to soak up enough energy (= 1.8 eV) from the beam to eject an electron? Assume that the ejected photoelectron collected its energy from a circular area of the foil whose radius equals the radius of a potassium atom (1.3×10^{-10} m).

Solution : If the source radiates uniformly in all directions, the intensity I of the light at a distance r is given by

$I = \frac{P_0}{4\pi r^2} = \frac{1.0 \quad W}{4\pi(0.5 \quad m)^2}$ = 0.32 W/m^2.

The target area A is $\pi(1.3 \times 10^{-10}$ m$)^2$ or 5.3×10^{-20} m^2, so that the rate at which energy falls on the target is given by

P = IA = (0.32 W/m^2) (5.3×10^{-20} m^2)

= 1.7×10^{-20} J/s.

If all this incoming energy is absorbed, the time required to accumulate enough energy for the electron to escape is

$$t = \left(\frac{1.8 \quad eV}{1.7 \times 10^{-20} J/s}\right)\left(\frac{1.6 \quad \times 10^{-19} J}{1 \quad eV}\right) = 17 \text{ s}.$$

Our selection of a radius for the effective target area was some-what arbitrary, but no matter what reasonable area we choose, we should still calculate a “soak-up time” within the range of easy measurement. However, no time delay has ever been observed under any circumstances, the early experiments setting an upper limit of about 10^{-9} s for such delays.

Example 4. A metallic surface is irradiated with monochromatic light of variable wavelength. Above a wavelength of 5000 Å, no photoelectrons are emitted from the surface. With an unknown wavelength, stopping potential is 3 V. Find the unknown wavelength.

Solution : Using equation of photoelectric effect

$K_{max} = E - W \quad (K_{max} = eV_s)$

$$\therefore \quad 3 \text{ eV} = \frac{12400}{\lambda} - \frac{12400}{5000} = -2.48 \text{ eV} \qquad \text{or} \qquad \lambda = 2262 \text{ Å}$$

Example 5. Illuminating the surface of a certain metal alternately with light of wavelengths $\lambda_1 = 0.35$ μm and $\lambda_2 = 0.54$ μm, it was found that the corresponding maximum velocities of photo electrons have a ratio $\eta = 2$. Find the work function of that metal.

Solution : Using equation for two wavelengths

$$\frac{1}{2}mv_1^2 = \frac{hc}{\lambda_1} - W \qquad \text{....(i)}$$

$$\frac{1}{2}mv_2^2 = \frac{hc}{\lambda_2} - W \qquad \text{....(ii)}$$

Dividing Eq. (i) with Eq. (ii), with $v_1 = 2v_2$, we have $4 = \dfrac{\frac{hc}{\lambda_1} - W}{\frac{hc}{\lambda_2} - W}$

$$3W = 4\left(\frac{hc}{\lambda_2}\right) - \left(\frac{hc}{\lambda_1}\right) = \frac{4 \times 12400}{5400} - \frac{12400}{3500} = 5.64 \text{ eV}$$

$$W = \frac{5.64}{3} \text{ eV} = 1.88 \text{ eV}$$ **Ans.**

Example 6. A photocell is operating in saturation mode with a photocurrent 4.8 μA when a monochromatic radiation of wavelength 3000 Å and power 1 mW is incident. When another monochromatic radiation of wavelength 1650 Å and power 5 mW is incident, it is observed that maximum velocity of photoelectron increases to two times. Assuming efficiency of photoelectron generation per incident to be same for both the cases, calculate,

(a) threshold wavelength for the cell (b) efficiency of photoelectron generation.
[(No. of photoelectrons emitted per incident photon) × 100]

(c) saturation current in second case

Solution : (a) $K_1 = \frac{12400}{3000} - W = 4.13 - W$ .....(i)

$K_2 = \frac{12400}{1650} - W = 7.51 - W$ .....(ii)

Since $v_2 = 2v_1$ so, $K_2 = 4K_1$ .....(iiii)

Solving above equations, we get

$W = 3$ eV

$\therefore$ Threshold wavelegth $\lambda_0 = \frac{12400}{3} = 4133$ Å **Ans.**

(b) Energy of a photon in first case = $\frac{12400}{3000} = 4.13$ eV

or $E_1 = 6.6 \times 10^{-19}$ J

Rate of incident photons (number of photons per second)

$\frac{P_1}{E_1} = \frac{10^{-3}}{6.6 \times 10^{-19}} = \ = 1.5 \times 10^{15}$ per second

Number of electrons ejected = $\frac{4.8 \times 10^{-6}}{1.6 \times 10^{-19}}$ per second = 3.0×10^{13} per second

$\therefore$ Efficiency of photoelectron generation

$(\eta) = \frac{3.0 \times 10^{13}}{1.5 \times 10^{15}} \times 100 = 2\%$ **Ans.**

(c) Energy of photon in second case

$E_2 = \frac{12400}{1650} = 7.51$ eV $= 12 \times 10^{-19}$ J

Therefore, number of photons incident per second

$n_2 = \frac{P_2}{E_2} = \frac{5.0 \times 10^{-3}}{12 \times 10^{-19}} = 4.17 \times 10^{15}$ per second

Number of electrons emitted per second = $\frac{2}{100} \times 4.7 \times 10^{15} = 9.4 \times 10^{13}$ per second

$\therefore$ Saturation current in second case i = (9.4×10^{13}) (1.6×10^{-19}) amp = 15 μA **Ans.**

Example 7 Light described at a place by the equation $E = (100 \text{ V/m}) [\sin (5 \times 10^{15} \text{ s}^{-1}) t + \sin (8 \times 10^{15} \text{ s}^{-1})t]$ falls on a metal surface having work function 2.0 eV. Calculate the maximum kinetic energy of the photoelectrons.

Solution : The light contains two different frequencies. The one with larger frequency will cause photoelectrons with largest kinetic energy. This larger frequency is

$$\nu = \frac{\omega}{2\pi} = \frac{8 \times 10^{15} \text{ s}^{-1}}{2\pi}$$

The maximum kinetic energy of the photoelectrons is

$K_{max} = h\upsilon - W$

$$= (4.14 \times 10^{-15} \text{ eV-s}) \times \left(\frac{8 \times 10^{15}}{2\pi} \text{s}^{-1}\right) - 2.0 \text{ eV}$$

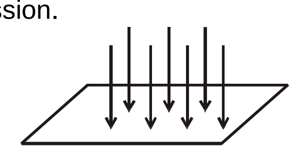
$= 5.27$ eV $- 2.0$ eV $= 3.27$ eV.

6 FORCE DUE TO RADIATION (PHOTON)

Each photon has a definite energy and a definite linear momentum. All photons of light of a particular wavelength λ have the same energy $E = hc/\lambda$ and the same magnitude of momentum $p = h/\lambda$.

When light of intensity I falls on a surface, it exerts force on that surface. Assume absorption and reflection coefficient of surface be 'a' and 'r' and assuming no transmission.

Assume light beam falls on surface of surface area 'A' perpendicularly as shown in figure.



For calculating the force exerted by beam on surface, we consider following cases.

Case : (I) a = 1, r = 0

initial momentum of the photon = $\frac{h}{\lambda}$

final momentum of photon = 0

change in momentum of photon = $\frac{h}{\lambda}$ (upward)

$\Delta P = \frac{h}{\lambda}$

energy incident per unit time = IA

no. of photons incident per unit time $= \frac{IA}{h\nu} = \frac{IA\lambda}{hc}$

$\therefore$ total change in momentum per unit time $= n\,\Delta P$

$$= \frac{IA\lambda}{hc} \times \frac{h}{\lambda} = \frac{IA}{c} \text{ (upward)}$$

force on photons = total change in momentum per unit time = $\frac{IA}{c}$ (upward)

$\therefore$ force on plate due to photons(F) = $\frac{IA}{c}$ (downward)

pressure = $\frac{F}{A} = \frac{IA}{cA} = \frac{I}{c}$

Case : (II)

when r = 1, a = 0

intial momentum of the photon $= \frac{h}{\lambda}$ (downward)

final momentum of photon $= \frac{h}{\lambda}$ (upward)

change in momentum $= \frac{h}{\lambda} + \frac{h}{\lambda} = \frac{2h}{\lambda}$

$\therefore$ energy incident per unit time = IA

no. of photons incident per unit time = $\frac{IA\lambda}{hc}$

$\therefore$ total change in momentum per unit time = n . $\Delta P = \frac{IA\lambda}{hc} \cdot \frac{2h}{\lambda} = \frac{2IA}{C}$

force = total change in momentum per unit time

$F = \frac{2IA}{c}$ (upward on photons and downward on the plate)

pressure $P = \frac{F}{A} = \frac{2IA}{cA} = \frac{2I}{c}$

Case : (III)

When o < r < 1 a + r = 1

change in momentum of photon when it is reflected = $\frac{2h}{\lambda}$ (upward)

change in momentum of photon when it is absorbed = $\frac{h}{\lambda}$ (upward)

no. of photons incident per unit time = $\frac{IA\lambda}{hc}$

No. of photons reflected per unit time = . $\frac{IA\lambda}{hc}$ r

No. of photon absorbed per unit time = $\dfrac{IA\lambda}{hc}$ (1 – r)

force due to absorbed photon (F_a) = $\dfrac{IA\lambda}{hc}$ (1 – r) . $\dfrac{h}{\lambda} = \dfrac{IA}{c}$ (1 – r) (downward)

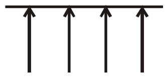
Force due to reflected photon (F_r) = $\dfrac{IA\lambda}{hc}$ .r $\dfrac{2h}{\lambda}$ = $\dfrac{2IA\lambda}{c}$ (downward)

total force = $F_a + F_r$ (downward)

$$= \frac{IA}{c}(1-r) + \frac{2IAr}{c} = \frac{IA}{c}(1+r)$$

Now pressure P = $\dfrac{IA}{c}(1+r) \times \dfrac{1}{A} = \dfrac{I}{c}(1+r)$

Example 8. A plate of mass 10 gm is in equilibrium in air due to the force exerted by light beam on plate. Calculate power of beam. Assume plate is perfectly absorbing.



Solution : Since plate is in air, so gravitational force will act on this

$F_{gravitational} = mg$ (downward)

$= 10 \times 10^{-3} \times 10 = 10^{-1}$ N

for equilibrium force exerted by light beam should be equal to $F_{gravitational}$

$F_{photon} = F_{gravitational}$

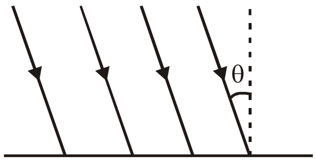
Let power of light beam be P

$\therefore$ $F_{photon} = \dfrac{P}{c}$

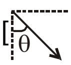
$\therefore$ $\dfrac{P}{c} = 10^{-1}$ P = $3.0 \times 10^{8} \times 10^{-1}$

P = 3×10^{7} W

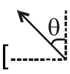
Example 9 Calculate force exerted by light beam if light is incident on surface at an angle θ as shown in figure. Consider all cases.



Solution : **Case - I** a = 1, r = 0

initial momentum of photon(in downward direction at an angle θ with vertical) = $\dfrac{h}{\lambda}$ []

final momentum of photon = 0

change in momentum (in upward direction at an angle θ with vertical) = $\dfrac{h}{\lambda}$ []

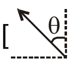
energy incident per unit time = IA cos θ

Intensity = power per unit normal area

$I = \dfrac{P}{A\cos\theta}$ P = IA cos θ

No. of photons incident per unit time = $\dfrac{IA\cos\theta}{hc}.\lambda$

total change in momentum per unit time (in upward direction at an angle θ with vertical)

$= \dfrac{IA\cos\theta.\lambda}{hc} . \dfrac{h}{\lambda} = \dfrac{IA\cos\theta}{c}$ []

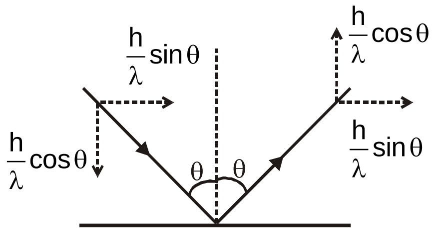
Force (F) = total change in momentum per unit time

F = $\dfrac{IA\cos\theta}{c}$ (direction on photon and on the plate)

Pressure = normal force per unit Area

Pressure = $\dfrac{F\cos\theta}{A}$ P = $\dfrac{IA\cos^2\theta}{cA} = \dfrac{I}{c}\cos^2\theta$

Case II : When r = 1, a = 0



$\therefore$ change in momentum of one photon

$= \dfrac{2h}{\lambda}\cos\theta$ (upward)

No. of photons incident per unit time

$= \dfrac{\text{energy incident per unit time}}{h\nu}$

$= \dfrac{IA\cos\theta\cdot\lambda}{hc}$

$\therefore$ total change in momentum per unit time = $\dfrac{IA\cos\theta\cdot\lambda}{hc}\times\dfrac{2h}{\lambda}\cos\theta = \dfrac{2IA\cos^2\theta}{c}$ (upward)

$\therefore$ force on the plate = $\dfrac{2IA\cos^2\theta}{c}$ (downward)

Pressure = $\dfrac{2IA\cos^2\theta}{cA}$ P = $\dfrac{2I\cos^2\theta}{c}$

Case III : $0 < r < 1$, $a + r = 1$

change in momentum of photon when it is reflected = $\dfrac{2h}{\lambda}\cos\theta$ (downward)

change in momentum of photon when it is absorbed = $\dfrac{h}{\lambda}$ (in the opposite direction of incident beam)

energy incident per unit time = IA cos θ

no. of photons incident per unit time = $\dfrac{IA\cos\theta\cdot\lambda}{hc}$

no. of reflected photon (n_r) = $\dfrac{IA\cos\theta.\lambda r}{hc}$

no. of absorbed photon (n_a) = $\dfrac{IA\cos\theta.\lambda}{hc}(1-r)$

force on plate due to absorbed photons $F_a = n_a \,.\, \Delta P_a$

$= \dfrac{IA\cos\theta.\lambda}{hc}(1-r)\dfrac{h}{\lambda}$

$= \dfrac{IA\cos\theta}{c}(1-r)$ (at an angle θ with vertical)

force on plate due to reflected photons $F_r = n_r\,\Delta P_r$

$= \dfrac{IA\cos\theta\cdot\lambda}{hc}\times\dfrac{2h}{\lambda}\cos\theta$ (vertically downward)

$= \dfrac{IA\cos^2\theta}{c}\cdot 2r$

now resultant force is given by $F_R = \sqrt{F_r^2 + F_a^2 + 2F_aF_r\cos\theta}$

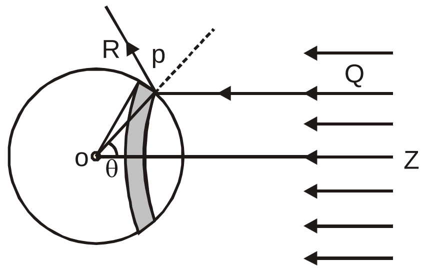
$= \dfrac{IA\cos\theta}{c}\sqrt{(1-r)^2+(2r)^2\cos^2\theta+4r(r-1)\cos^2\theta}$

and, pressure P = $\dfrac{F_a\cos\theta+F_r}{A} = \dfrac{IA\cos\theta(1-r)\cos\theta}{cA} + \dfrac{IA\cos^2\theta\cdot 2r}{cA}$

$= \dfrac{I\cos^2\theta}{c}(1-r) + \dfrac{I\cos^2\theta}{c}2r = \dfrac{I\cos^2\theta}{c}(1+r)$

Example 10. A perfectly reflecting solid sphere of radius r is kept in the path of a parallel beam of light of large aperture. If the beam carries an intensity I, find the force exerted by the beam on the sphere.

Solution : Let O be the centre of the sphere and OZ be the line opposite to the incident beam (figure). Consider a radius OP of the sphere making an angle θ with OZ. Rotate this radius about OZ to get a circle on the sphere. Change θ to $\theta + d\theta$ and rotate the radius about OZ to get another circle on the sphere. The part of the sphere between these circles is a ring of area $2\pi r^2 \sin\theta\, d\theta$. Consider a small part ΔA of this ring at P.



Energy of the light falling on this part in time Δt is

$\Delta U = I\Delta t(\Delta A\cos\theta)$

The momentum of this light falling on ΔA is $\Delta U/c$ along QP. The light is reflected by the sphere along PR. The change in momentum is

$$\Delta p = 2\frac{\Delta U}{c}\cos\theta = \frac{2}{c} I\,\Delta t\,(\Delta A\cos^2\theta) \qquad \text{(direction along } \overrightarrow{OP}\text{)}$$

The force on ΔA due to the light faling on it, is

$$\frac{\Delta p}{\Delta t} = \frac{2}{c} I\,\Delta A\cos^2\theta. \qquad \text{(direction along } \overrightarrow{PO}\text{)}$$

The resultant force on the ring as well as on the sphere is along ZO by symmetry. The component of the force on ΔA along ZO

$$\frac{\Delta p}{\Delta t}\cos\theta = \frac{2}{c} I\Delta A\cos^3\theta. \qquad \text{(along } \overrightarrow{ZO}\text{)}$$

The force acting on the ring is $dF = \dfrac{2}{c} I(2\pi r^2\sin\theta\, d\theta)\cos^3\theta$.

The force on the entire sphere is $F = \displaystyle\int_0^{\pi/2} \frac{4\pi r^2 I}{c}\cos^3\theta\sin\theta\, d\theta$

$$= -\int_0^{\pi/2}\frac{4\pi r^2 I}{c}\cos^3\theta\, d(\cos\theta) = -\int_{\theta=0}^{\pi/2}\frac{4\pi r^2 I}{c}\left[\frac{\cos^4\theta}{4}\right]_0^{\pi/2} = \frac{\pi r^2 I}{c}$$

Note that integration is done only for the hemisphere that faces the incident beam.

7. De-BROGLIE WAVELENGTH OF MATTER WAVE

A photon of frequency ν and wavelength λ has energy.

$$E = h\nu = \frac{hc}{\lambda}$$

By Einstein's energy mass relation, $E = mc^2$ the equivalent mass m of the photon is given by,

$$m = \frac{E}{c^2} = \frac{h\nu}{c^2} = \frac{h}{\lambda c} \quad .....(i)$$

or $\lambda = \frac{h}{mc}$ or $\lambda = \frac{h}{p}$ .....(ii)

Here p is the momentum of photon. By analogy de-Broglie suggested that a particle of mass m moving with speed v behaves in some ways like waves of wavelength λ (called de-Broglie wavelength and the wave is called matter wave) given by,

$$\lambda = \frac{h}{mv} = \frac{h}{p} \quad .... (iii)$$

where p is the momentum of the particle. Momentum is related to the kinetic energy by the equation,

$$p = \sqrt{2Km}$$

and a charge q when accelerated by a potential difference V gains a kinetic energy K = qV. Combining all these relations Eq. (iii), can be written as,

$$\lambda = \frac{h}{mv} = \frac{h}{p} = \frac{h}{\sqrt{2Km}} = \frac{h}{\sqrt{2qVm}} \text{ (de-Broglie wavelength)} \quad ....(iv)$$

7.1 de-Broglie wavelength for an electron

If an electron (charge = e) is accelerated by a potential of V volts, it acquires a kinetic energy,

$$K = eV$$

Substituting the values of h, m and q in Eq. (iv), we get a simple formula for calculating de-Broglie wavelength of an electron.

$$\lambda(\text{in Å}) = \sqrt{\frac{150}{V(\text{in volts})}} \quad ....(v)$$

7.2 de-Broglie wavelength of a gas molecule :

Let us consider a gas molecule at absolute temperature T. Kinetic energy of gas molecule is given by

$$K.E. = \frac{3}{2}kT \text{ ;} \qquad k = \text{Boltzman constant}$$

$$\therefore \qquad \lambda_{\text{gas molecule}} = \frac{h}{\sqrt{3mkT}}$$

Example 11. An electron is accelerated by a potential difference of 50 volt. Find the de-Broglie wavelength associated with it.

Solution : For an electron, de-Broglie wavelength is given by, $\lambda = \sqrt{\frac{150}{V}} = \sqrt{\frac{150}{50}} = \sqrt{3}$

= 1.73 Å **Ans.**

Example 12. Find the ratio of De-Broglie wavelength of molecules of hydrogen and helium which are at temperatures 27ºC and 127ºC respectively.

Solution : de-Broglie wavelength is given by

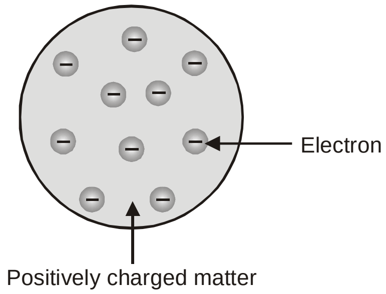
$$\therefore \quad \frac{\lambda_{H_2}}{\lambda_{He}} = \sqrt{\frac{m_{He}T_{He}}{m_{H_2}T_{H_2}}} = \sqrt{\frac{4}{2}.\frac{(127+273)}{(27+273)}} = \sqrt{\frac{8}{3}}$$

8. THOMSON'S ATOMIC MODEL :

J.J. Thomson suggested that atoms are just positively charge lumps of matter with electrons embedded in them like raisins in a fruit cake. Thomson's model called the 'plum pudding' model is illustrated in figure.

Thomson played an important role in discovering the electron, through gas discharge tube by discovering cathode rays. His idea was taken seriously.

But the real atom turned out to be quite different.



9. RUTHERFORD'S NUCLEAR ATOM :

Rutherford suggested that; " All the positive charge and nearly all the mass were concentrated in a very small volume of nucleus at the centre of the atom. The electrons were supposed to move in circular orbits round the nucleus (like planets round the sun). The electronstatic attraction between the two opposite charges being the required centripetal force for such motion.

Hence $\frac{mv^2}{r} = \frac{kZe^2}{r^2}$

and total energy = potential energy + kinetic energy = $\frac{-kZe^2}{2r}$

Rutherford's model of the atom, although strongly supported by evidence for the nucleus, is inconsistent with classical physics. This model suffer's from two defects

9.1 Regarding stability of atom : An electron moving in a circular orbit round a nucleus is accelerating and according to electromagnetic theory it should therefore, emit radiation continuously and thereby lose energy. If total energy decreases then radius increases as given by above formula. If this happened the radius of the orbit would decrease and the electron would spiral into the nucleus in a fraction of second. But atoms do not collapse. In 1913 an effort was made by Neil Bohr to overcome this paradox.

9.2 Regarding explanation of line spectrum : In Rutherford's model, due to continuously changing radii of the circular orbits of electrons, the frequency of revolution of the electrons must be changing. As a result, electrons will radiate electromagnetic waves of all frequencies, i.e., the spectrum of these waves will be 'continuous' in nature. But experimentally the atomic spectra are not continuous. Instead they are line spectra.

10. THE BOHR'S ATOMIC MODEL

In 1913, Prof. Niel Bohr removed the difficulties of Rutherford's atomic model by the application of Planck's quantum theory. For this he proposed the following postulates

(1) An electron moves only in certain circular orbits, called stationary orbits. In stationary orbits electron does not emit radiation, contrary to the predictions of classical electromagnetic theory.

(2) According to Bohr, there is a definite energy associated with each stable orbit and an atom radiaties energy only when it makes a transition from one of these orbits to another. If the energy of electron in the higher orbit be E_2 and that in the lower orbit be E_1, then the frequency ν of the radiated waves is given by

$$h\nu = E_2 - E_1 \quad \text{or} \quad \nu = \frac{E_2 - E_1}{h} \qquad \text{...(i)}$$

(3) Bohr found that the magnitude of the electron's angular momentum is quantized, and this magnitude for the electron must be integral multiple of $\frac{h}{2\pi}$. The magnitude of the angular momentum is L = mvr for a particle with mass m moving with speed v in a circle of radius r. So, according to Bohr's postulate, (n = 1, 2, 3....)

Each value of n corresponds to a permitted value of the orbit radius, which we will denote by r_n The value of n for each orbit is called **principal quantum number** for the orbit. Thus,

$$mv_nr_n = mvr = \frac{nh}{2\pi} \qquad ...(ii)$$

According to Newton's second law a radially inward centripetal force of magnitude F = $\frac{mv^2}{r_n}$ is needed by the electron which is being provided by the electrical attraction between the positive proton and the negative electron.

Thus, $$\frac{mv_n^2}{r_n} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n^2} \qquad ....(iii)$$

Solving Eqs. (ii) and (iii), we get

$$r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2} \qquad ...(iv)$$

and $$v_n = \frac{e^2}{2\varepsilon_0 nh} \qquad ...(v)$$

The smallest orbit radius corresponds to n = 1. We'll denote this minimum radius, called the **Bohr radius** as a_0. Thus,

$$a_0 = \frac{\varepsilon_0 h^2}{\pi m e^2}$$

Substituting values of ε_0, h, p, m and e, we get

$$a_0 = 0.529 \times 10^{-10}\text{ m} = 0.529\text{ Å} \qquad ...(vi)$$

Eq. (iv), in terms of a_0 can be written as,

$$r_n = n^2 a_0 \quad \text{or} \quad r_n \propto n^2 \qquad ...(vii)$$

Similarly, substituting values of e, ε_0 and h with n = 1 in Eq. (v), we get

$$v_1 = 2.19 \times 10^6\text{ m/s} \qquad ...(viii)$$

This is the greatest possible speed of the electron in the hydrogen atom. Which is approximately equal to c/137 where c is the speed of light in vacuum.

Eq. (v), in terms of v_1 can be written as,

$$v_n = \frac{v_1}{n} \quad \text{or} \quad v_n \propto \frac{1}{n}$$

Energy levels : Kinetic and potential energies K_n and U_n in nth orbit are given by

$$K_n = \frac{1}{2}mv_n^2 = \frac{me^4}{8\varepsilon_0^2 n^2 h^2} \text{ and } U_n = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n} = -\frac{me^4}{4\varepsilon_0^2 n^2 h^2}$$

(assuming infinity as a zero potential energy level)

The total energy E_n is the sum of the kinetic and potential energies.

so, $$E_n = K_n + U_n = -\frac{me^4}{8\varepsilon_0^2 n^2 h^2}$$

Substituting values of m, e, ε_0 and h with n = 1, we get the least energy of the atom in first orbit, which is –13.6 eV. Hence,

$$E_1 = -13.6\text{ eV} \qquad ....(x)$$

and $$E_n = \frac{E_1}{n^2} = -\frac{13.6}{n^2}\text{ eV} \qquad ....(xi)$$

Substituting n = 2, 3, 4, ..., etc., we get energies of atom in different orbits.

$$E_2 = -3.40\text{ eV}, E_3 = -1.51\text{ eV}, ... E_\infty = 0$$

10.1 Hydrogen Like Atoms

The Bohr model of hydrogen can be extended to hydrogen like atoms, i.e., one electron atoms, the nuclear charge is +ze, where z is the atomic number, equal to the number of protons in the nucleus. The effect in the previous analysis is to replace e^2 every where by ze^2. Thus, the equations for, r_n, v_n and E_n are altered as under:

$$r_n = \frac{\varepsilon_0 n^2 h^2}{nmze^2} = a_0 \frac{n^2}{z} \quad \text{or} \quad r_n \propto \frac{n^2}{z} \quad \text{....(i)}$$

where $a_0 = 0.529$ Å (radius of first orbit of H)

$$v_n = \frac{ze^2}{2\varepsilon_0 nh} = \frac{z}{n} v_1 \quad \text{or} \quad v_n \propto \frac{z}{n} \quad \text{....(ii)}$$

where $v_1 = 2.19 \times 10^6$ m/s (speed of electron in first orbit of H)

$$E_n = -\frac{mz^2e^4}{8\varepsilon_0^2 n^2 h^2} = \frac{z^2}{n^2} E_1 \quad \text{or} \quad E_n \propto \frac{z^2}{n^2} \quad \text{....(iii)}$$

where $E_1 = -13.60$ eV (energy of atom in first orbit of H)

10.2 Definations valid for single electron system

(1) Ground state : Lowest energy state of any atom or ion is called ground state of the atom.
Ground state energy of H atom = –13.6 eV
Ground state energy of He^+ Ion = –54.4 eV
Ground state energy of Li^{++} Ion = –122.4 eV

(2) Excited State : State of atom other than the ground state are called its excited states.
n = 2 first excited state
n = 3 second excited state
n = 4 third excited state
$n = n_0 + 1$ n_0^{th} excited state

(3) Ionisation energy (I.E.) : Minimum energy required to move an electron from ground state to $n = \infty$ is called ionisation energy of the atom or ion
Ionisation energy of H atom = 13.6 eV
Ionisation energy of He^+ Ion = 54.4 eV
Ionisation energy of Li^{++} Ion = 122.4 eV

(4) Ionisation potential (I.P.) : Potential difference through which a free electron must be accelerated from rest such that its kinetic energy becomes equal to ionisation energy of the atom is called ionisation potential of the atom.
I.P of H atom = 13.6 V
I.P. of He^+ Ion = 54.4 V

(5) Excitation energy : Energy required to move an electron from ground state of the atom to any other exited state of the atom is called excitation energy of that state.
Energy in ground state of H atom = –13.6 eV
Energy in first excited state of H-atom = –3.4 eV
I^{st} excitation energy = 10.2 eV.

(6) Excitation Potential : Potential difference through which an electron must be accelerated from rest so that its kinetic energy becomes equal to excitation energy of any state is called excitation potential of that state.
I^{st} excitation energy = 10.2 eV.
I^{st} excitation potential = 10.2 V.

(7) Binding energy or Seperation energy : Energy required to move an electron from any state to $n = \infty$ is called binding energy of that state. or energy released during formation of an H-like atom/ion from $n = \infty$ to some particular n is called binding energy of that state.
Binding energy of ground state of H-atom = 13.6 eV

Example 13. First excitation potential of a hypothetical hydrogen like atom is 15 volt. Find third excitation potential of the atom.

Solution : Let energy of ground state = E_0

$E_0 = -13.6\ Z^2$ eV and $E_n = \dfrac{E_0}{n^2}$

$n = 2,\ E_2 = \dfrac{E_0}{4}$

given $\dfrac{E_0}{4} - E_0 = 15$

$-\dfrac{3E_0}{4} = 15$

for n = 4, $E_4 = \dfrac{E_0}{16}$

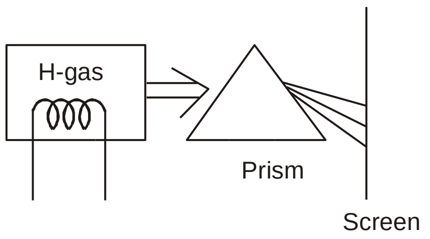
third exicitation energy = $\dfrac{E_0}{16} - E_0 = -\dfrac{15}{16}E_0 = -\dfrac{15}{16}\cdot\left(\dfrac{-4\times15}{3}\right) = \dfrac{75}{4}$ eV

$\therefore$ third excitation potential is $\dfrac{75}{4}$ V

10.3 Emission spectrum of hydrogen atom :

Under normal conditions the single electron in hydrogen atom stays in ground state (n = 1). It is excited to some higher energy state when it acquires some energy from external source. But it hardly stays there for more than 10^{-8} second.

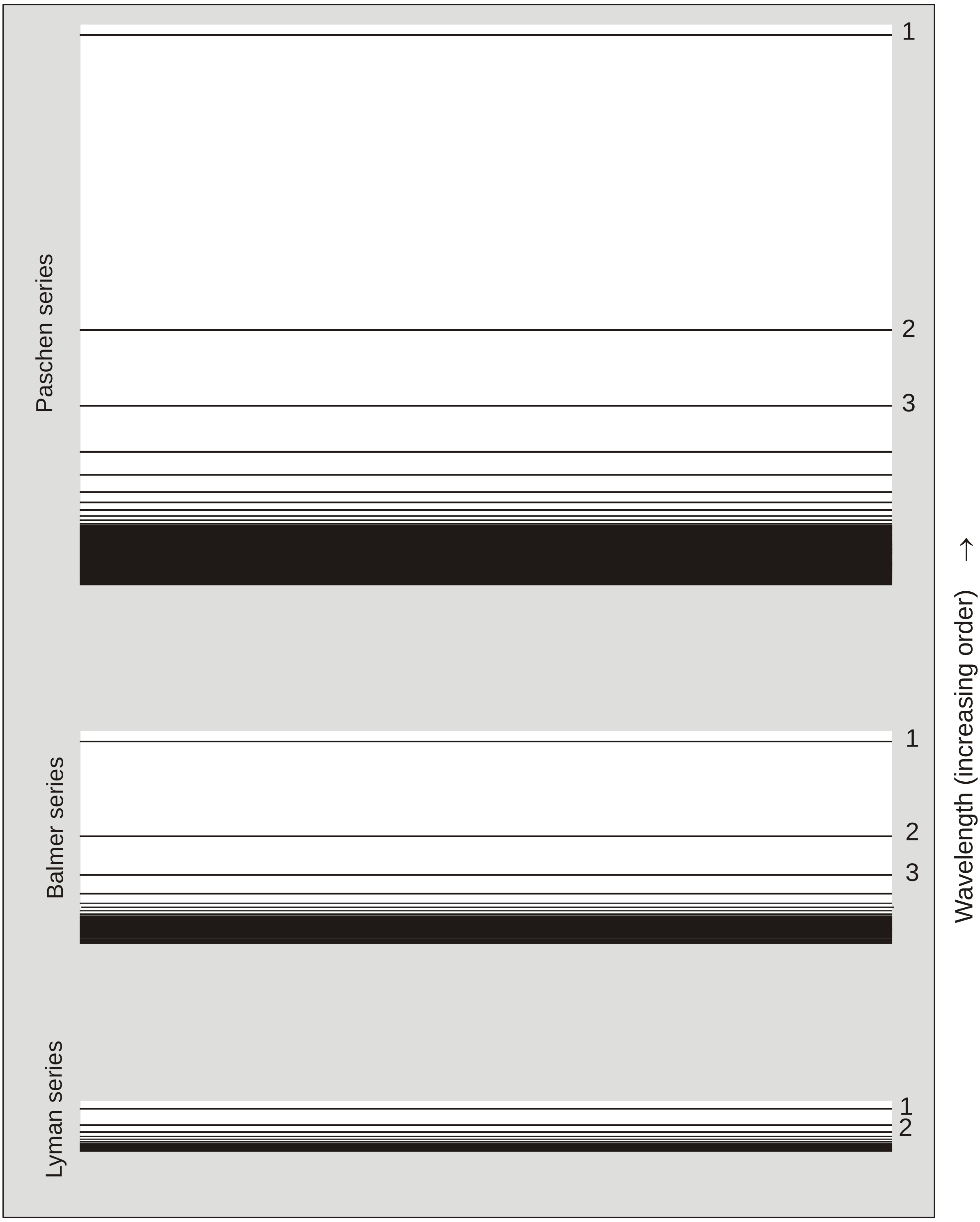
A photon corresponding to a particular spectrum line is emitted when an atom makes a transition from a state in an excited level to a state in a lower excited level or the ground level.



Let n_i be the initial and n_f the final energy state, then depending on the final energy state following series are observed in the emission spectrum of hydrogen atom.

On Screen :

A photograph of spectral lines of the Lyman, Balmer, Paschen series of atomic hydrogen.

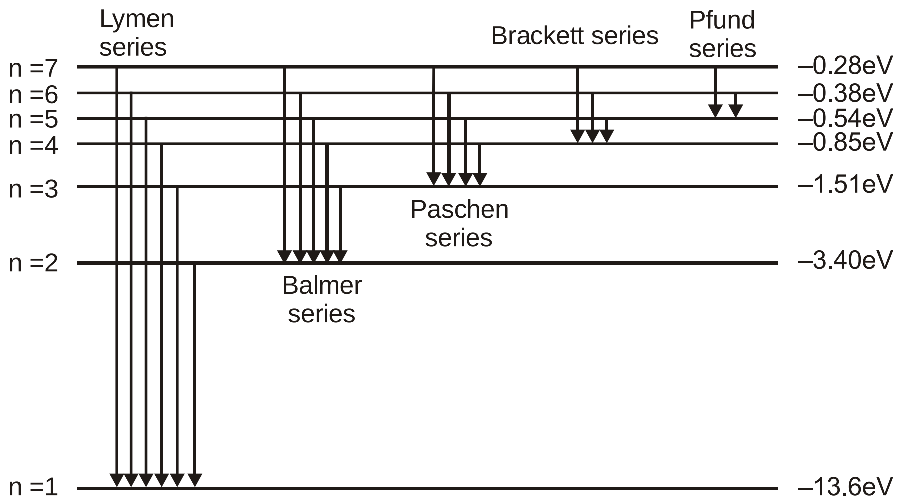


1, 2, 3..... represents the I, II & III line of Lyman, Balmer, Paschen series.

The hydrogen spectrum (some selected lines)

Name of series	Number of Line	Quantum Number			
		n_i (Lower State	n_f (Upper State)	Wavelength (nm)	Energy
Lymen	I	1	2	121.6	10.2 eV
	II	1	3	102.6	12.09 eV
	III	1	4	97	12.78 eV
	series limit	1	∞ (series limit)	91.2	13.6 eV
Balmer	I	2	3	656.3	1.89 eV
	II	2	4	486.1	2.55 eV
	III	2	5	434.1	2.86 eV
	series limit	2	∞ (series limit)	364.6	3.41 eV
Paschen	I	3	4	1875.1	0.66 eV
	II	3	5	1281.8	0.97 eV
	III	3	6	1093.8	1.13 eV
	series limit	3	∞ (series limit)	822	1.51 eV

Series limit : Line of any group having maximum energy of photon and minimum wavelength of that group is called series limit.



For the Lyman series n_f = 1, for Balmer series n_f = 2 and so on.

10.4 Wavelength of Photon Emitted in De-excitation

According to Bohr when an atom makes a transition from higher energy level to a lower level it emits a photon with energy equal to the energy difference between the initial and final levels. If E_i is the initial energy of the atom before such a transition, E_f is its final energy after the transition, and the photon's energy is $h\nu = \frac{hc}{\lambda}$, then conservation of energy gives,

$$h\nu = \frac{hc}{\lambda} = E_i - E_f \text{ (energy of emitted photon)} \qquad \text{....(i)}$$

By 1913, the spectrum of hydrogen had been studied intensively. The visible line with longest wavelength, or lowest frequency is called H_α, the next line is called H_β and so on.

In 1885, Johann Balmer, a Swiss teacher found a formula that gives the wave lengths of these lines. This is now called the Balmer series. The Balmer's formula is,

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \qquad \text{....(ii)}$$

Here, n = 3, 4, 5 ...., etc.

R = Rydberg constant = 1.097×10^7 m^{-1}

and λ is the wavelength of light/photon emitted during transition,
For n = 3, we obtain the wavelength of H_α line.
Similarly, for n = 4, we obtain the wavelength of H_β line. For n = ∞, the smallest wavelength (= 3646 Å) of this series is obtained. Using the relation, $E = \frac{hc}{\lambda}$ we can find the photon energies corresponding to the wavelength of the Balmer series.

$$E = \frac{hc}{\lambda} = hcR\left(\frac{1}{2^2} - \frac{1}{n^2}\right) = \frac{Rhc}{2^2} - \frac{Rhc}{n^2}$$

This formula suggests that,

$$E_n = -\frac{Rhc}{n^2}, \; n = 1, 2, 3..... \qquad ....(iii)$$

The wavelengths corresponding to other spectral series (Lyman, Paschen, (etc.) can be represented by formula similar to Balmer's formula.

Lyman Series : $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$, n = 2, 3, 4.....

Paschen Series : $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$, n = 4, 5, 6.....

Brackett Series : $\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$, n = 5, 6, 7.....

Pfund Series : $\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right)$, n = 6, 7, 8

The Lyman series is in the ultraviolet, and the Paschen. Brackett and Pfund series are in the infrared region.

Example 14. Calculate (a) the wavelength and (b) the frequency of the H_β line of the Balmer series for hydrogen.

Solution : (a) H_β line of Balmer series corresponds to the transition from n = 4 to n = 2 level. The corresponding wavelength for H_β line is,

$$\frac{1}{\lambda} = (1.097 \times 10^7)\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = 0.2056 \times 10^7 \quad \therefore \quad \lambda = 4.9 \times 10^{-7} \text{ m}$$ **Ans.**

(b) $\nu = \frac{c}{\lambda} = \frac{3.0 \times 10^8}{4.9 \times 10^{-7}} = 6.12 \times 10^{14}$ Hz **Ans.**

Example 15. Find the largest and shortest wavelengths in the Lyman series for hydrogen. In what region of the electromagnetic spectrum does each series lie?

Solution : The transition equation for Lyman series is given by,

$$\frac{1}{\lambda} = R\left[\frac{1}{(1)^2} - \frac{1}{n^2}\right] \qquad n = 2, 3, ...$$

for largest wavelength, n = 2

$$\frac{1}{\lambda_{max}} = 1.097 \times 10^7\left(\frac{1}{1} - \frac{1}{4}\right) = 0.823 \times 10^7$$

$\therefore$ $\lambda_{max} = 1.2154 \times 10^{-7}$ m = 1215 Å **Ans.**

The shortest wavelength corresponds to n = ∞

$$\therefore \quad \frac{1}{\lambda_{max}} = 1.097 \times 10^7\left(\frac{1}{1} - \frac{1}{\infty}\right)$$

or $\lambda_{min} = 0.911 \times 10^{-7}$ m = 911 Å **Ans.**

Both of these wavelengths lie in ultraviolet (UV) region of electromagnetic spectrum.

Example 16. How may different wavelengths may be observed in the spectrum from a hydrogen sample if the atoms are excited to states with principal quantum number n ?

Solution : From the nth state, the atom may go to (n – 1)th state, ..., 2nd state or 1st state. So there are (n – 1) possible transitions starting from the nth state. The atoms reaching (n – 1)th state may make (n – 2) different transitions. Similarly for other lower states. The total number of possible transitions is

(n – 1) + (n – 2) + (n – 3) +...........2 + 1

$= \frac{n(n-1)}{2}$ **(Remember)**

Example 17 (a) Find the wavelength of the radiation required to excite the electron in Li^{++} from the first to the third Bohr orbit.

(b) How many spectral linea are observed in the emission spectrum of the above excited system?

Solution : (a) The energy in the first orbit = $E_1 = Z^2 E_0$ where $E_0 = -13.6$ eV is the energy of a hydrogen atom in ground state thus for Li^{++},

$E_1 = 9E_0 = 9 \times (-13.6 \text{ eV}) = -122.4$ eV

The energy in the third orbit is $E_3 = \frac{E_1}{n^2} = \frac{E_1}{9} = -13.6$ eV

Thus, $E_3 - E_1 = 8 \times 13.6$ eV = 108.8 eV.

Energy required to excite Li^{++} from the first orbit to the third orbit is given by

$E_3 - E_1 = 8 \times 13.6$ eV = 108.8 eV.

The wavelength of radiation required to excite Li^{++} from the first orbit to the third orbit is given by

$$\frac{hc}{\lambda} = E_3 - E_1 \quad \text{or,} \quad \lambda = \frac{hc}{E_3 - E_1} = \frac{1240 \text{eV} - \text{nm}}{108.8 \text{eV}} \approx 11.4 \text{nm}$$

(b) The spectral lines emitted are due to the transitions n = 3 $\rightarrow$ n = 2, n = 3 $\rightarrow$ n = 1 and n = 2 $\rightarrow$ n = 1. Thus, there will be three spectral lines in the spectrum.

Example 18. Find the kinetic energy potential energy and total energy in first and second orbit of hydrogen atom if potential energy in first orbit is taken to be zero.

Solution : $E_1 = -13.60$ eV $\quad K_1 = -E_1 = 13.60$ eV $\quad U_1 = 2E_1 = -27.20$ eV

$E_2 = \frac{E_1}{(2)^2} = -3.40$ eV $\quad K_2 = 3.40$ eV and $\quad U_2 = -6.80$ eV

Now $U_1 = 0$, i.e., potential energy has been increased by 27.20 eV while kinetic energy will remain unchanged. So values of kinetic energy, potential energy and total energy in first orbit are 13.60 eV, 0, 13.60 respectively and for second orbit these values are 3.40 eV, 20.40 eV and 23.80 eV.

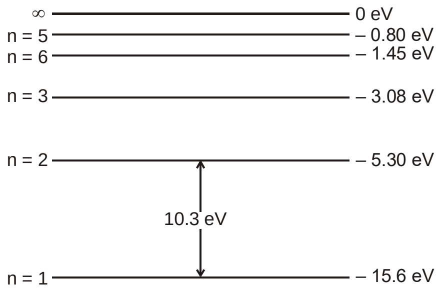
Example 19. A lithium atom has three electrons, Assume the following simple picture of the atom. Two electrons move close to the nucleus making up a spherical cloud around it and the third moves outside this cloud in a circular orbit. Bohr's model can be used for the motion of this third electron but n = 1 states are not available to it. Calculate the ionization energy of lithium in ground state using the above picture.

Solution : In this picture, the third electron moves in the field of a total charge + 3e – 2e = + e. Thus, the energies are the same as that of hydrogen atoms. The lowest energy is :

$E_2 = \frac{E_1}{4} = \frac{-13.6 \text{ eV}}{4} = -3.4$ eV

Thus, the ionization energy of the atom in this picture is 3.4 eV.

Example 20. The energy levels of a hypothetical one electron atom are shown in the figure.



(a) Find the ionization potential of this atom.
(b) Find the short wavelength limit of the series terminating at n = 2
(c) Find the excitation potential for the state n = 3.
(d) Find wave number of the photon emitted for the transition n = 3 to n = 1.
(e) What is the minimum energy that an electron will have after interacting with this atom in the ground state if the initial kinetic energy of the electron is
(i) 6 eV (ii) 11 eV

Solution : (a) Ionization potential = 15.6 V

(b) $\lambda_{min} = \dfrac{12400}{5.3} = 2340$ Å

(c) $\Delta E_{31} = -3.08 - (-15.6) = 12.52$ eV
Therefore, excitation potential for state n = 3 is 12.52 volt.

(d) $\dfrac{1}{\lambda_{31}} = \dfrac{\Delta E_{31}}{12400}$ Å^{-1} = $\dfrac{12.52}{12400}$ Å^{-1}
$\approx 1.01 \times 10^7\ m^{-1}$

(e) (i) $E_2 - E_1 = 10.3$ eV > 6 eV.
Hence electron cannot excite the atoms. So, K_{min} = 6 eV.
(ii) $E_2 - E_1 = 10.3$ eV < 11 eV.
Hence electron can excite the atoms. So, K_{min} = (11 – 10.3) = 0.7 eV.

Example 21. A small particle of mass m moves in such a way that the potential energy $U = ar^2$ where a is a constant and r is the distance of the particle from the origin. Assuming Bohr's model of quantization of angular momentum and circular orbits, find the radius of n^{th} allowed orbit.

Solution : The force at a distance r is, $F = -\dfrac{dU}{dr} = -2ar$

Suppose r be the radius of nth orbit. The necessary centripetal force is provided by the above force. Thus, $\dfrac{mv^2}{r} = 2ar$

Further, the quantization of angular momentum gives, $mvr = \dfrac{nh}{2\pi}$

Solving Eqs. (i) and (ii) for r, we get $r = \left(\dfrac{n^2h^2}{8am\pi^2}\right)^{1/4}$ **Ans.**

Example 22. An imaginary particle has a charge equal to that of an electron and mass 100 times the mass of the electron. It moves in a circular orbit around a nucleus of charge + 4e. Take the mass of the nucleus to be infinite. Assuming that the Bohr's model is applicable to the system.
(a) Derive and expression for the radius of n^{th} Bohr orbit.
(b) Find the wavelength of the radiation emitted when the particle jumps from fourth orbit to the second.

Solution : (a) We have $\dfrac{m_p v^2}{r_n} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{ze^2}{r_n^2}$ .....(i)

The quantization of angular momentum gives, $m_p vr_n = \frac{nh}{2\pi}$ ......(ii)

Solving Eqs. (i) and (ii), we get

$r = \frac{n^2h^2\varepsilon_0}{z\pi m_p e^2}$

Substituting $m_p = 100$ m

where m = mass of electron and z = 4

we get, $r_n = \frac{n^2h^2\varepsilon_0}{400\pi me^2}$ **Ans.**

(b) As we know,

Energy of hydrogen atom in ground state = – 13.60 eV

and $E_n \propto \left(\frac{z^2}{n^2}\right)m$

For the given particle, $E_4 = \frac{(-13.60)\ \ (4)^2}{(4)^2} \times 100 = -1360$ eV

and $E_2 = \frac{(-13.60)\ \ (4)^2}{(2)^2} \times 100 = -5440$ eV

$\Delta E = E_4 - E_2 = 4080$ eV

$\therefore \quad \lambda$ (in Å) $= \frac{12400}{4080} = 3.0$ Å **Ans.**

Example 23. A particle known as μ-meson, has a charge equal to that of an electron and mass 208 times the mass of the electron. It moves in a circular orbit around a nucleus of charge +3e. Take the mass of the nucleus to be infinite. Assuming that the Bohr's model is applicable to this system,

(a) derive an expression for the radius of the nth Bohr orbit,

(b) find the value of n for which the radius of the orbit is approximately the same as that of the first Bohr orbit for a hydrogen atom and

(c) find the wavelength of the radiation emitted when the μ–meson jumps from the third orbit to the first orbit.

Solution : (a) We have, $\frac{mv^2}{r} = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$

or, $v^2 r = \frac{Ze^2}{4\pi\varepsilon_0 m}$ .....(i)

The quantization rule is $vr = \frac{nh}{2\pi m}$

The radius is $r = \frac{(vr)^2}{v^2 r} = \frac{4\pi\varepsilon_0 m}{Ze^2}$

$= \frac{n^2h^2\varepsilon_0}{Z\pi me^2}$ ....(ii)

For the given system, Z = 3 and m = 208 m_e.

Thus $r_\mu = \frac{n^2h^2\varepsilon_0}{624\pi m_e e^2}$

(b) From (ii), the radius of the first Bohr orbit for the hydrogen atom is $r_h = \frac{h^2\varepsilon_0}{\pi m_e e^2}$

For $r_\mu = r_h$, $\frac{n^2h^2\varepsilon_0}{624\pi m_e e^2} = \frac{h^2\varepsilon_0}{\pi m_e e^2}$

or, $n^2 = 624$

or, $n = 25$

(c) From (i), the kinetic energy of the atom is $\frac{mv^2}{2} = \frac{Ze^2}{8\pi\varepsilon_0 r}$

and the potential energy is $-\frac{Ze^2}{4\pi\varepsilon_0 r}$

The total energy is $E_n = \frac{Ze^2}{8\pi\varepsilon_0 r}$

Using (ii), $E_n = -\frac{Z^2\pi me^4}{8\pi\varepsilon_0^2 n^2h^2} = -\frac{9\times 208 m_e{}^4}{8\varepsilon_0^2 n^2 h^2} = \frac{1872}{n^2}\left(-\frac{m_e e^4}{8\varepsilon_0^2 \; h^2}\right)$

But $\left(-\frac{m_e e^4}{8\varepsilon_0^2 \; h^2}\right)$ is the ground state energy of hydrogen atom and hence is equal to – 13.6 eV.

From (iii), $E_n = -\frac{1872}{n^2} \times 13.6 \text{ eV} = \frac{-25459.2 \text{ eV}}{n^2}$

Thus, $E_1 = -25459.2$ eV and $E_3 = \frac{E_1}{9} = -2828.8$ eV. The energy difference is $E_3 - E_1 = 22630.4$eV.

The wavelength emitted is $\lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV} - \text{nm}}{22630.4 \text{ eV}}$ = 55 pm.

Example 24. A gas of hydrogen like atoms can absorb radiations of 68 eV. Consequently, the atoms emit radiations of only three different wavelength. All the wavelengths are equal or smaller than that of the absorbed photon.

(a) Determine the initial state of the gas atoms.
(b) Identify the gas atoms.
(c) Find the minimum wavelength of the emitted radiations.
(d) Find the ionization energy and the respective wavelength for the gas atoms.

Solution : (a) $\frac{n(n-1)}{2} = 3$

$\therefore$ n = 3

i.e., after excitation atom jumps to second excited state.

Hence $n_f = 3$. So n_i can be 1 or 2

If $n_i = 1$ then energy emitted is either equal to, greater than or less than the energy absorbed. Hence the emitted wavelength is either equal to, less than or greater than the absorbed wavelength.

Hence $n_i \neq 1$.

If $n_i = 2$, then $E_e \geq E_a$. Hence $\lambda_e \leq \lambda_0$

(b) $E_3 - E_2 = 68$ eV

$\therefore$ $(13.6)(Z^2)\left(\frac{1}{4}-\frac{1}{9}\right) = 68$

$\therefore$ Z = 6

(c) $\lambda_{min} = \frac{12400}{E_3 - E_1} = \frac{12400}{(13.6)\;(6)^2\left(1-\frac{1}{9}\right)} = \frac{12400}{435.2} = 28.49$ **Ans.**

(d) Ionization energy = $(13.6)\,(6)^2 = 489.6$ eV **Ans.**

$\lambda = \frac{12400}{489.6} = 25.33$ Å **Ans.**

Example 25. An electron is orbiting in a circular orbit of radius r under the influence of a constant magnetic field of strength B. Assuming that Bohr's postulate regarding the quantisation of angular momentum holds good for this electron, find
(a) the allowed values of the radius 'r' of the orbit.
(b) the kinetic energy of the electron in orbit
(c) The potential energy of interaction between the magnetic moment of the orbital current due to the electron moving in its orbit and the magnetic field B.
(d) The total energy of the allowed energy levels.

Solution : (a) radius of circular path

$$r = \frac{mv}{Be} \quad \text{....(i)}$$

$$mvr = \frac{nh}{2\pi} \quad \text{....(ii)}$$

Solving these two equations, we get

$$r = \sqrt{\frac{nh}{2\pi Be}} \text{ and } v = \sqrt{\frac{nhBe}{2\pi m^2}}$$

(b) $K = \frac{1}{2}mv^2 = \frac{nhBe}{4\pi m}$ **Ans.**

(c) $M = iA = \left(\frac{e}{T}\right)(\pi r^2) = \frac{evr}{2}$

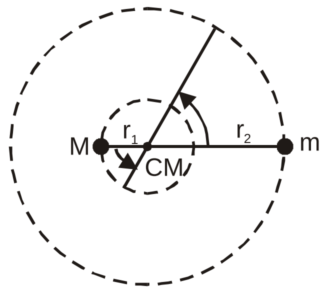
$$= \frac{e}{2}\sqrt{\frac{nh}{2\pi Be}}\sqrt{\frac{nhBe}{2\pi m^2}} = \frac{nhe}{4\pi m}$$

Now potential energy U = – **M . B**

$$= \frac{nheB}{4\pi m}$$

(d) $E = U + K = \frac{nheB}{2\pi m}$

11. EFFECT OF NUCLEUS MOTION ON ENERGY OF ATOM



Let both the nucleus of mass M, charge Ze and electron of mass m, and charge e revolve about their centre of mass (CM) with same angular velocity (ω) but different linear speeds. Let r_1 and r_2 be the distance of CM from nucleus and electron. Their angular velocity should be same then only their separation will remain unchanged in an energy level. Let r be the distance between the nucleus and the electron. Then

$Mr_1 = mr_2$

$r_1 + r_2 = r$

$\therefore \quad r_1 = \frac{mr}{M+m}$ and $\quad r_2 = \frac{Mr}{M+m}$

Centripetal force to the electron is provided by the electrostatic force. So,

$$mr_2\omega^2 = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}$$

or $m\left(\frac{Mr}{M+m}\right)\omega^2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2}{r^2}$

or $\left(\frac{Mm}{M+m}\right) r^3 \omega^2 = \frac{Ze^2}{4\pi\varepsilon_0}$ or $\mu r^3\omega^2 = \frac{e^2}{4\pi\varepsilon_0}$

where $\frac{Mm}{M+m} = \mu$

Moment of inertia of atom about CM, $I = Mr_1^2 + mr_2^2 = \left(\frac{Mm}{M+m}\right) r^2 = \mu r^2$

According to Bohr's theory, $\frac{nh}{2\pi} = I\,\omega$ or $\mu r^2\omega = \frac{nh}{2\pi}$

Solving above equations for r, we get

$r = \frac{\varepsilon_0 n^2 h^2}{\pi\mu e^2 Z}$ and **$r = (0.529\ \text{Å})$** $\frac{n^2}{Z}\cdot\frac{m}{\mu}$

Further electrical potential energy of the system, $U = \frac{-Ze^2}{4\pi\varepsilon_0 r}$ $U = \frac{-Z^2e^4\mu}{4\varepsilon_0^2 n^2 h^2}$

and kinetic energy, $K = \frac{1}{2} I\omega^2 = \frac{1}{2}\mu r^2\omega^2$ and $K = \frac{1}{2}\mu v^2$

v-speed of electron with respect to nucleus. ($v = r\omega$)

here $\omega^2 = \frac{Ze^2}{4\pi\varepsilon_0 \mu r^3}$

$\therefore$ $K = \frac{Ze^2}{8\pi\varepsilon_0 r} = \frac{Z^2e^4\mu}{8\pi\varepsilon_0^2 n^2 h^2}$

$\therefore$ Total energy of the system $E_n = K + U$, $E_n = -\frac{\mu e^4}{8\varepsilon_0{}^2 n^2 h^2}$

this expression can also be written as **$E_n = -(13.6\ \text{eV})$** $\frac{Z^2}{n^2}\cdot\left(\frac{\mu}{m}\right)$

The expression for E_n without considering the motion of proton is $E_n = -\frac{me^4}{8\varepsilon_0{}^2 n^2 h^2}$, i.e., m is replaced by μ while considering the motion of nucleus.

Example 26. A positronium 'atom' is a system that consists of a positron and an electron that orbit each other. Compare the wavelength of the spectral lines of positronium with those of ordinary hydrogen.

Solution : Here the two particle have the same mass m, so the reduced mass is $\mu = \frac{mM}{m+M} = \frac{m^2}{2m} = \frac{m}{2}$

where m is the electron mass. We know that $E_n \propto m$

$\therefore$ $\frac{E'_n}{E_n} = \frac{\mu}{m} = \frac{1}{2}$ energy of each level is halved.

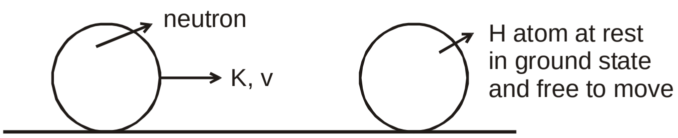
$\therefore$ Their difference will also be halved.

Hence $\lambda'_n = 2\lambda_n$

12. ATOMIC COLLISION

In such collisions assume that the loss in the kinetic energy of system is possible only if it can excite or ionise.

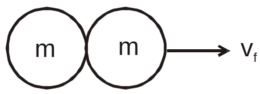
Example 27



head on collision

What will be the type of collision, if K = 14eV, 20.4 eV, 22 eV, 24.18 eV (elastic/inelastic/perectly inelastic)

Solution : Loss in energy (ΔE) during the collision will be used to excite the atom or electron from one level to another.



According to quantum Mechanics, for hydrogen atom.

ΔE = {0, 10.2 eV, 12.09 eV, ..., 13.6 eV)

According to Newtonion mechanics

minimum loss = 0. (elastic collsion)

for maximum loss collision will be perfectly inelastic if neutron collides perfectly inelastically then,

Applying momentum conservation $m\nu_0 = 2m\nu_f$

$$v_f = \frac{v_0}{2}$$

$$\text{final K.E.} = \frac{1}{2} \times 2m \times \frac{\nu_0^2}{4} = \frac{\frac{1}{2}m\nu_0^2}{2} = \frac{K}{2} \quad \text{maximum loss} = \frac{K}{2}$$

According to classical mechanics (ΔE) = $[0, \frac{K}{2}]$

(a) If K = 14 eV,
According to quantum mechanics (ΔE) = {0, 10.2eV, 12.09 eV}
According to classical mechanics ΔE = [0, 7 eV]
loss = 0, hence it is elastic collision speed of particle changes.

(b) If K = 20.4 eV
According to classical mechanics
loss = [0, 10.2 eV]
According to quantum mechanics
loss = {0, 10.2eV, 12.09eV,.........}
loss = 0 elastic collision.
loss = 10.2eV perfectly inelastic collision

(c) If K = 22 eV
Classical mechanics ΔE =[0, 11]
Quantum mechanics ΔE = {0, 10.2eV, 12.09eV, ...}
loss = 0 elastic collision
loss = 10.2 eV inelastic collsion

(d) If K = 24.18 eV
According to classical mechanics ΔE =[0, 12.09eV]
According to quantum mechanics ΔE = {0, 10.2eV, 12.09eV, ... 13.6eV}
loss = 0 elastic collision
loss = 10.2 eV inelastic collision
loss = 12.09 eV perfectly inelastic collision

Example 28. A He^+ ion is at rest and is in ground state. A neutron with initial kinetic energy K collides head on with the He^+ ion. Find minimum value of K so that there can be an inelastic collision between these two particles.

Solution :

Here the loss during the collision can only be used to excite the atoms or electrons.
So according to quantum mechanics
loss = {0, 40.8eV, 48.3eV, ..., 54.4eV} ....(1)

$$E_n = -\frac{Z^2}{n^2}\ 13.6\ eV$$

Now according to newtonion mechanics
Minimum loss = 0
maximum loss will be for perfectly inelastic collision.
let v_0 be the initial speed of neutron and v_f be the final common speed.

so by momentum conservation $mv_0 = mv_f + 4mv_f$ $\quad v_f = \frac{v_0}{5}$

where m = mass of Neutron
$\therefore$ mass of He^+ ion = 4m
so final kinetic energy of system

$$K.E. = \frac{1}{2} m v_f^2 + \frac{1}{2} 4m v_f^2 = \frac{1}{2}.(5m).\frac{v_0^2}{25} = \frac{1}{5}.(\frac{1}{2}mv_0^2) = \frac{K}{5}$$

maximum loss = $K - \frac{K}{5} = \frac{4K}{5}$

so loss will be $\left[0,\ \frac{4K}{5}\right]$ ....(2)

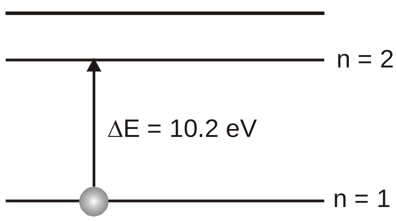
For inelastic collision there should be at least one common value other than zero in set (1) and (2)

$\therefore \quad \frac{4K}{5} > 40.8$ eV

K > 51 eV
minimum value of K = 51 eV.

Example 29 A moving hydrogen atom makes a head on collision with a stationary hydrogen atom. Before collision both atoms are in ground state and after collision they move together. What is the minimum value of the kinetic energy of the moving hydrogen atom, such that one of the atoms reaches one of the excitation state.

Solution : Let K be the kinetic energy of the moving hydrogen atom and K', the kinetic energy of combined mass after collision.



From conservation of linear momentum,

p = p' or $\sqrt{2Km} = \sqrt{2K'(2m)}$

or K = 2K' ....(i)

From conservation of energy, $K = K' + \Delta E$ ....(ii)

Solving Eqs. (i) and (ii), we get $\Delta E = \frac{K}{2}$

Now minimum value of ΔE for hydrogen atom is 10.2 eV.

or $\Delta E \geq 10.2$ eV

$\therefore \quad \frac{K}{2} \geq 10.2$

$\therefore \quad K \geq 20.4$ eV

Therefore, the minimum kinetic energy of moving hydrogen is 20.4 eV **Ans.**

Example 30. A neutron moving with speed v makes a head-on collision with a hydrogen atom in ground state kept at rest. Find the minimum kinetic energy of the neutron for which inelastic (completely or partially) collision may take place. The mass of neutron = mass of hydrogen = 1.67×10^{-27} kg.

Solution : Suppose the neutron and the hydrogen atom move at speed v_1 and v_2 after the collision. The collision will be inelastic if a part of the kinetic energy is used to excite the atom. Suppose an energy ΔE is used in this way. Using conservation of linear momentum and energy.

$mv = mv_1 + mv_2$ ....(i)

and $\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \Delta E$ ....(ii)

From (i), $v^2 = v_1^2 + v_2^2 + 2v_1v_2$,

From (ii), $v^2 = v_1^2 + v_2^2 + \frac{2\Delta E}{m}$

Thus, $2v_1v_2 = \frac{2\Delta E}{m}$

Hence, $(v_1 - v_2)^2 - 4v_1v_2 = v^2 - \frac{4\Delta E}{m}$

As $v_1 - v_2$ must be real, $v^2 - \frac{4\Delta E}{m} \geq 0$ or $\frac{1}{2}mv^2 > 2\Delta E$.

The minimum energy that can be absorbed by the hydrogen atom in ground state to go in an excited state is 10.2 eV. Thus, the minimum kinetic energy of the neutron needed for an inelastic collision is

$$\frac{1}{2}mv_{min}^2 = 2 \times 10.2 \text{ eV} = 20.4 \text{ eV}$$

Example 31. How many head-on, elastic collisions must a neutron have with deuterium nucleus to reduce its energy from 1 MeV to 0.025 eV.

Solution : Let mass of neutron = m and mass of deuterium = 2m

initial kinetic energy of neutron = K_0

Let after first collision kinetic energy of neutron and deuterium be K_1 and K_2.

Using C.O.L.M. along direction of motion $\sqrt{2mK_0} = \sqrt{2mK_1} + \sqrt{4mK_2}$

velocity of seperation = velocity of approach $\frac{\sqrt{4mK_2}}{2m} - \frac{\sqrt{2mK_1}}{m} = \frac{\sqrt{2mK_0}}{m}$

Solving equaiton (i) and (ii) we get ; $K_1 = \frac{K_0}{9}$

Loss in kinetic eneryg after first collision $\Delta K_1 = K_0 - K_1$

$\Delta K_1 = \frac{8}{9} K_0$ ....(1)

After second collision $\Delta K_2 = \frac{8}{9}K_1 = \frac{8}{9}.\frac{K_0}{9}$

$\therefore$ Total energy loss $\Delta K = \Delta K_1 + \Delta K_2 + ... + \Delta K_n$

As, $\Delta K = \frac{8}{9}K_0 + \frac{8}{9^2}K_0 + ... + \frac{8}{9^n}K_0$

$\Delta K = \frac{8}{9}K_0 (1 + \frac{1}{9} + ... + \frac{1}{9^{n-1}})$

$$\frac{\Delta K}{K_0} = \frac{8}{9}\left[\frac{1-\frac{1}{9^n}}{1-\frac{1}{9}}\right] = 1 - \frac{1}{9^n}$$

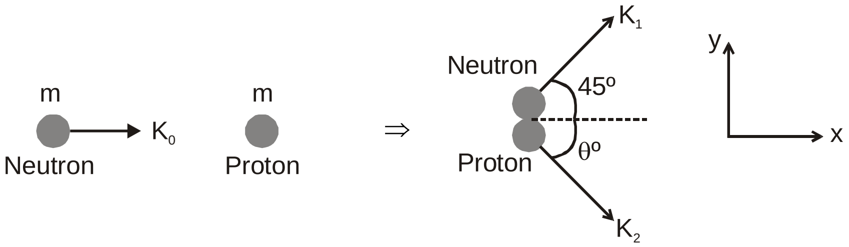
Here, $K_0 = 10^6$ eV, $\Delta K = (10^6 - 0.025)$ eV

$\therefore \frac{1}{9^n} = \frac{K_0 - \Delta K}{K_0} = \frac{0.025}{10^6}$ or $9^n = 4 \times 10^7$

Taking log both sides and solving, we get n = 8

Example 32. A neutron with an energy of 4.6 MeV collides with protons and is retarded. Assuming that upon each collision neutron is deflected by 45º find the number of collisions which will reduce its energy to 0.23 eV.

Solution : Mass of neutron $\approx$ mass of proton = m



From conservation of momentum in y-direction

$\sqrt{2mK_1}$ sin 45º = $\sqrt{2mK_2}$ sin θ ....(i)

In x-direction $\sqrt{2mK_0} - \sqrt{2mK_1}$ cos 45º = $\sqrt{2mK_2}$ cos θ ....(ii)

Squaring and adding equation (i) and (ii), we have

$K_2 = K_1 + K_0 - \sqrt{2K_0K_1}$ ....(iii)

From conservation of energy

$K_2 = K_0 - K_1$ ....(iv)

Solving equations (iii) and (iv), we get

$K_1 = \frac{K_0}{2}$

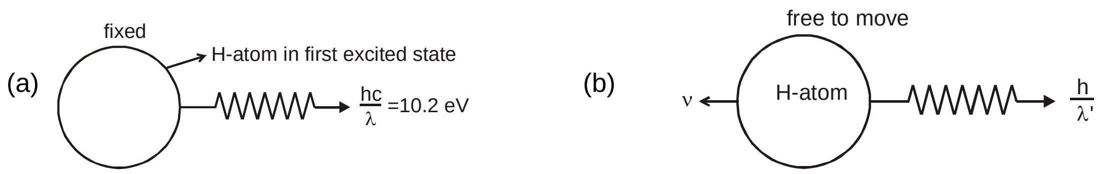
i.e., after each collision energy remains half. Therefore, after n collisions, $K_n = K_0 \left(\frac{1}{2}\right)^n$

$\therefore$ $0.23 = (4.6 \times 10^6)\left(\frac{1}{2}\right)^n$ $\Rightarrow$ $2^n = \frac{4.6 \times 10^6}{0.23}$

Taking log and solving, we get $n \approx 24$ **Ans.**

12.1 Calculation of recoil speed of atom on emission of a photon

momentum of photon = mc = $\frac{h}{\lambda}$



m - mass of atom

According to momentum conservation $mv = \frac{h}{\lambda'}$ ....(i)

According to energy conservation $\frac{1}{2}mv^2 + \frac{hc}{\lambda'}$ = 10.2 eV

Since mass of atom is very large than photon

hence $\frac{1}{2}mv^2$ can be neglected

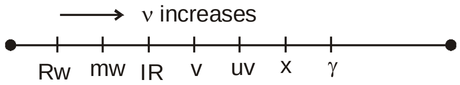
$\frac{hc}{\lambda'}$ = 10.2 eV $\qquad$ $\frac{h}{\lambda} = \frac{10.2}{c}$eV

$mv = \frac{10.2}{c}$eV $\qquad$ $v = \frac{10.2}{cm}$

recoil speed of atom = $\frac{10.2}{cm}$

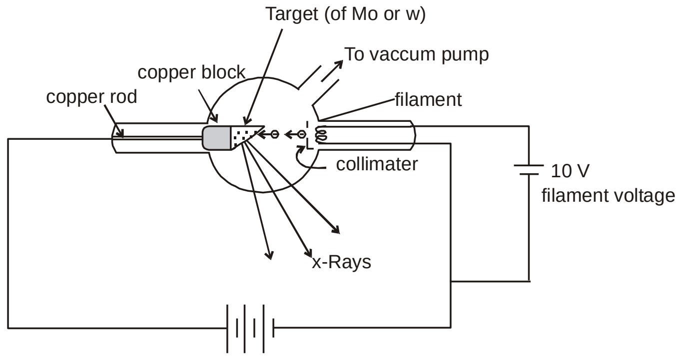
13. X-RAYS

It was discovered by **ROENTGEN**. The wavelength of x-rays is found between 0.1 Å to 10 Å. These rays are invisible to eye. They are electromagnetic waves and have speed c = 3 × 10^8 m/s in vacuum. Its photons have energy around 1000 times more than the visible light.



When fast moving electrons having energy of order of several KeV strike the metallic target then x-rays are produced.

13.1 Production of x-rays by coolidge tube :



The melting point, specific heat capacity and atomic number of target should be high. When voltage is applied across the filament then filament on being heated emits electrons from it. Now for giving the beam shape of electrons, collimator is used. Now when electron strikes the target then x-rays are produced.

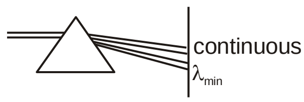
When electrons strike with the target, some part of energy is lost and converted into heat. Since, target should not melt or it can absorb heat so that the melting point, specific heat of target should be high.

Here copper rod is attached so that heat produced can go behind and it can absorb heat and target does not get heated very high.

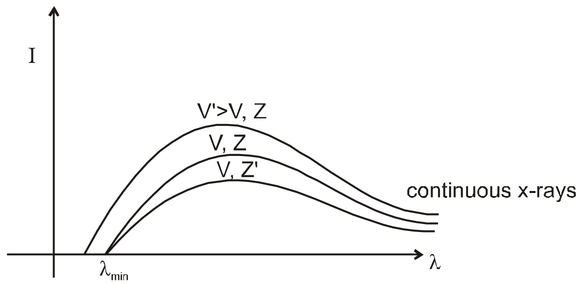
For more energetic electron, accelerating voltage is increased.

For more no. of photons voltage across filament is increased.

The x-ray were analysed by mostly taking their spectrum



13.2 Variation of Intensity of x-rays with λ is plotted as shown in figure :



13.2.1 The minimum wavelength corresponds to the maximum energy of the x-rays which in turn is equal to the maximum kinetic energy eV of the striking electrons thus

$$eV = h\nu_{max} = \frac{hc}{\lambda_{min}} \;; \qquad \lambda_{min} = \frac{hc}{eV} = \frac{12400}{V(\text{involts})}\,\text{Å}.$$

We see that cutoff wavelength λ_{min} depends only on accelerating voltage applied between target and filament. It does not depend upon material of target, it is same for two different metals (Z and Z')

Example 33. An X-ray tube operates at 20 kV. A particular electron loses 5% of its kinetic energy to emit an X-ray photon at the first collision. Find the wavelength corresponding to this photon.

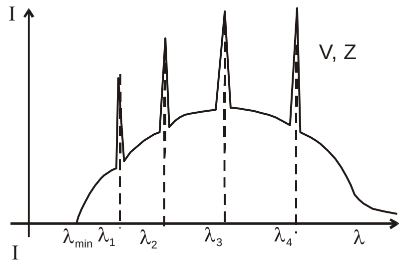
Solution : Kinetic energy acquired by the electron is K = eV = 20 × 10^3 eV.

The energy of the photon = 0.05 × 20 = 10^3 eV = 10^3 eV.

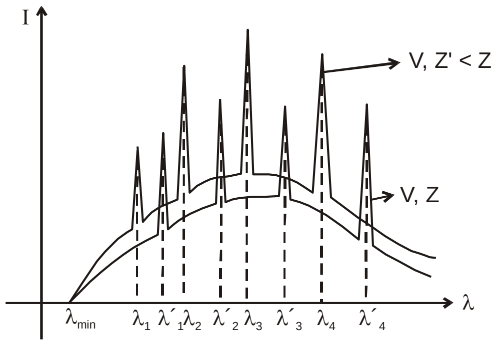
$$\text{Thus,}\ \frac{h\nu}{\lambda} = 10^3\,eV = \frac{(4.14\times10^{-15}\,eV-s)\times(3\times10^8\,m/s)}{10^3\,eV} = \frac{1242\ \ eV-nm}{10^3\,eV} = 1.24\ \ nm$$

13.2.2 Charactristic X-rays

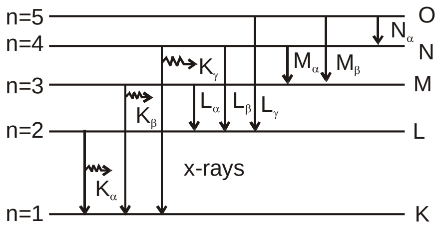
The sharp peaks obtained in graph are known as characteristic x-rays because they are characteristic of target material.



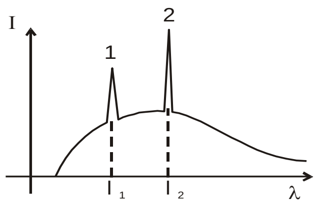
λ_1, λ_2, λ_3, λ_4, ... = charecteristic wavelength of material having atomic number Z are called **characteristic x-rays** and the spectrum obtained is called **characteristic spectrum**. If target of atomic number Z' is used then peaks are shifted.



Characteristic x-ray emission occurs when an energetic electron collides with target and remove an inner shell electron from atom, the vacancy created in the shell is filled when an electron from higher level drops into it. Suppose vacancy created in innermost K-shell is filled by an electron droping from next higher level L-shell then K_α characteristic x-ray is obtained. If vaccany in K-shell is filled by an electron from M-shell, K_β line is produced and so on similarly L_α, L_β,.....M_α, M_β lines are produced.



Example 34. Find which is K_α and K_β



Solution : $\Delta E = \dfrac{hc}{\lambda}$, $\lambda = \dfrac{hc}{\Delta E}$

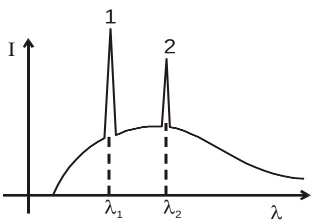
since energy difference of K_α is less than K_β

$\Delta E_{k\alpha} < \Delta E_{k\beta}$

$\lambda_{k\beta} < \lambda_{k\alpha}$

1 is K_β and 2 is K_α

Example 35



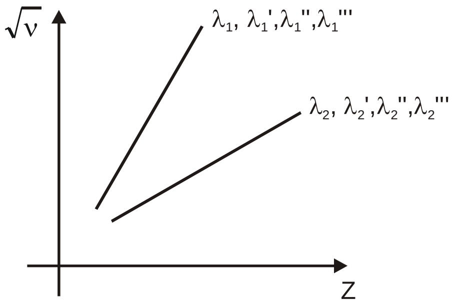
Find which is K_α and L_α

Solution : $\therefore$ $\Delta E_{K\alpha} > \Delta E_{L\alpha}$

1 is K_α and 2 is L_α

14. MOSELEY'S LAW :

Moseley measured the frequencies of characteristic x-rays for a large number of elements and plotted the sqaure root of frequency against position number in periodic table. He discovered that plot is very closed to a straight line not passing through origin.



Z_1	λ_1	λ_2
Z_2	λ_1'	λ_2'
Z_3	λ_1''	λ_2''
Z_4	λ_1'''	λ_2'''

Wavelength of charactristic wavelengths.

Moseley's observations can be mathematically expressed as $\sqrt{\nu} = a(Z - b)$

a and b are positive constants for one type of x-rays & for all elements (independent of Z).

Moseley's Law can be derived on the basis of Bohr's theory of atom, frequency of x-rays is given by

$$\sqrt{\nu} = \sqrt{CR\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)} \, . \, (Z - b)$$

by using the formula $\frac{1}{\lambda} = R\, z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ with modification for multi electron system.

b $\rightarrow$ known as screening constant or shielding effect, and (Z – b) is effective nuclear charge.

for K_α line

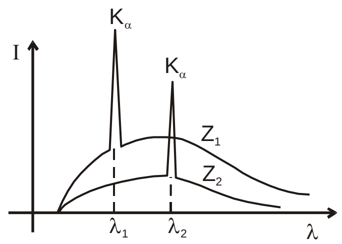
$n_1 = 1, \quad n_2 = 2$

$$\therefore \quad \sqrt{\nu} = \sqrt{\frac{3RC}{4}}\,(Z - b)$$

$$\sqrt{\nu} = a(Z - b)$$

Here $a = \sqrt{\frac{3RC}{4}}$, [b = 1 for K_α lines]

Example 36



Find in Z_1 and Z_2 which one is greater.

Solution : $\because \sqrt{\nu} \equiv \sqrt{cR\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)} \, . \, (Z - b)$

If Z is greater then ν will be greater, λ will be less

$\therefore \quad \lambda_1 < \lambda_2$

$\therefore \quad Z_1 > Z_2$.

Example 37 A cobalt target is bombarded with electrons and the wavelength of its characteristic spectrum are measured. A second, fainter, characteristic spectrum is also found because of an impurity in the target. The wavelength of the K_α lines are 178.9 pm (cobalt) and 143.5 pm (impurity). What is the impurity?

Solution : Using Moseley's law and putting c/λ for ν (and assuming b = 1), we obtain

$$\sqrt{\frac{c}{\lambda_{c_0}}} = aZ_{c_0} - a$$

and $\sqrt{\frac{c}{\lambda_x}} = aZ_x - a$

Dividing yields $\sqrt{\frac{\lambda_{c_0}}{\lambda_x}} = \frac{Z_x - 1}{Z_{c_0} - 1}$

Substituting gives us $\sqrt{\frac{178.9\,\text{pm}}{143.5\,\text{pm}}} = \frac{Z_x - 1}{27 - 1}$.

Solving for the unknown, we find $Z_x = 30.0$; the impurity is zinc.

Example 38 Find the constants a and b in Moseley's equation $\sqrt{\nu} = a(Z - b)$ from the following data.

Element	Z	Wavelength of K_α X-ray
Mo	42	71 pm
Co	27	178.5 pm

Solution : Moseley's equation is $\sqrt{\nu} = a(Z - b)$

Thus, $\sqrt{\frac{c}{\lambda_1}} = a(Z_1 - b)$ ....(i)

and $\sqrt{\frac{c}{\lambda_2}} = a(Z_2 - b)$ ....(ii)

From (i) and (ii) $\sqrt{c}\left(\frac{1}{\sqrt{\lambda_1}} - \frac{1}{\sqrt{\lambda_2}}\right) = a(Z_1 - Z_2)$ or $a = \frac{\sqrt{c}}{(Z_1 - Z_2)}\left(\frac{1}{\sqrt{\lambda_1}} - \frac{1}{\sqrt{\lambda_2}}\right)$

$$= \frac{(3\times10^8\,\text{m/s})^{1/2}}{42-27}\left[\frac{1}{(71\times10^{-12}\text{m})^{1/2}} - \frac{1}{(178.5\times10^{-12}\text{m})^{1/2}}\right] = 5.0 \times 10^7\ (\text{Hz})^{1/2}$$

Dividing (i) by (ii),

$\sqrt{\frac{\lambda_2}{\lambda_1}} = \frac{Z_1 - b}{Z_2 - b}$ or $\sqrt{\frac{178.5}{71}} = \frac{42 - b}{27 - b}$ or b = 1.37

Problem 1. Find the momentum of a 12.0 MeV photon.

Solution : $p = \frac{E}{c}$ = 12 MeV/c.

Problem 2. Monochromatic light of wavelength 3000 Å is incident nornally on a surface of area 4 cm^2. If the intensity of the light is 15×10^{-2} W/m^2, determine the rate at which photons strike the surface.

Solution : Rate at which photons strike the surface

$$= \frac{IA}{hc/\lambda} = \frac{6\times10^{-5}\,\text{J/s}}{6.63\times10^{-19}\,\text{J/photon}} = 9.05 \times 10^{13}\ \text{photon/s}.$$

Problem 3. The kinetic energies of photoelectrons range from zero to 4.0×10^{-19} J when light of wavelength 3000 Å falls on a surface. What is the stopping potential for this light ?

Solution : $K_{max} = 4.0 \times 10^{-19}\ \text{J} \times \frac{1\text{eV}}{1.6\times10^{-19}\,\text{J}} = 2.5$ eV.

Then, from $eV_s = K_{max}$, $V_s = 2.5$ V.

Problem 4. What is the threshold wavelength for the material in above problem ?

Solution : 2.5 eV = $\dfrac{12.4\times10^3\,\text{eV.Å}}{3000\text{ Å}} - \dfrac{12.4\times10^3\,\text{eV.Å}}{\lambda_{th}}$

Solving, λ_{th} = 7590 Å.

Problem 5. Find the de Broglie wavelength of a 0.01 kg pellet having a velocity of 10 m/s.

Solution : $\lambda = h/p = \dfrac{6.63\times10^{-34}\,\text{J.s}}{0.01\,\text{kg}\times10\text{ m/s}} = 6.63\times10^{-23}$ Å .

Problem 6. Determine the accelerating potential necessary to give an electron a de Broglie wavelength of 1 Å, which is the size of the interatomic spacing of atoms in a crystal.

Solution : $V = \dfrac{h^2}{2m_0e\lambda^2}$ = 151 V.

Problem 7. Determine the wavelength of the second line of the Paschen series for hydrogen.

Solution . $\dfrac{1}{\lambda} = (1.097\times10^{-3}\text{ Å}^{-1})\left(\dfrac{1}{3^2}-\dfrac{1}{5^2}\right)$ or λ = 12,820 Å.

Problem 8. How many different photons can be emitted by hydrogen atoms that undergo transitions to the ground state from the n = 5 state ?

Solution : No of possible transition from n = 5 are ${}^5C_2$ = 10

Answer. 10 photons.

Problem 9. An electron rotates in a circle around a nucleus with positive charge Ze. How is the electrons' velocity releated to the radius of its orbit ?

Solution : The force on the electron due to the nuclear provides the required centripetal force

$$\frac{1}{4\pi\varepsilon_0}\frac{Ze.\ \ e}{r^2} = \frac{mv^2}{r}$$

$$\Rightarrow\ v = \sqrt{\frac{Ze^2}{4\pi\varepsilon_0.rm}}$$

Ans. $v = \sqrt{\dfrac{Ze^2}{4\pi\varepsilon_0.rm}}$.

Problem 10. (i) Calculate the first three energy levels for positronium.

(ii) Find the wavelength of the H_a line (3 $\rightarrow$ 2 transition) of positronium.

Solution : In positronium electron and positron revolve around their centre of mass

$$\frac{1}{4p\varepsilon_0}\frac{e^2}{r^2} = \frac{mv^2}{r/2} \qquad ......(1)$$

$$\frac{nh}{2\pi} = 2\times mv_{k/2} \qquad ......(2)$$

From (1) & (2)

$$V = \frac{1}{2} \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{nh} \times 2p = \frac{e^2}{4\varepsilon_0 nh}$$

$$TE = -\frac{1}{2} mv^2 \times 2 = -m.\frac{e^4}{16\varepsilon_0^{\,2} n^2 h^2}$$

$$= -6.8 \frac{1}{n^2} \text{ eV}$$

(i) $E_1 = -6.8$ ev

$$E_2 = -6.8 \times \frac{1}{2^2} \text{ eV} = -1.70 \text{ eV}$$

$$E_3 = -6.8 \times \frac{1}{3^2} \text{ eV} = -0.76 \text{ eV}$$

(ii) $\Delta E\ (3 \rightarrow 2) = E_3 - E_2 = -0.76 - (-1.70)$ eV

$= 0.94$ eV

The corresponding wave length

$$\lambda = \frac{1.24 \times 10^4}{0.94} \text{ Å} = 1313 \text{ Å}$$

Ans. (i) –6.8 eV, –1.7 eV, –0.76 eV ;
(ii) 1313 Å .

Problem 11. A H-atom in ground state is moving with intial kinetic energy K. It collides head on with a He^+ ion in ground state kept at rest but free to move. Find minimum value of K so that both the particles can excite to their first excited state.

Solution : Energy available for excitation = $\frac{4k}{5}$

Total energy required for excitation

= 10.2 ev + 40.8 eV

= 51.0 ev

$$\therefore \quad \frac{4k}{5} = 51 \quad \Rightarrow \quad k = 63.75 \text{ eV}$$

Problem 12. A TV tube operates with a 20 kV accelerating potential. What are the maximum–energy X–rays from the TV set ?

Solution : The electrons in the TV tube have an energy of 20 keV, and if these electrons are brought to rest by a collision in which one X–ray photon is emitted, the photon energy is 20 keV.

Problem 13. In the Moseley relation, $\sqrt{v} = a(Z - b)$ which will have the greater value for the constant a for K_α or K_β transition ?

Solution : A is larger for the K_β transitions than for the K_α transitions.