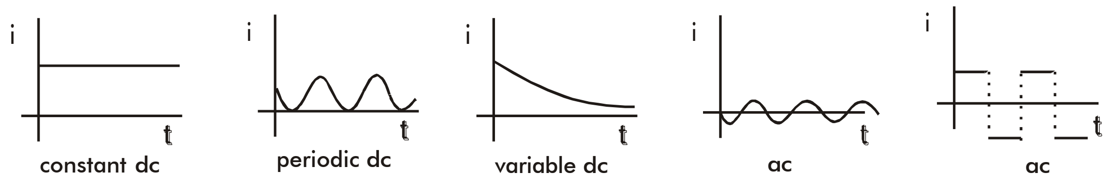
# CHAPTER-27
# ALTERNATING CURRENT

## AC AND DC CURRENT :

A current that changes its direction periodically is called alternating current (AC). If a current maintains its direction constant it is called direct current (DC).

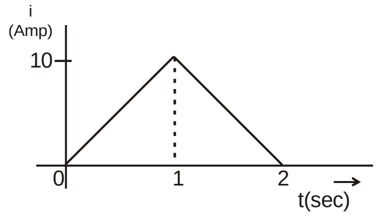



If a function suppose current, varies with time as $i = I_m \sin(\omega t+\phi)$, it is called sinusoidally varying function. Here $I_m$ is the peak current or maximum current and i is the instantaneous current. The factor $(\omega t+\phi)$ is called phase. $\omega$ is called the angular frequency, its unit rad/s. Also $\omega = 2\pi f$ where f is called the frequency, its unit $s^{-1}$ or Hz. Also frequency f = 1/T where T is called the time period.

## 1. AVERAGE VALUE :

Average value of a function, from $t_1$ to $t_2$, is defined as $< f > = \dfrac{\int_{t_1}^{t_2} f\,dt}{t_2 - t_1}$. We can find the value of $\int_{t_1}^{t_2} f\,dt$ graphically if the graph is simple. It is the area of f-t graph from $t_1$ to $t_2$.

the average value of current shown graphically, from t = 0 to t = 2 sec.

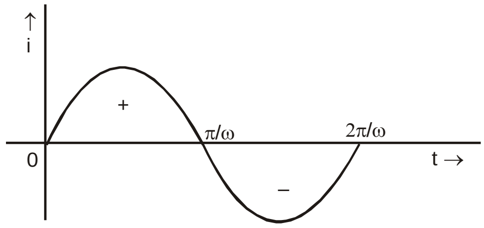



**Solution :** From the i – t graph, area from t = 0 to t = 2 sec = $\dfrac{1}{2} \times 2 \times 10 = 10$ Amp. sec.

$\therefore$ Average Current = $\dfrac{10}{2}$ = 5 Amp.

**Example 2.** Find the average value of current from t = 0 to t = $\dfrac{2\pi}{\omega}$ if the current varies as $i = I_m \sin \omega t$.

**Solution :** $$< i > = \frac{\int_0^{2\pi/\omega} I_m \sin\omega t\,dt}{\frac{2\pi}{\omega}} = \frac{-\frac{I_m}{\omega}\left(1 - \cos\omega\frac{2\pi}{\omega}\right)}{\frac{2\pi}{\omega}} = 0.$$

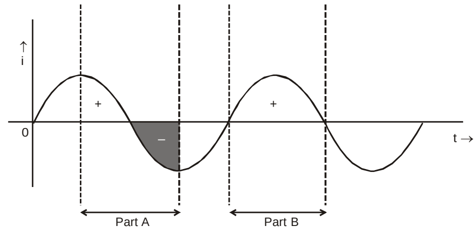



It can be seen graphically that the area of i – t graph of one cycle is zero.

$\therefore$ < i > in one cycle = 0.

**Example 3.** Show graphically that the average of sinusoidally varying current in half cycle may or may not be zero.

**Solution :** Figure shows two parts A and B, each half cycle.
In part A we can see that the net area is zero
$\therefore$ < i > in part A is zero.
In part B,
area is positive hence in this part $\langle i \rangle \neq 0$.




**Example 4.** Find the average value of current $i = I_m \sin \omega t$ from (i) t = 0 to t = $\frac{\pi}{\omega}$ (ii) t = $\frac{\pi}{2\omega}$ to t = $\frac{3\pi}{2\omega}$.

**Solution :** (i) $\langle i \rangle = \dfrac{\int_0^{\pi/\omega} I_m \sin \omega t\,dt}{\frac{\pi}{\omega}} = \dfrac{\frac{I_m}{\omega}\left(1 - \cos \omega \frac{\pi}{\omega}\right)}{\frac{\pi}{\omega}} = \dfrac{2I_m}{\pi}$ (ii) $\langle i \rangle = \dfrac{\int_{\pi/2\omega}^{3\pi/2\omega} I_m \sin \omega t\,dt}{\frac{\pi}{\omega}} = 0.$

**Example 5.** Current in an A.C. circuit is given by $i = 2\sqrt{2} \sin(\pi t + \pi/4)$, then the average value of current during time t = 0 to t = 1 sec is:

**Solution :** $\langle i \rangle = \dfrac{\int_0^1 i\,dt}{1} = 2\sqrt{2} \int_0^1 \sin\left(\pi t + \dfrac{\pi}{4}\right) = \dfrac{4}{\pi}$ **Ans.**

## 2. ROOT MEAN SQUARE VALUE :

Root Mean Square Value of a function, from $t_1$ to $t_2$, is defined as $f_{rms} = \sqrt{\dfrac{\int_{t_1}^{t_2} f^2 dt}{t_2 - t_1}}$.

**Example 6.** Find the rms value of current from t = 0 to t = $\frac{2\pi}{\omega}$ if the current varies as $i = I_m \sin \omega t$.

**Solution :** $i_{rms} = \sqrt{\dfrac{\int_0^{2\pi/\omega} I_m^2 \sin^2 \omega t\,dt}{\frac{2\pi}{\omega}}} = \sqrt{\dfrac{I_m^2}{2}} = \dfrac{I_m}{\sqrt{2}}$

**Example 7.** Find the rms value of current $i = I_m \sin \omega t$ from (i) t = 0 to t = $\frac{\pi}{\omega}$ (ii) t = $\frac{\pi}{2\omega}$ to t = $\frac{3\pi}{2\omega}$.

**Solution :** (i) $i_{rms} = \sqrt{\dfrac{\int_0^{\pi/\omega} I_m^2 \sin^2 \omega t\,dt}{\frac{\pi}{\omega}}} = \sqrt{\dfrac{I_m^2}{2}} = \dfrac{I_m}{\sqrt{2}}$

(ii) $\langle i \rangle = \sqrt{\dfrac{\int_{\pi/2\omega}^{3\pi/2\omega} I_m^2 \sin^2 \omega t\,dt}{\frac{\pi}{\omega}}} = \sqrt{\dfrac{I_m^2}{2}} = \dfrac{I_m}{\sqrt{2}}$

**Note:**
- The r m s values for one cycle and half cycle (either positive half cycle or negative half cycle) is same.
- From the above two examples note that for sinusoidal functions **rms value** (Also called **effective value**) = $\frac{\text{peak value}}{\sqrt{2}}$ or $I_{rms} = \frac{I_m}{\sqrt{2}}$

**Example 8.** Find the effective value of current $i = 2 \sin 100\,\pi t + 2 \cos (100\,\pi t + 30^{\circ})$.

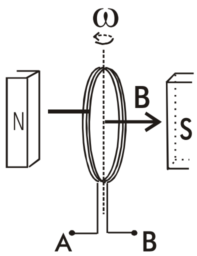
**Solution :** The equation can be written as $i = 2 \sin 100\,\pi t + 2 \sin (100\,\pi t + 120^{\circ})$

so phase difference $\phi = 120^{\circ}$ $\quad I_m)_{res} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$= \sqrt{4 + 4 + 2 \times 2 \times 2\left(-\frac{1}{2}\right)}$ **=** 2, so effective value or rms value = $2/\sqrt{2} = \sqrt{2}$ A

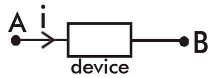
## 3. AC SINUSOIDAL SOURCE :

Figure shows a coil rotating in a magnetic field. The flux in the coil changes as $\varphi = NBA\cos(\omega t + \phi)$. Emf induced in the coil, from Faraday's law is $\frac{-d\varphi}{dt} = N\,B\,A\,\omega \sin(\omega t + \phi)$. Thus the emf between the points A and B will vary as $E = E_0 \sin(\omega t + \phi)$ .The potential difference between the points A and B will also vary as $V = V_0 \sin(\omega t + \phi)$. The symbolic notation of the above arrangement is A •—(~)—• B. We do not put any + or – sign on the AC source.




## 4. POWER CONSUMED OR SUPPLIED IN AN AC CIRCUIT:

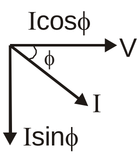
Consider an electrical device which may be a source, a capacitor, a resistor, an inductor or any combination of these. Let the potential difference be $V = V_A - V_B = V_m \sin \omega t$. Let the current through it be $i = I_m \sin(\omega t + \phi)$. Instantaneous power P consumed by the device = $V\,i = (V_m \sin \omega t)\,(I_m \sin(\omega t + \phi))$




Average power consumed in a cycle = $\dfrac{\int_0^{2\pi/\omega} P\,dt}{\frac{2\pi}{\omega}} = V_m I_m \cos\phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cdot \cos\phi = V_{rms} I_{rms} \cos\phi$.

Here $\cos\phi$ is called **power factor**.

**Note :** $I\sin\phi$ is called "wattless current".



**Example 9.** When a voltage $v_s = 200\sqrt{2}\sin(\omega t + 15^o)$ is applied to an AC circuit the current in the circuit is found to be $i = 2\sin(\omega t + \pi/4)$ then average power consumed in the circuit is

(A) 200 watt (B) $400\sqrt{2}$ watt (C) $100\sqrt{6}$ watt (D) $200\sqrt{2}$ watt

**Solution :** $P_{av} = v_{rms}\, I_{rms} \cos\phi = \frac{200\sqrt{2}}{\sqrt{2}} \cdot \frac{2}{\sqrt{2}} \cos(30^o) = 100\sqrt{6}$ watt

## 5. SOME DEFINITIONS :

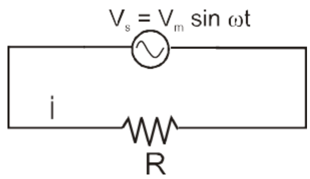
The factor $\cos\phi$ is called Power factor.

$I_m \sin\phi$ is called wattless current.

Impedance Z is defined as $Z = \frac{V_m}{I_m} = \frac{V_{rms}}{I_{rms}}$

$\omega L$ is called inductive reactance and is denoted by $X_L$. $\frac{1}{\omega C}$ is called capacitive reactance and is denoted by $X_C$.

## 6. PURELY RESISTIVE CIRCUIT:




Writing KVL along the circuit,

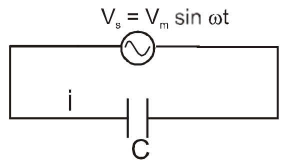
$V_s - iR = 0$

or $i = \frac{V_s}{R} = \frac{V_m \sin\omega t}{R} = I_m \sin\omega t$

$\Rightarrow$ We see that the phase difference between potential difference across resistance, $V_R$ and $i_R$ is 0.

$I_m = \frac{V_m}{R}$ $\Rightarrow$ $I_{rms} = \frac{V_{rms}}{R}$ ; $\langle P\rangle = V_{rms} I_{rms} \cos\phi = \frac{V_{rms}^2}{R}$

## 7. PURELY CAPACITIVE CIRCUIT:

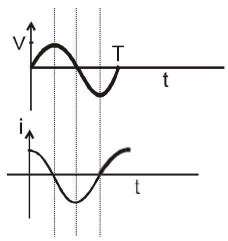



Writing KVL along the circuit, $V_s - \frac{q}{C} = 0$

or $i = \frac{dq}{dt} = \frac{d(CV)}{dt} = \frac{d(CV_m \sin\omega t)}{dt}$

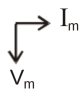
$= CV_m\omega \cos\omega t = \frac{V_m}{1/\omega C} \cos\omega t = \frac{V_m}{X_C} \cos\omega t = I_m \cos\omega t.$

$X_C = \frac{1}{\omega C}$ and is called capacitive reactance. Its unit is ohm $\Omega$.




From the graph of current versus time and voltage versus time $\frac{T}{4}$, it is clear that current attains its peak value at a time before the time at which voltage attains its peak value. Corresponding to $\frac{T}{4}$ the phase difference

$= \omega\Delta t = \frac{2\pi}{T}\frac{T}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$. $i_C$ leads $v_C$ by $\pi/2$ Diagrammatically (phasor diagram) it is represented as




Since $\phi = 90^o$, $\langle P \rangle = V_{rms}\, I_{rms} \cos\phi = 0$

**Example 10.** An alternating voltage $E = 200\sqrt{2}\sin(100\,t)$ V is connected to a $1\mu F$ capacitor through an ac ammeter (it reads rms value). What will be the reading of the ammeter?

**Solution :** Comparing $E = 200\sqrt{2}\sin(100\,t)$ with $E = E_0 \sin \omega t$ we find that,

$E_0 = 200\sqrt{2}$ V and $\omega = 100$ (rad/s)

So, $X_C = \frac{1}{\omega C} = \frac{1}{100 \times 10^{-6}} = 10^4 \Omega$

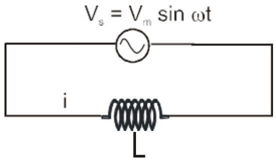
And as ac instruments reads rms value, the reading of ammeter will be,

$I_{rms} = \frac{E_{rms}}{X_C} = \frac{E_0}{\sqrt{2}X_C}$ $\left[\text{as } E_{rms} = \frac{E_0}{\sqrt{2}}\right]$

i.e. $I_{rms} = \frac{200\sqrt{2}}{\sqrt{2} \times 10^4} = 20\text{mA}$ **Ans.**

## 8. PURELY INDUCTIVE CIRCUIT:

Writing KVL along the circuit,

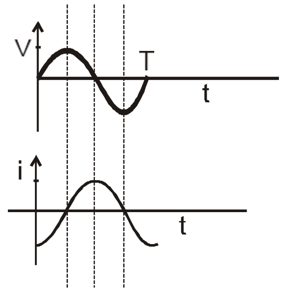



$V_s - L\frac{di}{dt} = 0 \quad \Rightarrow \quad L\frac{di}{dt} = V_m \sin \omega t$

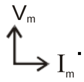
$\int L di = \int V_m \sin\omega t \, dt \Rightarrow \quad i = -\frac{V_m}{\omega L}\cos \omega t + C$

$< i > = 0 \qquad C = 0$

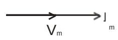
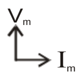
$\therefore \; i = -\frac{V_m}{\omega L}\cos\omega t \quad \Rightarrow \quad I_m = \frac{V_m}{X_L}$



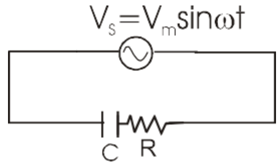

From the graph of current versus time and voltage versus time $\frac{T}{4}$, it is clear that voltage attains its peak value at a time before the time at which current attains its peak value. Corresponding to $\frac{T}{4}$ the phase difference $= \omega\Delta t = \frac{2\pi}{T}\frac{T}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$.

Diagrammatically (phasor diagram) it is represented as . $i_L$ lags behind $v_L$ by $\pi/2$. Since $\phi = 90^o$, $< P > = V_{rms}I_{rms}\cos\phi = 0$

**Summary :**

| AC source connected with | $\phi$ | | Z | Phasor Diagram |
|---|---|---|---|---|
| Pure Resistor | 0 | $V_R$ is in same phase with $i_R$ | R |  |
| Pure Inductor | $\pi/2$ | $V_L$ leads $i_L$ | $X_L$ |  |
| Pure Capacitor | $\pi/2$ | $V_C$ lags $i_C$ | $X_C$ | $I_m$ $V_m$ |

## 9. RC SERIES CIRCUIT WITH AN AC SOURCE :




Let $i = I_m \sin(\omega t + \phi)$ $\Rightarrow$ $V_R = iR = I_m R \sin(\omega t + \phi)$

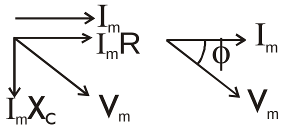
$V_C = (I_m X_C)\sin\left(\omega t + \phi - \frac{\pi}{2}\right)$ $\Rightarrow$ $V_S = V_R + V_C$

or $V_m \sin(\omega t + \phi) = I_m R \sin(\omega t + \phi) + I_m X_C \sin\left(\omega t + \phi - \frac{\pi}{2}\right)$

$$V_m = \sqrt{(I_m R)^2 + (I_m X_C)^2 + 2(I_m R)(I_m X_C)\cos\frac{\pi}{2}}$$

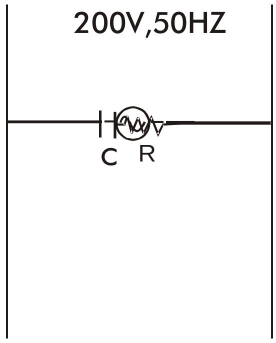
OR $I_m = \dfrac{V_m}{\sqrt{R^2 + X_C^2}}$ $\Rightarrow$ $Z = \sqrt{R^2 + X_C^2}$

Using phasor diagram also we can find the above result.




$$\tan\phi = \frac{I_m X_C}{I_m R} = \frac{X_C}{R}.$$

**Example 11.** In an RC series circuit, the rms voltage of source is 200V and its frequency is 50 Hz. If R =100 Ω and C= $\frac{100}{\pi}$ μF, find




(i) Impedance of the circuit (ii) Power factor angle
(iii) Power factor (iv) Current
(v) Maximum current (vi) voltage across R
(vii) voltage across C (viii) max voltage across R
(ix) max voltage across C (x) < P >
(xi) < $P_R$ > (xii) < $P_C$ >

**Solution :** $X_C = \dfrac{10^6}{\frac{100}{\pi}(2\pi 50)} = 100\ \Omega$

(i) $Z = \sqrt{R^2 + X_C^2} = \sqrt{100^2 + (100)^2} = 100\sqrt{2}\ \Omega$

(ii) $\tan\phi = \dfrac{X_C}{R} = 1$ $\therefore \phi = 45^o$

(iii) Power factor = $\cos\phi = \dfrac{1}{\sqrt{2}}$

(iv) Current $I_{rms} = \dfrac{V_{rms}}{Z} = \dfrac{200}{100\sqrt{2}} = \sqrt{2}$ A

(v) Maximum current $= I_{rms}\sqrt{2} = 2$A

(vi) voltage across R = $V_{R,rms} = I_{rms} R = \sqrt{2} \times 100$ Volt

(vii) voltage across C = $V_{C,rms} = I_{rms} X_C = \sqrt{2} \times 100$ Volt

(viii) max voltage across R = $\sqrt{2}\, V_{R,rms} = 200$ Volt

(ix) max voltage across C = $\sqrt{2}\, V_{C,rms} = 200$ Volt

(x) $\langle P \rangle = V_{rms} I_{rms} \cos\phi = 200 \times \sqrt{2} \times \dfrac{1}{\sqrt{2}} = 200$ Watt

(xi) $\langle P_R \rangle = I_{rms}^2 R = 200$ W

(xii) $\langle P_C \rangle = 0$

**Example 12.** In the above question if $V_s(t) = 220\sqrt{2}\,\sin(2\pi\,50\,t)$, find (a) i (t), (b) $v_R$ and (c) $v_C$ (t)

**Solution :** (a) $i(t) = I_m \sin(\omega t + \phi) = 2\sin(2\pi\,50\,t + 45^\circ)$

(b) $V_R = i_R\,.\,R = i(t)\,R = 2\times 100\sin(100\,\pi t + 45^\circ)$

(c) $V_C\,(t) = i_C X_C$ (with a phase lag of 90º) $= 2\times 100\sin(100\,\pi t + 45 - 90)$

**Example 13.** An ac source of angular frequency $\omega$ is fed across a resistor R and a capacitor C in series. The current registered is I. If now the frequency of source is changed to $\omega/3$ (but maintaining the same voltage), the current in the circuit is found to be halved. Calculate the ratio of reactance to resistance at the original frequency $\omega$.

**Solution :** According to given problem,

$$I = \frac{V}{Z} = \frac{V}{[R^2 + (1/C\omega)^2]^{1/2}} \qquad .....(1)$$

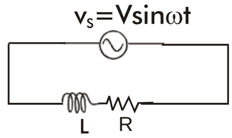
and, $$\frac{I}{2} = \frac{V}{[R^2 + (3/C\omega)^2]^{1/2}} \qquad .....(2)$$

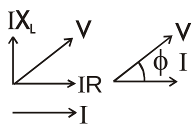
Substituting the value of I from Equation (1) in (2),

$$4\left(R^2 + \frac{1}{C^2\omega^2}\right) = R^2 + \frac{9}{C^2\omega^2}\,.\ \text{i.e.,}\ \frac{1}{C^2\omega^2} = \frac{3}{5}R^2$$

So that, $$\frac{X}{R} = \frac{(1/C\omega)}{R} = \frac{\left(\frac{3}{5}R^2\right)^{1/2}}{R} = \sqrt{\frac{3}{5}} \qquad \textbf{Ans.}$$

## 10. LR SERIES CIRCUIT WITH AN AC SOURCE :




From the phasor diagram $V = \sqrt{(IR)^2 + (IX_L)^2} = I\sqrt{(R)^2 + (X_L)^2} = IZ \Rightarrow \tan\phi = \frac{IX_L}{IR} = \frac{X_L}{R}$

**Example 14.** A $\frac{9}{100\pi}$ H inductor and a 12 ohm resistance are connected in series to a 225 V, 50 Hz ac source. Calculate the current in the circuit and the phase angle between the current and the source voltage.

**Solution :** Here $X_L = \omega L = 2\pi f L = 2\pi \times 50 \times \frac{9}{100\pi} = 9\ \Omega$

So, $Z = \sqrt{R^2 + X_L^2} = \sqrt{12^2 + 9^2} = 15\ \Omega$

So (a) $I = \frac{V}{Z} = \frac{225}{15} = 15$ A **Ans.**

and (b) $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{9}{12}\right) = \tan^{-1} 3/4 = 37^\circ$

i.e., the current will lag the applied voltage by 37º in phase. **Ans.**

**Example 15.** When an inductor coil is connected to an ideal battery of emf 10 V, a constant current 2.5 A flows. When the same inductor coil is connected to an AC source of 10 V and 50 Hz then the current is 2A. Find out inductance of the coil.

**Solution :** When the coil is connected to dc source, the final current is decided by the resistance of the coil .

$\therefore \quad r = \frac{10}{2.5} = 4\ \Omega$

When the coil is connected to ac source, the final current is decided by the impedance of the coil.

$\therefore \quad Z = \frac{10}{2} = 5\,\Omega$

But Z = $\sqrt{(r)^2 + (X_L)^2}$ $\qquad X_L^2 = 5^2 - 4^2 = 9$

$X_L = 3\ \Omega$

$\therefore \quad \omega L = 2\pi fL = 3$

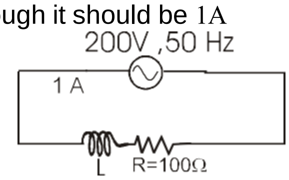
$\therefore \quad 2\pi\, 50\, L = 3 \qquad \therefore \quad L = 3/100\pi$ Henry

**Example 16.** A bulb is rated at 100 V, 100 W, it can be treated as a resistor .Find out the inductance of an inductor (called choke coil) that should be connected in series with the bulb to operate the bulb at its rated power with the help of an ac source of 200 V and 50 Hz.

**Solution :** From the rating of the bulb , the resistance of the bulb is $R = \frac{V_{rms}^2}{P} = 100\ \Omega$

For the bulb to be operated at its rated value the rms current through it should be 1A

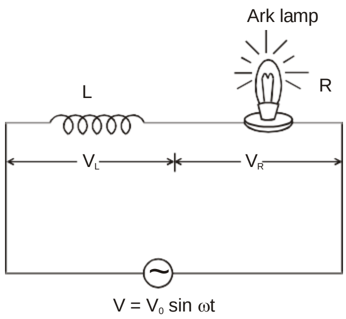
Also, $\quad I_{rms} = \frac{V_{rms}}{Z}$




$\therefore \quad 1 = \frac{200}{\sqrt{100^2 + (2\pi 50L)^2}} \qquad ; L = \frac{\sqrt{3}}{\pi}$ H

**Example 17.** A choke coil is needed to operate an arc lamp at 160 V (rms) and 50 Hz. The arc lamp has an effective resistance of 5 Ω when running of 10 A (rms). Calculate the inductance of the choke coil. If the same arc lamp is to be operated on 160 V (dc), what additional resistance is required? Compare the power losses in both cases.

**Solution :** As for lamp $V_R = IR = 10 \times 5 = 50$ V, so when it is connected to 160 V ac source through a choke in series,




$V^2 = V_R^2 + V_L^2$,

$V_L = \sqrt{160^2 - 50^2} = 152$ V

and as, $V_L = IX_L = I\omega L = 2\pi fLI$

So, $L = \frac{V_L}{2\pi fI} = \frac{152}{2 \times \pi \times 50 \times 10} = 4.84 \times 10^{-2}$ H **Ans.**

Now the lamp is to be operated at 160 V dc; instead of choke if additional resistance r is put in series with it,

V = I(R + r), i.e., 160 = 10(5 + r) $\qquad$ i.e., $\qquad$ r = 11 Ω $\qquad$ **Ans.**

In case of ac, as choke has no resistance, power loss in the choke will be zero while the bulb will consume,

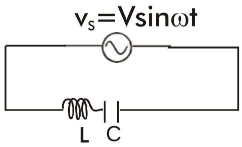
$P = I^2 R = 10^2 \times 5 = 500$ W

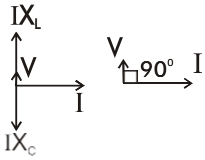
However, in case of dc as resistance r is to be used instead of choke, the power loss in the resistance r will be.

$PL = 10^2 \times 11 = 1100$ W

while the bulb will still consume 500 W, i.e., when the lamp is run on resistance r instead of choke more than double the power consumed by the lamp is wasted by the resistance r.

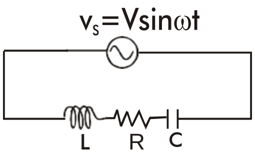
## 12. LC SERIES CIRCUIT WITH AN AC SOURCE :

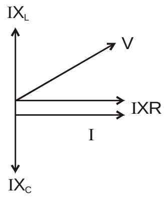


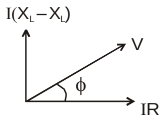




From the phasor diagram $V = I\left|(X_L - X_C)\right| = IZ$ $\phi = 90^o$

## 13. RLC SERIES CIRCUIT WITH AN AC SOURCE :




From the phasor diagram

$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2} = I\sqrt{(R)^2 + (X_L - X_C)^2} = IZ \quad Z = \sqrt{(R)^2 + (X_L - X_C)^2}$$

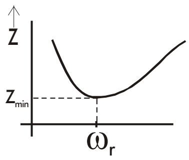
$$\tan\phi = \frac{I(X_L - X_C)}{IR} = \frac{(X_L - X_C)}{R}$$

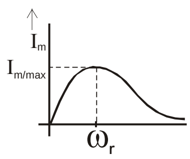
### 13.1 Resonance :

Amplitude of current (and therefore $I_{rms}$ also) in an RLC series circuit is maximum for a given value of $V_m$ and R , if the impedance of the circuit is minimum, which will be when $X_L$-$X_C$ =0. This condition is called **resonance**.

So at resonance: $X_L$-$X_C$ =0.

or $\omega L = \frac{1}{\omega C}$ or $\omega = \frac{1}{\sqrt{LC}}$. Let us denote this $\omega$ as $\omega_r$.




**Example 18.** In the circuit shown in the figure, find :

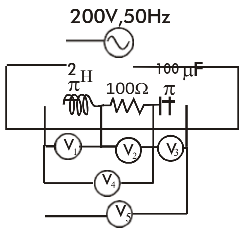
(a) the reactance of the circuit .

(b) impedance of the circuit

(c) the current

(d) readings of the ideal AC voltmeters

(these are hot wire instruments and read rms values).




**Solution :** (a) $X_L = 2\pi fL = 2\pi \times 50 \times \frac{2}{\pi} = 200\Omega$

$$X_C = \frac{1}{2\pi 50 \frac{100}{\pi} \times 10^{-6}} = 100\Omega$$

$\therefore$ The reactance of the circuit $X = X_L - X_C = 200\text{-}100 = 100\ \Omega$

Since $X_L > X_C$, the circuit is called inductive.

(b) impedance of the circuit $Z = \sqrt{R^2 + X^2} = \sqrt{100^2 + 100^2} = 100\Omega$

(c) the current $I_{rms} = \frac{V_{rms}}{Z} = \frac{200}{100\sqrt{2}} = \sqrt{2}$ A

(d) readings of the ideal voltmeter

$V_1$ : $I_{rms}X_L = 200\sqrt{2}$ Volt

$V_2$ : $I_{rms}R = 100\sqrt{2}$ Volt

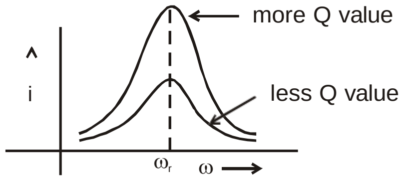
$V_3$ : $I_{rms}X_c = 100\sqrt{2}$ Volt

$V_4$ : $I_{rms}\sqrt{R^2 + X_L^2} = 100\sqrt{10}$ Volt

$V_5$ : $I_{rms}Z = 200$ Volt, which also happens to be the voltage of source.

## 13.2 Q VALUE (QUALITY FACTOR) OF LCR SERIES CIRCUIT (NOT IN IIT SYLLABUS) :

Q value is defined as $\frac{X_L}{R}$ where $X_L$ is the inductive reactance of the circuit, at resonance. More Q value implies more sharpness of I v/s $\omega$ curve.




Quality factor : $Q = \frac{X_L}{R} = \frac{X_C}{R}$

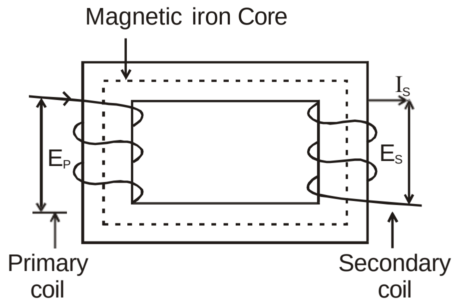
$$Q = \frac{\text{Resonance freq.}}{\text{Band width}} = \frac{\omega_R}{\Delta\omega} = \frac{f_R}{f_2 - f_1}$$

where $f_1$ & $f_2$ are half power frequencies.

## 14. TRANSFORMER

**A transformer** changes an alternating potential difference from one value to another of greater or smaller value using the principle of mutual induction. Two coils called the primary and secondary windings, which are not connected to one another in any way, are wound on a complete soft iron core. When an alternating voltage $E_P$ is applied to the primary, the resulting current produces a large alternating magnetic flux which links the secondary and induces an emf $E_S$ in it. It can be shown that for an ideal transformer

$$\frac{E_s}{E_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s} \;;$$




$\frac{N_s}{N_p}$ = turns ratio of the transformer.

$E_S$, N and I are the emf, number of turns and current in the coils.

$N_S > N_P \Rightarrow E_S > E_P \rightarrow$ step up transformer.

$N_S < N_P \Rightarrow E_S < E_P \rightarrow$ step down transformer.

**Note :** Phase difference between the primary and secondary voltage is $\pi$.

## 15. ENERGY LOSSES IN TRANSFORMER

Although transformers are very efficient devices, small energy losses do occur in them due to four main causes.

### 15.1. RESISTANCE OF THE WINDINGS :

The copper wire used for the windings has resistance and so $I^2R$ heat losses occur.

### 15.2. EDDY CURRENT :

Eddy current is induced in a conductor when it is placed in a changing magnetic field or when a conductor is moved in a magnetic field and/or both. Any imagined circuit within the conductor will change its magnetic flux linkage and the subsequent induced emf. will drive current around the circuit. Thus the alternating magnetic flux induces eddy currents in the iron core and causes heating. The effect is reduced by **laminating** the core, i.e., the core is made of this sheets of iron with insulating sheets between them so that the circuits for the eddy currents are broken.

### 15.3. HYSTERESIS :

The magnetization of the core is repeatedly reversed by the alternating magnetic field. The resulting expenditure of energy in the core appears as heat and is kept to a minimum by using a magnetic material which has a low hysteresis loss.

### 15.4. FLUX LEAKAGE :

The flux due to the primary may not all link the secondary if the core is badly designed or has air gaps in it .Very large transformers have to be oil cooled to prevent overheating.

**Example 19.** In a step-up transformer the turns ratio is 10. If the frequency of the current in the primary coil is 50 Hz then the frequency of the current in the secondary coil will b

(A) 500 Hz (B) 5 Hz (C) 60 Hz (D*) 50 Hz

**Solution :** Frequency of the current remains same, only magnitudes of current changes in a tranformer.

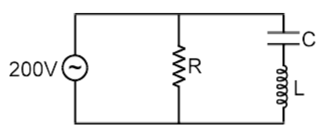
**Example 20.** A power transformer is used to step up an alternating emf of 220 volt to11 kv to transmit 4.4 kw of power. If the primary coil has 1000 turns, what is the current in the secondary?

(A) 4 A (B) 0.4 A (C) 0.04 A (D) 0.2 A

**Answer :** **(C)**

**Solution :** $I_s = P_S/V_S \Rightarrow I_S = \dfrac{4.4\times10^3}{11\times10^3} = 0.4A$ **Ans.**

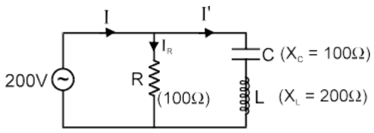
**Example 21.** In the circuit diagram shown, $X_C = 100\Omega$, $X_L = 200\Omega$ & $R = 100\Omega$. The effective current through the source is:

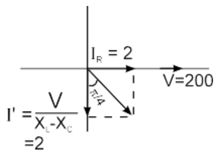



(A) 2 A (B) $2\sqrt{2}$ A

(C) 0.5 A (D) $\sqrt{0.4}$ A

**Solution :** $I_R = \frac{V}{R} = \frac{200}{100} = 2A$

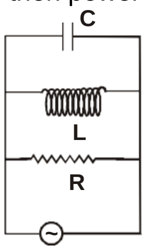



$I' = \frac{V}{X_L - X_C} = \frac{200}{100} = 2A$

$I = \sqrt{I_R^2 + I'^2} = 2\sqrt{2}$ Amp.

**Example 22.** If for above circuit the capacitive reactance is two times of Inductive Reactance, and resistance R is equal to Inductive Reactance then power factor of circuit is.




(A) 1 (B) Zero (C) $\frac{1}{2}$ (D) $\frac{2}{\sqrt{5}}$

**Answer :** **(D)**

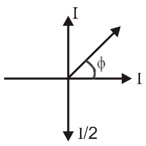
**Solution :** $X_C = 2X_L = 2R$

$$\frac{1}{z^2} = \frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2 = \frac{1}{R^2} + \left(\frac{1}{2R} - \frac{1}{R}\right)^2 = \frac{5}{4R^2}$$

$$z = \frac{2R}{\sqrt{5}}$$

Power factor in parallel combination = $\cos\theta = \frac{z}{R} = \frac{2}{\sqrt{5}}$.

**Alternate solution:**




$$\cos\phi = \frac{I}{\sqrt{I^2 + \frac{I^2}{4}}} = \frac{2}{\sqrt{5}}$$

**Problem 1.** The peak voltage in a 220 V AC source is

(A) 220 V (B) about 160 V (C) about 310 V (D) 440 V

**Solution :** $V_0 = \sqrt{2}\,V_{rms} = \sqrt{2} \times 220 \simeq 330$ V

**Ans.** is (C)

**Problem 2.** An AC source is rated 220V, 50 Hz. The average voltage is calculated in a time interval of 0.01 s. It

(A) must be zero (B) may be zero (C) is never zero (D) is (220/√2)V

**Solution :** May be zero

**Ans.** is (B)

**Problem 3.** Find the effective value of current i = 2 + 4 cos 100 $\pi$ t.

**Solution :** $I_{rms} = \left[\int_0^T \frac{(2 + 4\cos 100\pi t)^2\,dt}{T}\right]^{1/2} = 2\sqrt{3}$

**Problem 4.** The peak value of an alternating current is 5 A and its frequency is 60 Hz. Find its rms value. How long will the current take to reach the peak value starting from zero?

**Solution :** $I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{5}{\sqrt{2}}$ A, $t = \frac{T}{4} = \frac{1}{240}$ s

**Problem 5.** An alternating current having peak value 14 A is used to heat a metal wire. To produce the same heating effect, a constant current i can be used where i is

(A) 14 A (B) about 20 A (C) 7 A (D) about 10 A

**Solution :** $I_{RMS} = \frac{I_0}{\sqrt{2}} = \frac{14}{\sqrt{2}} \simeq 10$

**Ans.** is (D)

**Problem 6.** Find the average power concumed in the circuit if a voltage $v_s = 200\sqrt{2} \sin \omega t$ is applied to an AC circuit and the current in the circuit is found to be $i = 2 \sin (\omega t + \pi/4)$ .

**Solution :** $P = V_{RMS} I_{RMS} \cos \phi = \frac{200\sqrt{2}}{\sqrt{2}} \times \frac{2}{\sqrt{2}} \times \cos \frac{\pi}{4} = 200$ W

**Problem 7.** A capacitor acts as an infinite resistance for

(A) DC (B) AC (C) DC as well as AC (D) neither AC nor DC

**Solution :** $x_C = \frac{1}{\omega c}$ for DC $\omega = 0$. So, $x_C = \infty$

**Ans.** is (A)

**Problem 8.** A 10 $\mu$F capacitor is connected with an ac source $E = 200\sqrt{2} \sin (100\ t)$ V through an ac ammeter (it reads rms value). What will be the reading of the ammeter?

**Solution :** $I_0 = \frac{V_0}{x_C} = \frac{200\sqrt{2}}{1/\omega C}$ ; $I_{RMS} = \frac{I_0}{\sqrt{2}} = 200$ mA

**Problem 9.** Find the reactance of a capacitor (C = 200 $\mu$F) when it is connected to (a) 10 Hz AC source, (b) a 50 Hz AC source and (c) a 500 Hz AC source.

**Solution :** (a) $x_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \simeq 80\Omega$ for f = 10 Hz AC source,

(b) $x_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \simeq 16\Omega$ for f = 50 Hz and

(c) $x_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \simeq 1.6\Omega$ for f= 500 Hz.

**Problem 10.** An inductor (L = 200mH) is connected to an AC source of peak current. What is the instantaneous voltage of the source when the current is at its peak value?

**Solution :** Because phase difference between voltage and current is $\pi/2$ for pure inductor.

So, Ans. is zero

**Problem 11.** An AC source producing emf $E = E_0[\cos(100\pi\ s^{-1})t + \cos(500\ \pi s^{-1})t]$ is connected in series with a capacitor and a resistor. The current in the circuit is found to be

$i = i_1 \cos[(100\ \pi s^{-1})t + \varphi_1] + i_2 \cos[(500\ \pi s^{-1})t + \varphi_1]$

(A) $i_1 > i_2$ (B) $i_1 = i_2$ (C) $i_1 < i_2$

(D) the information is insufficient to find the relation between $i_1$ and $i_2$

**Solution :** Impedence z is given by $z = \sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2}$

For higher $\omega$, z will be lower so current will be higher. Ans is (C)

**Problem 12.** An alternating voltage of 220 volt r.m.s. at a frequency of 40 cycles/sec is supplied to a circuit containing a pure inductance of 0.01 H and a pure resistance of 6 ohms in series. Calculate (i) the current, (ii) potential difference across the resistance, (iii) potential difference across the inductance, (iv) the time lag, (v) power factor.

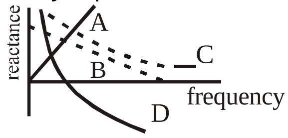
**Solution :** (i) $z = \sqrt{(\omega L)^2 + R^2} = \sqrt{(2\pi \times 40 \times 0.01^2)^2 + 6^2} = \sqrt{(42.4)}$

$I_{rms} = \dfrac{220}{z} = 33.83$ amp.

(ii) $V_{rms} = I_{rms} \times R = 202.98$ volts (iii) $\omega L \times I_{rms} = 96.83$ volts

(iv) $t = T\dfrac{\phi}{2\pi} = 0.01579$ sec (v) $\cos\phi = \dfrac{R}{Z} = 0.92$

**Problem 13.** Which of the following plots may represents the reactance of a series LC combination ?




**Answer :** (D)

**Problem 14.** A series AC circuit has resistance of 4Ω and a reactance of 3Ω. The impedance of the circuit is

(A) 5Ω (B) 7Ω (C) 12/7 Ω (D) 7/12 Ω

**Solution :** $Z = \sqrt{4^2 + 3^2} = 5\Omega$

**Ans.** is (A)