CHAPTER-26
ELECTROMAGNETIC INDUCTION

1. FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

(i) When magnetic flux passing through a loop changes with time or magnetic lines of force are cut by a conducting wire then an emf is produced in the loop or in that wire. This emf is called induced emf.
If the circuit is closed then the current will be called induced current.

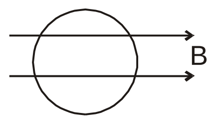
magnetic flux = $\int \vec{B}.\vec{ds}$

(ii) The magnitude of induced emf is equal to the rate of change of flux w.r.t. time in case of loop. In case of a wire it is equal to the rate at which magnetic lines of force are cut by a wire

$E = -\frac{d\phi}{dt}$

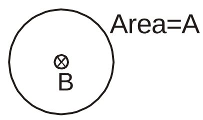
(–) sign indicates that the emf will be induced in such a way that it will oppose the change of flux.
SI unit of magnetic flux = Weber.

Example 1. A coil is placed in a constant magnetic field .The magnetic field is parallel to the plane of the coil as shown in figure. Find the emf induced in the coil.



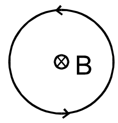
Solution : ϕ = 0 (always) since area is perpendicular to magnetic field. $\therefore$ emf = 0

Example 2. Find the emf induced in the coil shown in figure. The magnetic field is perpendicular to the plane of the coil and is constant.



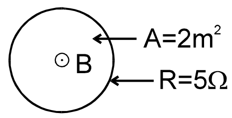
Solution : ϕ = BA (always)
= const. $\therefore$ emf = 0

Example 3. Find the direction of induced current in the coil shown in figure. Magnetic field is perpendicular to the plane of coil and it is increasing with time.



Solution : Inward flux is increasing with time. To opposite it outward magnetic field should be induced. Hence current will flow anticlockwise.

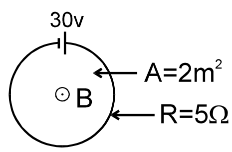
Example 4. Figure shows a coil placed in decreasing magnetic field applied perpendicular to the plane of coil .The magnetic field is decreasing at a rate of 10T/s. Find out current in magnitude and direction



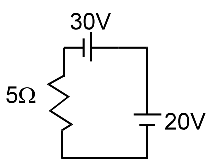
Solution : ϕ = B.A $\Rightarrow$ emf = A . $\frac{dB}{dt}$ = 2 × 10 = 20 v

$\therefore$ i = 20/ 5 = 4 amp. From Lenz's law direction of current will be anticlockwise.

Example 5. Figure shows a coil placed in a magnetic field decreasing at a rate of 10T/s. There is also a source of emf 30 V in the coil. Find the magnitude and direction of the current in the coil.

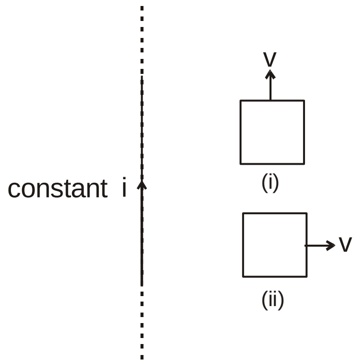


Solution :



Induce emf = 20V
equivalent i = 2A clockwise

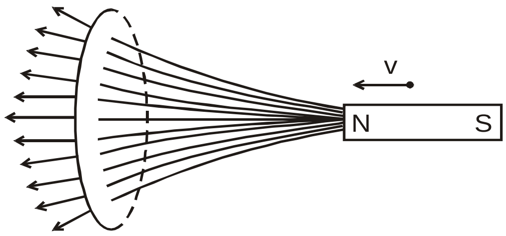
Example 6. Figure shows a long current carrying wire and two rectangular loops moving with velocity v. Find the direction of current in each loop.



Solution : In loop (i) no emf will be induced because there is no flux change.
In loop (ii) emf will be induced because the coil is moving in a region of decreasing magnetic field inward in direction. Therefore to oppose the flux decrease in inward direction, current will be induced such that its magnetic field will be inwards. For this direction of current should be clockwise.

2. LENZ'S LAW (CONSERVATION OF ENERGY PRINCIPLE)

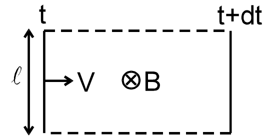
According to this law, emf will be induced in such a way that it will oppose the cause which has produced it.
Figure shows a magnet approaching a ring with its north pole towards the ring.



We know that magnetic field lines come out of the north pole and magnetic field intensity decreases as we move away from magnet. So the magnetic flux (here towards left) will increase with the approach of magnet. This is the cause of flux change. To oppose it, induced magnetic field will be towards right. For this the current must be anticlockwise as seen by the magnet.
If we consider the approach of North pole to be the cause of flux change, the Lenz's law suggests that the side of the coil towards the magnet will behave as North pole and will repel the magnet. We know that a current carrying coil will behave like North pole if it flows anticlockwise. Thus as seen by the magnet, the current will be anticlockwise.
If we consider the approach of magnet as the cause of the flux change, Lenz's law suggest that a force opposite to the motion of magnet will act on the magnet, whatever be the mechanism.
Lenz's law tells that if the coil is set free, it will move away from magnet, because in doing so it will oppose the 'approach' of magnet.
If the magnet is given some initial velocity towards the coil and is released, it will slow down .It can be explained as the following.
The current induced in the coil will produce heat. From the energy conservation, if heat is produced there must be an equal decrease of energy in some other form, here it is the kinetic energy of the moving magnet. Thus the magnet must slow down. So we can justify that the **Lenz's law is conservation of energy principle**.

3. MOTIONAL EMF

We can find emf induced in a moving rod by considering the number of lines cut by it per sec assuming there are 'B' lines per unit area. Thus when a rod of length ℓ moves with velocity v in a magnetic field B, as shown, it will sweep area per unit time equal to ℓv and hence it will cut $B \ell v$ lines per unit time.

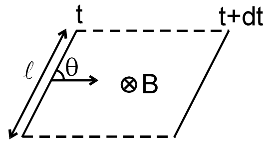


Hence emf induced between the ends of the rod = $Bv\ell$

Also emf = $\frac{d\phi}{dt}$. Here ϕ denotes flux passing through the area,swept by the rod.The rod sweeps an area equal to ℓ vdt in time interval dt.Flux through this area = $B\ell$ vdt. Thus $\frac{d\phi}{dt} = \frac{B\ell vdt}{dt} = Bv\ell$

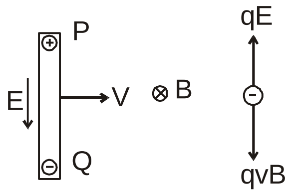
If the rod is moving as shown in the following figure, it will sweep area per unit time = $v\ell\sin\theta$ and hence it will cut $B v \ell \sin\theta$ lines per unit time.

Thus emf = $Bv\ell\sin\theta$.



3.1 EXPLANATION OF EMF INDUCED IN ROD ON THE BASIS OF MAGNETIC FORCE:

If a rod is moving with velocity v in a magnetic field B ,as shown , the free electrons in a rod will experience a magnetic force in downward direction and hence free electrons will accumulate at the lower end and there will be a deficiency of free electrons and hence a surplus of positive charge at the upper end. These charges at the ends will produce an electric field in downward direction which will exert an upward force on electron. If the rod has been moving for quite some time enough charges will accumulate at the ends so that the two forces qE and qvB will balance each other. Thus E = v B.



$V_P - V_Q = VB\ell$

The moving rod is equivalent to the following diagram, electrically.

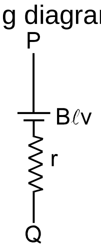
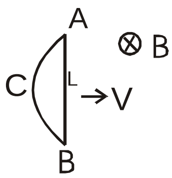
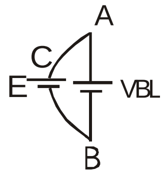


Figure shows a closed coil ABCA moving in a uniform magnetic field **B** with a velocity **v. The** flux passing through the coil is a constant and therefore the induced emf is zero.



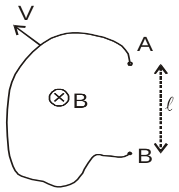
Now consider rod AB, which is a part of the coil. Emf induced in the rod =B L v

Suppose the emf induced in part ACB is E, as shown.



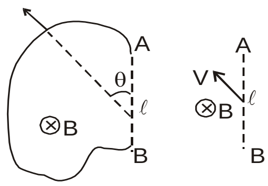
Since the emf in the coil is zero, Emf (in ACB) + Emf (in BA) = 0
or $-E + vBL = 0$ or $E = vBL$
Thus emf induced in any path joining A and B is same, provided the magnetic field is uniform. Also the equivalent emf between A and B is BLv (here the two emf's are in parallel)

Example 7. Figure shows an irregular shaped wire AB moving with velocity v, as shown.



Find the emf induced in the wire.

Solution : The same emf will be induced in the straight imaginary wire joining A and B , which is $Bv\ell \sin\theta$

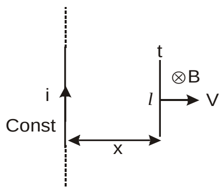


Example 8. A rod of length l is kept parallel to a long wire carrying constant current i. It is moving away from the wire with a velocity v. Find the emf induced in the wire when its distance from the long wire is x.

Solution : $E = B\,l\,V = \dfrac{\mu_0 i l v}{2\pi x}$

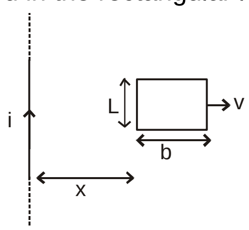
OR

Emf is equal to the rate with which magnetic field lines are cut. In dttime the area swept by the rod is l v dt. The magnetic field lines cut in dt time $= B\,l\,vdt = \dfrac{\mu_0 i l vdt}{2\pi x}$.

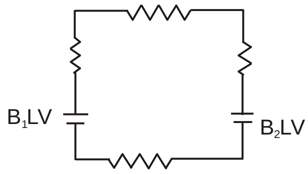


$\therefore$ The rate with which magnetic field lines are cut $= \dfrac{\mu_0 i l v}{2\pi x}$

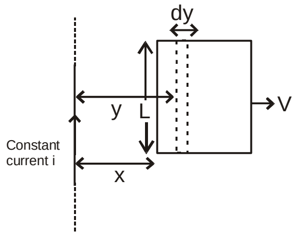
Example 9. A rectangular loop, as shown in the figure, moves away from an infinitely long wire carrying a current i. Find the emf induced in the rectangular loop.



Solution : $E = B_1 LV - B_2 Lv = \frac{\mu_0 i}{2\pi x}Lv - \frac{\mu_0 i}{2\pi(x+b)}Lv = \frac{\mu_0 iLbv}{2\pi x(x+b)}$



Aliter :
Consider a small segment of width dy at a distance y from the wire.
Let flux through the segment be $d\phi$.

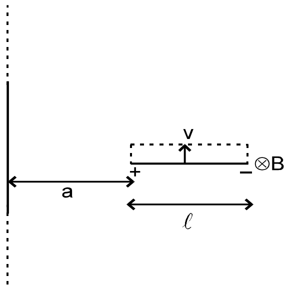


$\therefore \quad d\phi = \frac{\mu_0 i}{2\pi y} L\, dy \qquad \therefore \qquad \phi = \frac{\mu_0 iL}{2\pi}\int_x^{x+b}\frac{dy}{y} = \frac{\mu_0 iL}{2\pi}\left(\ell n(x+b) - \ell nx\right)$

Now $\frac{d\phi}{dt} = \frac{\mu_0 iL}{2\pi}\left[\frac{1}{x+b}\frac{dx}{dt} - \frac{1}{x}\frac{dx}{dt}\right]v = \frac{-\mu_0 ibLv}{2\pi x(x+b)}$

$\therefore \quad$ induced emf $= \frac{\mu_0 ibLv}{2\pi x(x+b)}$

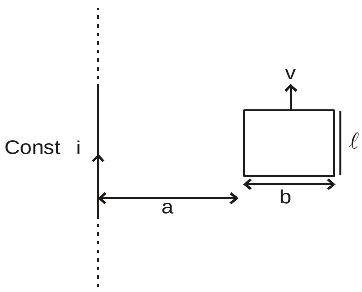
Example 10. A rod of length l is placed perpendicular to a long wire carrying current i. The rod is moved parallel to the wire with a velocity v. Find the emf induced in the rod, if its nearest end is at a distance 'a' from the wire.



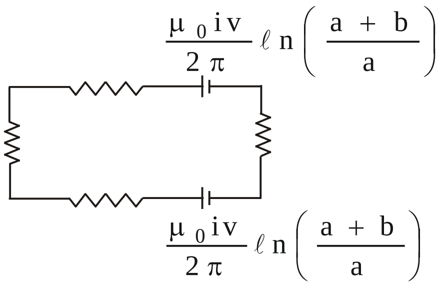
Solution : Consider a segment of rod of length dx , at a distance x from the wire. Emf induced in the segment

$dE = \frac{\mu_0 i}{2\pi x} dx.v \qquad \therefore \qquad E = \int_a^{a+\ell}\frac{\mu_0 ivdx}{2\pi x} = \frac{\mu_0 iv}{2\pi}\ell n\left(\frac{\ell + a}{a}\right)$

Example 11. A rectangular loop is moving parallel to a long wire carrying current i with a velocity v. Find the emf induced in the loop, if its nearest end is at a distance 'a' from the wire. Draw equivalent electrical diagram.



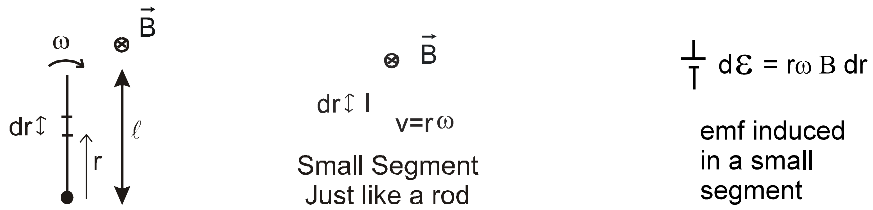
Solution : emf = 0 ;



4. INDUCED EMF DUE TO ROTATION

4.1 ROTATION OF THE ROD

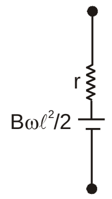
Consider a conducting rod of length l rotating in a uniform magnetic field.



Emf induced in a small segment of length dr, of the rod = v B dr = rω B dr

$\therefore$ emf induced in the rod = $\omega B\int_0^{\ell} r dr = \frac{1}{2}B\omega\ell^2$

equivalent of this rod is as following

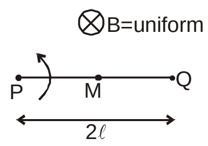


or $\varepsilon = \frac{d\Phi}{dt}$

$$\varepsilon = \frac{d\Phi}{dt} = \frac{\text{flux through the area swept by the rod in time dt}}{dt}$$

$$= \frac{B\frac{1}{2}\ell^2\omega dt}{dt} = \frac{1}{2}B\omega\ell^2$$

Example 12. A rod PQ of length 2ℓ is rotating about one end P in a uniform magnetic field B which is perpendicular to the plane of rotation of the rod. Point M is the mid point of the rod. Find the induced emf between M & Q if that between P & Q = 100V .

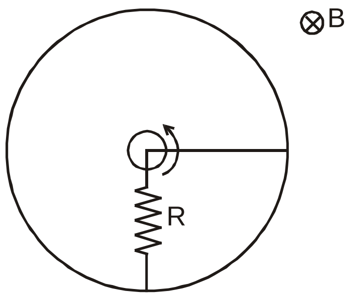


Solution : $E_{MQ} + E_{PM} = E_{PQ}$

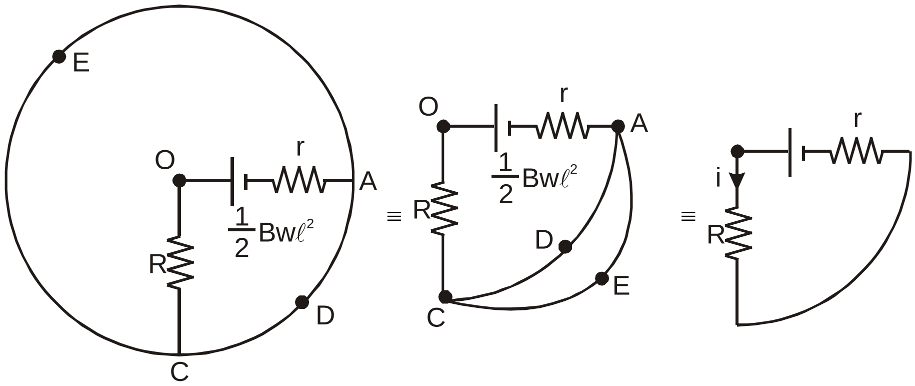
$$E_{PM} = \frac{Bw\ell^2}{2} = 100$$

$$E_{MQ} + \frac{B\omega\left(\frac{\ell}{2}\right)^2}{2} = \frac{B\omega\ell^2}{2} \quad \Rightarrow \quad E_{MQ} = \frac{3}{8}\,B\omega\ell^2 = \frac{3}{4} \times 100\text{ V} = 75\text{ V}$$

Example 13. A rod of length ℓ and resistance r rotates about one end as shown in figure. Its other end touches a conducting ring a of negligible resistance. A resistance R is connected between centre and periphery. Draw the electrical equivalence and find the current in the resistance R. There is a uniform magnetic field B directed as shown.

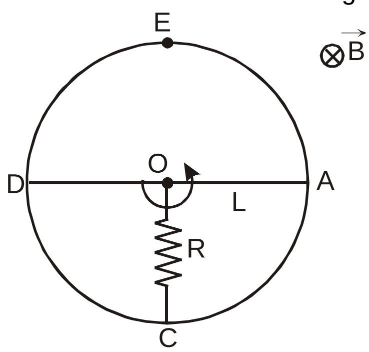


Solution :

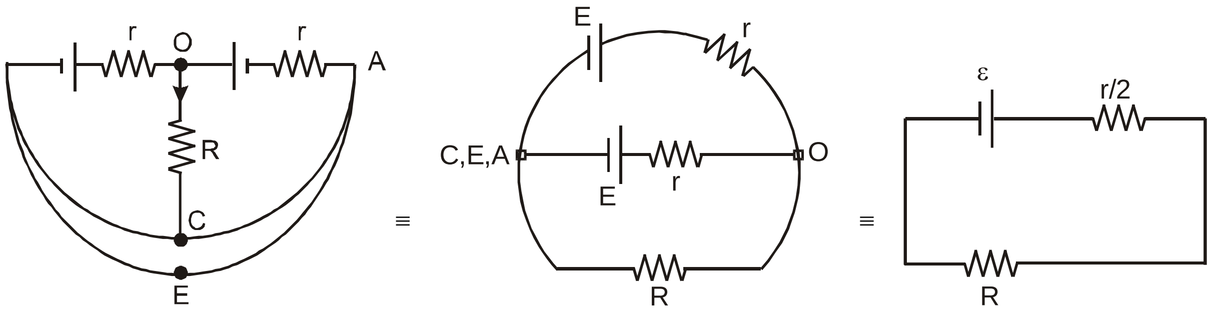


current i = $\dfrac{\frac{1}{2}B\omega\ell^2}{R+r}$

Example 14. Solve the above question if the length of rod is 2L and resistance 2r and it is rotating about its centre. Both ends of the rod now touch the conducting ring

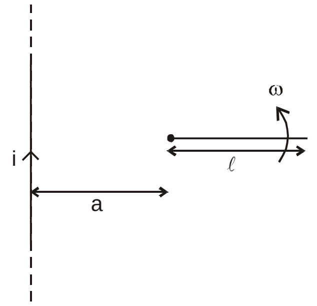


Solution :

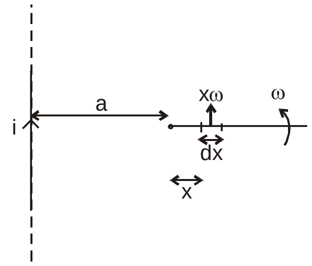


$$i = \frac{\varepsilon}{R + \frac{r}{2}} = \frac{\frac{1}{2}B\omega L^2}{R + \frac{r}{2}}$$

Example 15. A rod of length l is rotating with an angular speed ω about its one end which is at a distance 'a' from an infinitely long wire carrying current i. Find the emf induced in the rod at the instant shown in the figure.

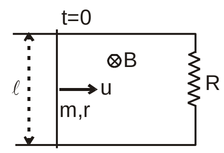


Solution : Consider a small segment of rod of length dx, at a distance x from one end of the rod. Emf induced in the segment

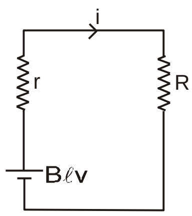


$$dE = \frac{\mu_0 i}{2\pi(x+a)}\ (x\omega)dx \qquad \therefore \qquad E = \int_0^{\ell} \frac{\mu_0 i}{2\pi(x+a)}\ (x\omega)dx = \frac{\mu_0 i\omega}{2\pi}\left[\ell - a.\ell n\left(\frac{\ell + a}{a}\right)\right].$$

Example 16. A rod of mass m and resistance r is placed on fixed, resistanceless, smooth conducting rails (closed by a resistance R) and it is projected with an initial velocity u .Find its velocity as a function of time.



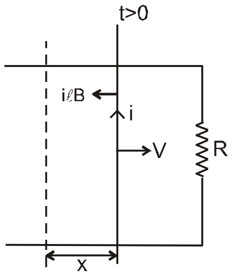
Solution : Let at an instant the velocity of the rod be v .The emf induced in the rod will be vBl. The electrically equivalent circuit is shown in the following diagram.



$\therefore$ Current in the circuit $i = \frac{B\ell v}{R + r}$

At time t

Magnetic force acting on the rod is $F = i\ell B$, opposite to the motion of the rod.



$i\ell B = -m\dfrac{dV}{dt}$ ...(1)

$i = \dfrac{B\ell v}{R+r}$ ...(2)

Now solving these two equation

$\dfrac{B^2\ell^2 V}{R+r} = -m \,.\, \dfrac{dV}{dt}$

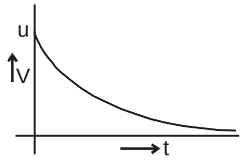
$-\dfrac{B^2\ell^2}{(R+r)m}\,.\,dt = \dfrac{dV}{V}$

let $\dfrac{B^2\ell^2}{(R+r)m} = k$

$-K\,.\,dt = \dfrac{dV}{V}$

$\int_u^v \dfrac{dV}{V} = \int_0^t -K.dt$

$V = ue^{-Kt}$



$\ln\left(\dfrac{v}{u}\right) = -Kt$

$V = ue^{-Kt}$

Example 17 In the above question find the force required to move the rod with constant velocity v, and also find the power delivered by the external agent .

Solution : The force needed to keep the velocity constant

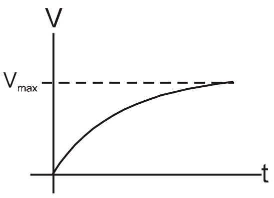
$F_{ext} = i\ell B = \dfrac{B^2\ell^2 v}{R+r}$

Power due to external force $= \dfrac{B^2\ell^2 v^2}{R+r} = \dfrac{\varepsilon^2}{R+r} = i^2(R+r)$

Note that the power delivered by the external agent is converted into joule heating in the circuit. That means magnetic field helps in converting the mechanical energy into joule heating.

Example 18 In the above question if a constant force F is applied on the rod. Find the velocity of the rod as a function of time assuming it started with zero initial velocity.

Solution :



$m\dfrac{dv}{dt} = F - i\ell B$ ....(1) $\qquad i = \dfrac{B\ell v}{R+r}$ ....(2)

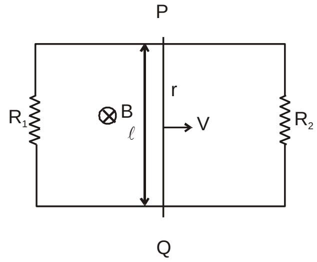
$m\dfrac{dv}{dt} = F - \dfrac{B^2\ell^2 v}{R+r}$

let $K = \dfrac{B^2\ell^2}{R+r}$ $\Rightarrow$ $\displaystyle\int_0^v \frac{dV}{F-Kv} = \int_0^t \frac{dt}{m}$

$-\left[\ell n(F-KV)\right]_0^V = \dfrac{t}{m}$ $\Rightarrow$ $\ell n\left(\dfrac{F-kV}{F}\right) = -\dfrac{Kt}{m}$

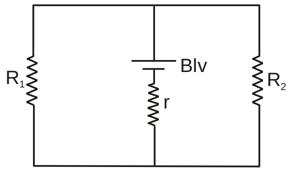
$F - KV = F e^{-kt/m}$ $\Rightarrow$ $V = \dfrac{F}{K}(1 - e^{-kt/m})$

Example 19. A rod PQ of mass m and resistance r is moving on two fixed, resistanceless, smooth conducting rails (closed on both sides by resistances R_1 and R_2) .Find the current in the rod at the instant its velocity is v.



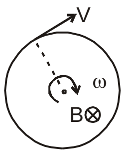
Solution : $i = \dfrac{B\ell V}{r + \dfrac{R_1R_2}{R_1+R_2}}$

this circuit is equivalent to the following diagram.



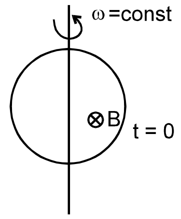
4.2. EMF INDUCED DUE TO ROTATION OF A COIL

Example 20. A ring rotates with angular velocity ω about an axis perpendicular to the plane of the ring passing through the center of the ring. A constant magnetic field B exists parallel to the axis. Find the emf induced in the ring



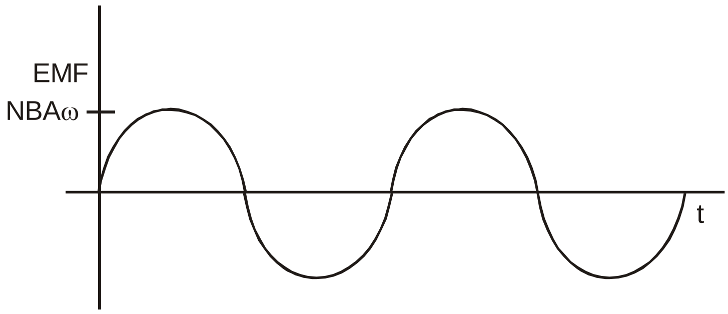
Solution : Flux passing through the ring ϕ = B.A is a constant here, therefore emf induced in the coil is zero. Every point of this ring is at the same potential, by symmetry.

Example 21. A ring rotates with angular velocity ω about an axis in the plane of the ring and which passes through the center of the ring. A constant magnetic field B exists perpendicular to the plane of the ring. Find the emf induced in the ring as a function of time.



Solution : At any time t , $\phi = BA \cos\theta = BA \cos\omega t$

Now induced emf in the loop



$$e = \frac{-d\phi}{dt} = BA\,\omega \sin\omega t$$

If there are N turns

emf = $BA\omega N \sin\omega t$

$BA\,\omega N$ is the amplitude of the emf

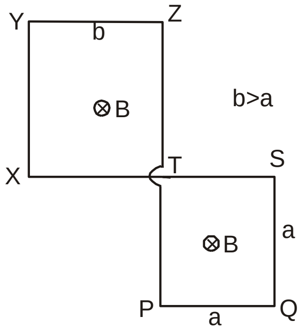
$e = e_m \sin\omega t$

$$i = \frac{e}{R} = \frac{e_m}{R} \sin wt = i_m \sin wt$$

$$i_m = \frac{e_m}{R}$$

The rotating coil thus produces a sinusoidally varying current or alternating current. This is also the principle used in generator.

Example 22. Figure shows a wire frame PQSTXYZ placed in a time varying magnetic field given as $B=\beta t$, where β is a positive constant. Resistance per unit length of the wire is λ. Find the current induced in the wire and draw its electrical equivalent diagram.

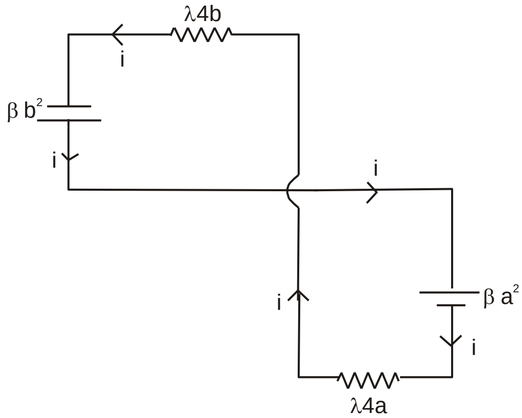


Solution : Induced emf in part PQST = βa^2 (in anticlockwise direction, from Lenz's Law)

Similarly Induced emf in part TXYZ = βb^2 (in anticlockwise direction, from Lenz's Law)

Total resistance of the part PQST =$\lambda 4a$.

Total resistance of the part TXYZ = $\lambda 4b$. The equivalent circuit is as shown in the following diagram.



writing KVL along the current flow

$\beta b^2 - \beta a^2 - \lambda\, 4ai - \lambda\, 4bi = 0$

$i = \dfrac{\beta}{4\lambda}\,(b - a)$

4.3 EMF INDUCED IN A ROTATING DISC :

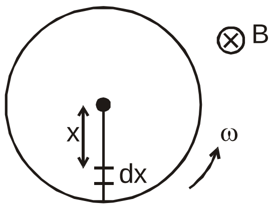
Consider a disc of radius r rotating in a magnetic field B.

Consider an element dx at a distance x form the centre. This element is moving with speed $v = \omega x$.

$\therefore$ Induced emf across dx

$= B(dx)\, v = Bdx\omega x = B\omega x dx$

$\therefore$ emf between the centre and the edge of disc.



$$= \int_0^r B\omega x dx = \frac{B\omega r^2}{2}$$

5. FIXED LOOP IN A VARYING MAGNETIC FIELD

Now consider a circular loop, at rest in a varying magnetic field. Suppose the magnetic field is directed inside the page and it is increasing in magnitude. The emf induced in the loop will be

$\varepsilon = -\dfrac{d\phi}{dt}$. Flux through the coil will be $\phi = -\pi r^2 B$; $\dfrac{d\phi}{dt} = -\pi r^2 \dfrac{dB}{dt}$; $\varepsilon = -\dfrac{d\phi}{dt}$ $\therefore$ $\varepsilon = \pi r^2 \dfrac{dB}{dt}$

$\therefore$ $E\, 2\pi r = \pi r^2 \dfrac{dB}{dt}$ or $E = \dfrac{r}{2}\dfrac{dB}{dt}$

Thus changing magnetic field produces electric field which is non conservative in nature. The lines of force associated with this electric field are closed curves.

6. SELF INDUCTION

Self induction is induction of emf in a coil due to its own current change.Total flux $N\phi$ passing through a coil due to its own current is proportional to the current and is given as $N\phi = Li$ where L is called coefficient of self induction or inductance**.**The inductance L is purely a geometrical property i.e., we can tell the inductance value even if a coil is not connected in a circuit. Inductance depends on the shape and size of the loop and the number of turns it has.

If current in the coil changes by ΔI in a time interval Δt, the average emf induced in the coil is given as

$$\varepsilon = -\frac{\Delta(N\phi)}{\Delta t} = -\frac{\Delta(LI)}{\Delta t} = -\frac{L\Delta I}{\Delta t}.$$

The instantaneous emf is given as $\varepsilon = -\frac{d(N\phi)}{dt} = -\frac{d(LI)}{dt} = -\frac{LdI}{dt}$

S.I Unit of inductance is wb/amp or Henry(H)

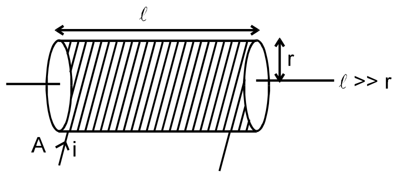
L - self inductance is +ve quantity .

L depends on : (1) Geometry of loop

(2) Medium in which it is kept. L does not depend upon current.

L is a scalar quantity.

6.1 SELF INDUCTANCE OF SOLENOID



Let the volume of the solenoid be V, the number of turns per unit length be n.

Let a current I be flowing in the solenoid.Magnetic field in the solenoid is given as B=$\mu_0$ni.The magnetic flux through one turn of solenoid $\phi = \mu_0$ niA.

The total magnetic flux through the solenoid = Nϕ = N$\mu_0$ni A = $\mu_0 n^2$i A ℓ

$\therefore$ L = $\mu_0 n^2 \ell A = \mu_0 n^2 V$ $\Rightarrow$ $\phi = \mu_0$ n i πr^2 (nℓ) $\Rightarrow$ L = $\frac{\phi}{i} = \mu_0 n^2 \pi r^2 \ell$.

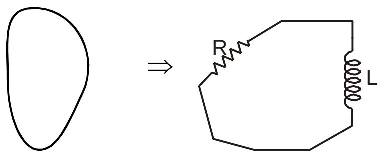
Inductance per unit volume = $\mu_0 n^2$.

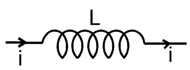
Self inductance is the physical property of the loop due to which it opposes the change in current that means it tries to keep the current constant.Current can not change suddenly in the inductor.

7. INDUCTOR :

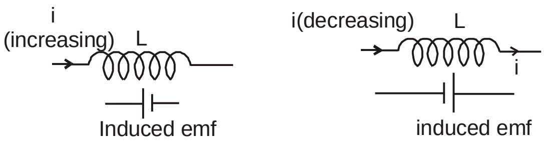
It is represent by 

electrical equivalence of loop

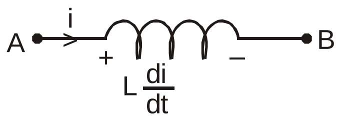




If current i through the inductor is increasing the induced emf will oppose the **increase** in current and hence will be opposite to the current.If current i through the inductor is decreasing the induced emf will oppose the **decrease** in current and hence will be in the direction of the current.

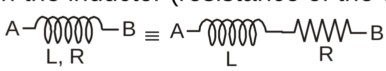


Over all result



$V_A - L\frac{di}{dt} = V_B$

Note : If there is a resistance in the inductor (resistance of the coil of inductor) then :

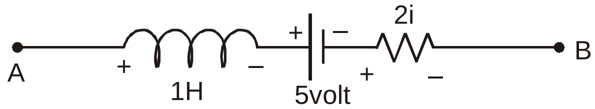


Example 23. A B is a part of circuit. Find the potential difference $v_A - v_B$ if

(i) current i = 2A and is constant
(ii) current i = 2A and is increasing at the rate of 1 amp/sec.
(iii) current i = 2A and is decreasing at the rate 1 amp/sec.

Solution : $L\frac{di}{dt} = 1\frac{di}{dt}$

writing KVL from A to B



$V_A - 1\frac{di}{dt} - 5 - 2i = V_B$.

(i) Put i = 2, $\frac{di}{dt} = 0$

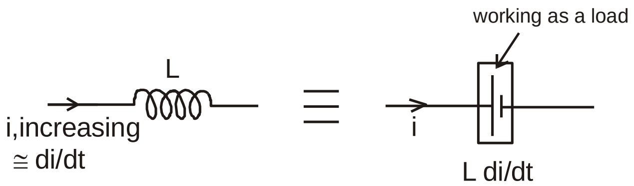
$V_A - 5 - 4 = V_B$ $\therefore$ $V_A - V_B = 9$ volt

(ii) Put i = 2, $\frac{di}{dt} = 1$; $V_A - 1 - 5 - 4 = V_B$ or $V_A - V_B = 10\ V_0$

(iii) Put i = 2 , $\frac{di}{dt} = -1$; $V_A + 1 - 5 - 2 \times 2 = V_B$ or $V_A = 8$ volt.

7.1 ENERGY STORED IN AN INDUCTOR:

If current in an inductor at an instant is i and is increasing at the rate di/dt, the induced emf will oppose the current. Its behaviour is shown in the figure.



Power consumed by the inductor = $i\,L\frac{di}{dt}$

Energy consumed in dt time = $i\,L\frac{di}{dt}dt$

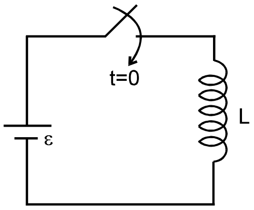
$\therefore$ total energy consumed as the current increases from 0 to I = $\int_0^I iLdi = \frac{1}{2}LI^2 = \frac{1}{2}Li^2 \Rightarrow U = \frac{1}{2}LI^2$

Note : This energy is stored in the magnetic field with energy density

$$\frac{dU}{dV} = \frac{B^2}{2\mu} = \frac{B^2}{2\mu_0\mu_r}$$

Total energy U = $\int \frac{B^2}{2\mu_0\mu_r}dV$

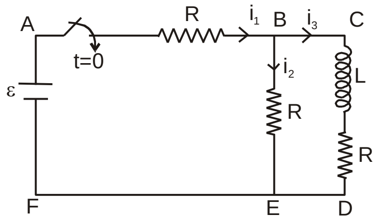
Example 24. A circuit contains an ideal cell and an inductor with a switch. Initially the switch is open .It is closed at t = 0.Find the current as a function of time.



Solution : $\varepsilon = L\dfrac{di}{dt} \Rightarrow \int_0^i \varepsilon dt = \int_0^i L di$

$\varepsilon t = Li \Rightarrow i = \dfrac{\varepsilon t}{L}$

Example 25. In the following circuit, the switch is closed at t = 0. Find the currents i_1, i_2, i_3 and $\dfrac{di_3}{dt}$ at t = 0 and at t = ∞. Initially all currents are zero.



Solution : At t = 0

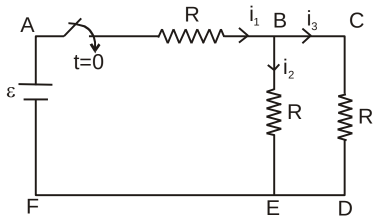
i_3 is zero ,since current cannot suddenly change due to the inductor.

∴ $i_1 = i_2$ (from KCL)

applying KVL in the part ABEF we get $i_1 = i_2 = \dfrac{\varepsilon}{2R}$.

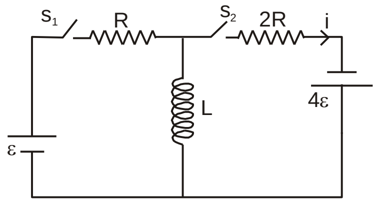
At t = ∞

i_3 will become constant and hence potential difference across the inductor will be zero.It is just like a simple wire and the circuit can be solved assuming it to be like shown in the following diagram.

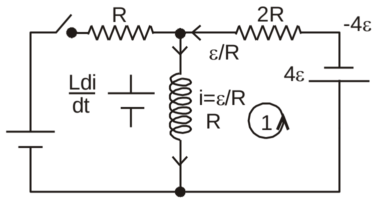


$i_2 = i_3 = \dfrac{\varepsilon}{3R}$, $i_1 = \dfrac{2\varepsilon}{3R}$.

Example 26. In the circuit shown in the figure, S_1 remains closed for a long time and S_2 remains open. Now S_2 is closed and S_1 is opened. Find out the di/dt just after that moment.



Solution : Before S_2 is closed and S_1 is opened current in the left part of the circuit = $\frac{\varepsilon}{R}$. Now when S_2 closed S_1 opened, current through the inductor can not change suddenly, current $\frac{\varepsilon}{R}$ will continue to move in the inductor.



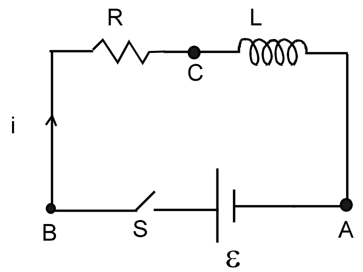
Applying KVL in loop 1.

$$L\frac{di}{dt} + \frac{\varepsilon}{R}(2R) + 4\varepsilon = 0$$

$$\frac{di}{dt} = -\frac{6\varepsilon}{L}$$

7.2 GROWTH OF CURRENT IN SERIES R–L CIRCUIT :

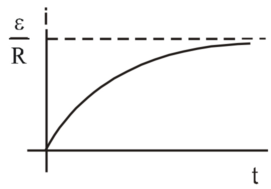
Figure shows a circuit consisting of a cell, an inductor L and a resistor R, connected in series. Let the switch S be closed at t=0.Suppose at an instant current in the circuit be i which is increasing at the rate di/dt.



Writing KVL along the circuit, we have $\varepsilon - L\frac{di}{dt} - iR = 0$

On solving we get, $i = \frac{\varepsilon}{R}(1 - e^{\frac{-Rt}{L}})$

The quantity L/R is called time constant of the circuit and is denoted by τ.The variation of current with time is as shown.

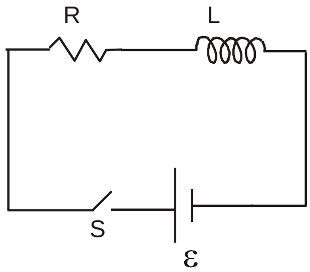


Note : 1. Final current in the circuit = $\frac{\varepsilon}{R}$, which is independent of L.

2. After one time constant, current in the circuit = 63% of the final current (verify yourself)
3. More time constant in the circuit implies slower rate of change of current.

4. If there is any change in the circuit containing inductor then there is no instantaneous effect on the flux of inductor.
$L_1 i_1 = L_2 i_2$

Example 27. At t = 0 switch is closed (shown in figure) after a long time suddenly the inductance of the inductor is made η times lesser ($\frac{L}{\eta}$) then its initial value, find out instant current just after the operation.

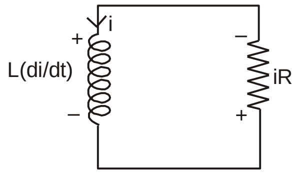


Solution : Using above result (note 4)

$L_1 i_1 = L_2 i_2 \Rightarrow i_2 = \frac{\eta\varepsilon}{R}$

DECAY OF CURRENT IN THE CIRCUIT CONTAINING RESISTOR AND INDUCTOR:

Let the initial current in the circuit be I_0. At any time t, let the current be i and let its rate of change at this instant be $\frac{di}{dt}$.

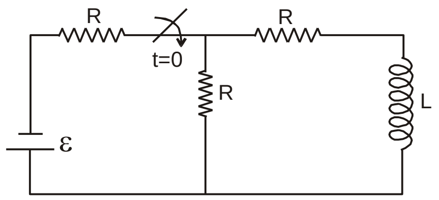


$L.\frac{di}{dt} + iR = 0, \qquad \frac{di}{dt} = -\frac{iR}{L}$

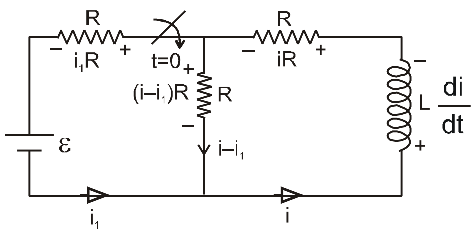
$\int_{I_0}^{i} \frac{di}{i} = -\int_0^t \frac{R}{L}.dt \Rightarrow \ln\left(\frac{i}{I_0}\right) = -\frac{Rt}{L} \qquad \text{or} \qquad i = I_0 e^{\frac{-Rt}{L}}$

Current after one time constant : $i = I_0 = 0.37\%$ of initial current.

Example 28 In the following circuit the switch is closed at t = 0. Intially there is no current in inductor. Find out current the inductor coil as a function of time.



Solution :



At any time t

$-\varepsilon + i_1R - (i - i_1)R = 0$

$-\varepsilon + 2i_1R - iR = 0$

$i_1 = \dfrac{iR + \varepsilon}{2R}$

$-\varepsilon + \left(\dfrac{iR+\varepsilon}{2}\right) + iR + L\,.\,\dfrac{di}{dt} = 0$

$\left(\dfrac{-\varepsilon+3iR}{2}\right)dt = -L\,.\,di$

$-\displaystyle\int_0^t \frac{dt}{2L} = \int_0^i \frac{di}{-\varepsilon+3iR}$

$-\ln\left(\dfrac{-\varepsilon+3iR}{-\varepsilon}\right) = \dfrac{3Rt}{2L}$

Now, $-\varepsilon + i_1R + iR + L.\dfrac{di}{dt} = 0$

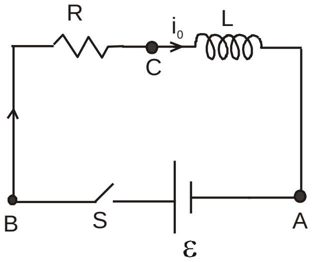
$\Rightarrow \quad -\dfrac{\varepsilon}{2} + \dfrac{3iR}{2} = -L\,.\,\dfrac{di}{dt}$

$\Rightarrow \quad -\dfrac{-dt}{2L} = \dfrac{di}{-\varepsilon+3iR}$

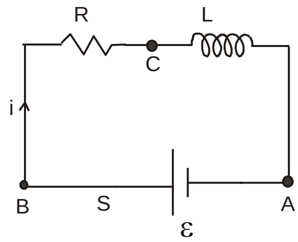
$\Rightarrow \quad -\dfrac{t}{2L} = \dfrac{1}{3R}\ln\left(\dfrac{-\varepsilon+3iR}{-\varepsilon}\right)$

$\Rightarrow \quad i = +\dfrac{\varepsilon}{3R}\left(1 - e^{-\frac{3Rt}{2L}}\right)$

Example 29. Figure shows a circuit consisting of a ideal cell, an inductor L and a resistor R, connected in series. Let the switch S be closed at t = 0. Suppose at t = 0 current in the inductor is i_0 then find out equation of current as a function of time



Solution : Let an instant t current in the circuit is i which is increasing at the rate di/dt.

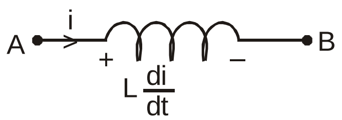


Writing KVL along the circuit , we have $\varepsilon - L\dfrac{di}{dt} - iR = 0$

$\Rightarrow \quad L\dfrac{di}{dt} = \varepsilon - iR \quad \Rightarrow \quad \displaystyle\int_{i_0}^{i} \frac{di}{\varepsilon - iR} = \int_0^t \frac{dt}{L}$

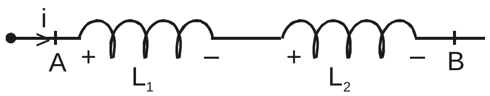
$\Rightarrow \quad \ln\left(\dfrac{\varepsilon - iR}{\varepsilon - i_0R}\right) = -\dfrac{Rt}{L} \Rightarrow \varepsilon - iR = (\varepsilon - i_0R)\,e^{-Rt/L} \Rightarrow \quad i = \dfrac{\varepsilon - (\varepsilon - i_0R)e^{-Rt/L}}{R}$

Equivalent self inductance :



$L = \dfrac{V_A - V_B}{di/dt}$ ..(1)

Series combination

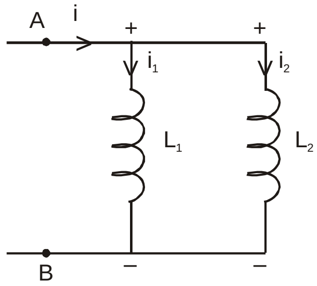


$V_A - L_1\dfrac{di}{dt} - L_2\dfrac{di}{dt} = V_B$ ....(2)

from (1) and (2)

$L = L_1 + L_2$ (neglecting mutual inductance)

Parallel Combination :



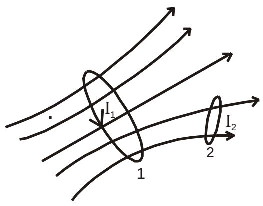
From figure $V_A - V_B = L_1 \frac{di_1}{dt} = L_2 \frac{di_2}{dt}$ ..... (3)

also $i = i_1 + i_2$

or $\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$ or $\frac{V_A - V_B}{L} = \frac{V_A - V_B}{L_1} + \frac{V_A - V_B}{L_2}$

$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$ (Neglecting mutual inductance)

8. MUTUAL INDUCTANCE



Consider two arbitrary conducting loops 1 and 2. Suppose that I_1 is the instantaneous current flowing around loop 1. This current generates a magnetic field $\mathbf{B_1}$ which links the second circuit, giving rise to a magnetic flux ϕ_2 through that circuit. If the current I_1doubles, then the magnetic field $\mathbf{B_1}$ doubles in strength at all points in space, so the magnetic flux ϕ_2 through the second circuit also doubles. Furthermore, it is obvious that the flux through the second circuit is zero whenever the current flowing around the first circuit is zero. It follows that the flux ϕ_2 through the second circuit is directly proportional to the current I_1 flowing around the first circuit. Hence, we can write $\phi_2=M_{21}I_1$ where the constant of proportionality M_{21} is called the mutual inductance of circuit 2 with respect to circuit 1. Similarly, the flux ϕ_2 through the first circuit due to the instantaneous current I_2 flowing around the second circuit is directly proportional to that current, so we can write $\phi_1=M_{12}I_2$ where $M_{12}$is the mutual inductance of circuit 1 with respect to circuit 2. It can be shown that **M_{21}= M_{12} (Reciprocity Theorem)**. Note that M is a purely geometric quantity, depending only on the size, number of turns, relative position, and relative orientation of the two circuits. The S.I. unit of mutual inductance is called Henry (H). One Henry is equivalent to a volt-second per ampere.

Suppose that the current flowing around circuit 1 changes by an amount ΔI_1 in a small time interval Δt. The flux linking circuit 2 changes by an amount $\Delta\phi_2=M\Delta I_1$in the same time interval. According to Faraday's law, an emf $\varepsilon_2 = -\frac{\Delta\phi_2}{\Delta t}$ is generated around the second circuit due to the changing magnetic flux linking that circuit. Since, $\Delta\phi_2 = M\Delta I_1$, this emf can also be written $\varepsilon_2 = -M\frac{\Delta I_1}{\Delta t}$.

Thus, the emf generated around the second circuit due to the current flowing around the first circuit is directly proportional to the rate at which that current changes. Likewise, if the current I_2 flowing around the second circuit changes by an amount ΔI_1in a time interval Δt then the emf generated around the first circuit is $\varepsilon_1 = -M\frac{\Delta I_2}{\Delta t}$ Note that there is no direct physical connection(coupling) between the two circuits: the coupling is due entirely to the magnetic field generated by the currents flowing around the circuits.

Note : (1) $M \le \sqrt{L_1L_2}$

(2) For two coils in series if mutual inductance is considered then
$L_{eq} = L_1 + L_2 \pm 2M$

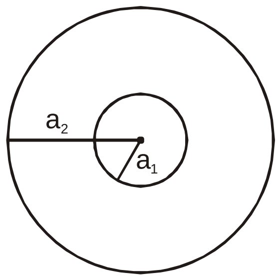
Example 30. Two insulated wires are wound on the same hollow cylinder, so as to form two solenoids sharing a common air-filled core. Let I be the length of the core, A the cross-sectional area of the core, N_1 the number of times the first wire is wound around the core, and N_2 the number of turns the second wire is wound around the core. Find the mutual inductance of the two solenoids, neglecting the end effects.

Solution : If a current I_1 flows around the first wire then a uniform axial magnetic field of strength $B_1 = \frac{\mu_0 N_1 I_1}{\ell}$ is generated in the core. The magnetic field in the region outside the core is of negligible magnitude. The flux linking a single turn of the second wire is B_1A. Thus, the flux linking all N_2 turns of the second wire is

$\phi_2 = N_2 B_1 A = \frac{\mu_0 N_1 N_2 A I_1}{\ell} = MI_1$ $\therefore$ $M = \frac{\mu_0 N_1 N_2 A}{\ell}$

As described previously, M is a geometric quantity depending on the dimensions of the core and the manner in which the two wires are wound around the core, but not on the actual currents flowing through the wires.

Example 31. Find the mutual inductance of two concentric coils of radii a_1 and a_2 ($a_1 << a_2$) if the planes of coils are same.



Solution : Let a current i flow in coil of radius a_2.

Magnetic field at the centre of coil = $\frac{\mu_0 i}{2a_2} \pi a_1^2$

or $Mi = \frac{\mu_0 i}{2a_2} \pi a_1^2$ or $M = \frac{\mu_0 \pi \; a_1^2}{2a_2}$

Example 32. Solve the above question, if the planes of coil are perpendicular.

Solution : Let a current i flow in the coil of radius a_1. The magnetic field at the centre of this coil will now be parallel to the plane of smaller coil and hence no flux will pass through it, hence M = 0.

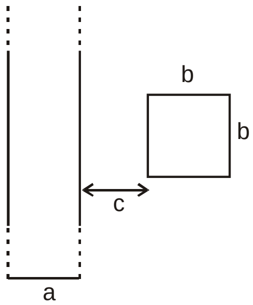
Example 33. Solve the above problem if the planes of coils make θ angle with each other.

Solution : If i current flows in the larger coil, magnetic field produced at the centre will be perpendicular to the plane of larger coil.

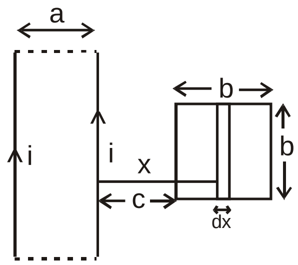
Now the area vector of smaller coil which is perpendicular to the plane of smaller coil will make an angle θ with the magnetic field.

Thus flux = $\vec{B}.\vec{A} = \frac{\mu_0 i}{2a_2}.\pi a_1^2.\cos\theta$ or $M = \frac{\mu_0 \pi a_1^2 \cos\theta_1}{2a_2}$

Example 34. Find the mutual inductance between two rectangular loops, shown in the figure



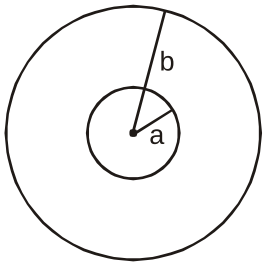
Solution :



Let current i flow in the loop having ∞-by long sides. Consider a segment of width dx at a distance x as shown flux through the regent

$$d\phi = \left[\frac{\mu_0 i}{2\pi x} - \frac{\mu_0 i}{2\pi(x+a)}\right] b\,dx \Rightarrow \quad \phi = \int_c^{c+b}\left[\frac{\mu_0 i}{2\pi x} - \frac{\mu_0 i}{2\pi(x+a)}\right] b\,dx = \frac{\mu_0 i b}{2\pi}\left[\ell n\frac{c+b}{c} - \ell n\frac{a+b+c}{a+c}\right].$$

Example 35. Figure shows two concentric coplanar coils with radii a and b (a << b). A current i = 2t flows in the smaller loop. Neglecting self inductance of larger loop



(a) Find the mutual inductance of the two coils
(b) Find the emf induced in the larger coil
(c) If the resistance of the larger loop is R find the current in it as a function of time

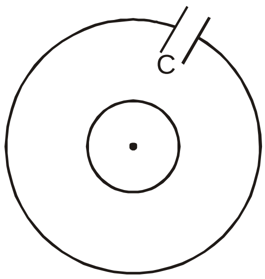
Solution : (a) To find mutual inductance, it does not matter in which coil we consider current and in which flux is calculated (Reciprocity theorem) Let current i be flowing in the larger coil. Magnetic field at the centre = $\frac{\mu_0 i}{2b}$.

flux through the smaller coil = $\frac{\mu_0 i}{2b}\pi a^2$ $\quad\therefore\quad$ $M = \frac{\mu_0}{2b}\pi a^2$

(b) $|\text{emf induced in larger coil}| = M\left[\left(\frac{di}{dt}\right) \text{ in smaller coil}\right] = \frac{\mu_0}{2b}\pi a^2 \ (2) = \frac{\mu_0 \pi a^2}{b}$

(c) current in the larger coil $= \frac{\mu_0 \pi a^2}{b\ \ R}$.

Example 36. If the current in the inner loop changes according to i = 2t^2 then, find the current in the capacitor as a function of time.



Solution : $M = \frac{\mu_0}{2b}\pi a^2$

$|\text{emf induced in larger coil}| = M\left[\left(\frac{di}{dt}\right) \text{ in smaller coil}\right] \quad \Rightarrow \quad e = \frac{\mu_0}{2b}\pi a^2 \ (4t) = \frac{2\mu_0 \pi a^2 t}{b}$

Applying KVL :-

$+e - \frac{q}{c} - iR = 0$

$\frac{2\mu_0\pi a^2 t}{b} - \frac{q}{c} - iR = 0$

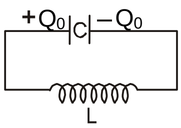
differentiate wrt time :-

$\frac{2\mu_0\pi a^2}{b} - \frac{i}{c} - \frac{di}{dt}R = 0$

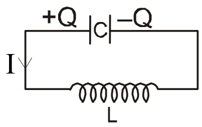
on solving it

$i = \frac{2\mu_0\pi a^2 C}{b}\left[1 - e^{-t/RC}\right]$

9. LC OSCILLATIONS



At t = 0



At t = t

When capacitor C is completely charged upto Q_0 and connected to an inductor L at t = 0 then at t = t

$L\frac{dI}{dt} - \frac{Q}{C} = 0,$ $\quad -L\frac{d^2Q}{dt^2} - \frac{Q}{C} = 0,$ $\quad Q = -LC\frac{d^2Q}{dt^2}$

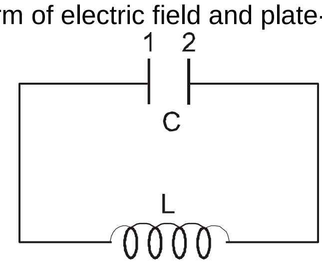
$\frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0$ therefore charge Q oscillates with $Q = Q_0 \cos \omega t$

Hence initial phase of oscillation is $\frac{\pi}{2}$ and angular frequency $\omega = \frac{1}{\sqrt{LC}}$

One can prove that the energy in the system remains conserved.

Therefore $\frac{Q_0^2}{2C} + 0 = \frac{Q^2}{2C} + \frac{1}{2}LI^2 = \frac{1}{2}LI_0^2 + 0$

Example 37. Consider a L – C oscillation circuit. Circuit elements has zero resistance. Initially at t = 0 all the energy is stored in the form of electric field and plate-1 is having positive charge :



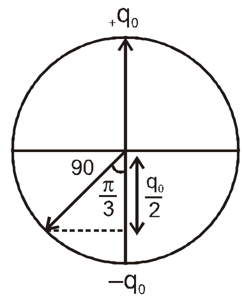
at time t = t_1 plate-2 attains half of the maximum +ve change for the first time. Value of t_1 is :

(A) $\frac{2\pi}{3}\sqrt{LC}$ (B) $\frac{\pi}{3}\sqrt{LC}$ (C) $\frac{4\pi}{3}\sqrt{LC}$ (D) $\pi\sqrt{LC}$

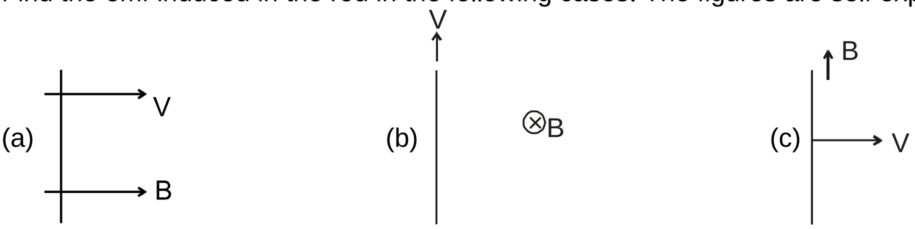
Solution : $q_1 = q_0 \sin(\omega t + \pi/2)$

at $t = t_1$ $q_1 = -\dfrac{q_0}{2}$

$$t_1 = \frac{\pi - \frac{\pi}{3}}{\omega} = \frac{2\pi}{3\omega} = \frac{2\pi}{3}\sqrt{LC}$$



Problem 1. Find the emf induced in the rod in the following cases. The figures are self explanatory.

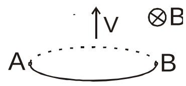


Solution : (a) here $\vec{v} \parallel \vec{B}$ so $\vec{v} \times \vec{B} = 0$ emf = $\vec{\ell} \cdot (\vec{v} \times \vec{B}) = 0$

(b) here $\vec{v} \parallel \vec{\ell}$ so emf = $\vec{\ell} \cdot (\vec{v} \times \vec{B}) = 0$

(c) here $\vec{B} \parallel \vec{\ell}$ so emf = $\vec{\ell} \cdot (\vec{v} \times \vec{B}) = 0$

Problem 2. A circular coil of radius R is moving in a magnetic field **B** with a velocity **v** as shown in the figure.

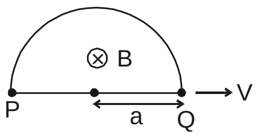


Find the emf across the diametrically opposite points A and B.

Solution : emf = $BVl_{effective}$

= 2RvB

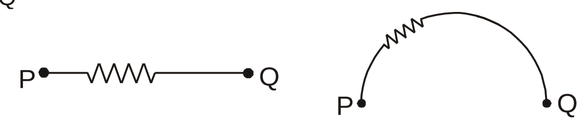
Problem 3. Find the emf across the points P and Q which are diametrically opposite points of a semicircular closed loop moving in a magnetic field as shown.Also draw the electrical equivalent circuit of each branch.



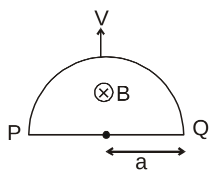
Solution : Here $\vec{v} \parallel \vec{\ell}$

so emf = $\vec{\ell} \cdot (\vec{v} \times \vec{B}) = 0$

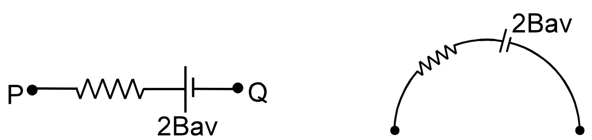
Induced emf = 0



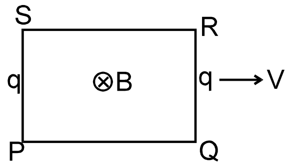
Problem 4. Find the emf across the points P and Q which are diametrically opposite points of a semicircular closed loop moving in a magnetic field as shown. Also draw the electrical equivalence of each branch.



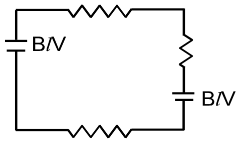
Solution : Induced emf = 2Bav



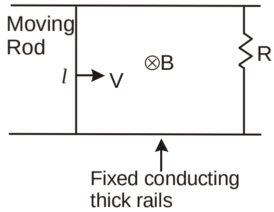
Problem 5. Figure shows a rectangular loop moving in a uniform magnetic field .Show the electrical equivalence of each branch.



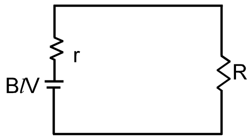
Solution :



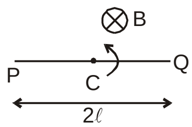
Problem 6. Figure shows a rod of length l and resistance r moving on two rails shorted by a resistance R.A uniform magnetic field B is present normal to the plane of rod and rails .Show the electrical equivalence of each branch.



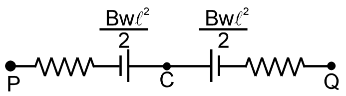
Solution :



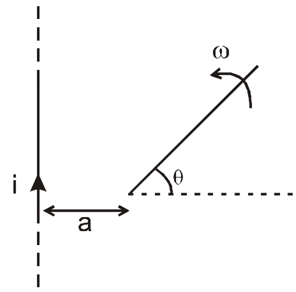
Problem 7. A rod PQ of length 2ℓ is rotating about its mid point C, in a uniform magnetic field B which is perpendicular to the plane of rotation of the rod. Find the induced emf between P Q and PC. Draw the circuit diagram of parts PC and CQ.



Solution : $emf_{PQ} = 0$; $emf_{PC} = \dfrac{B\omega\ell^2}{2}$



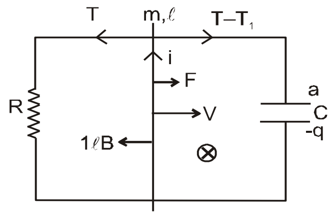
Problem 8. A rod of length ℓ is rotating with an angular speed ω about its one end which is at a distance 'a' from an infinitely long wire carrying current i. Find the emf induced in the rod at the instant shown in the figure.



Solution : $E = \displaystyle\int \frac{\mu_0 i}{2\pi \ (a + r\cos\theta)} \times (r\omega) \ . \ (dr)$

$$E = \frac{\mu_0 \omega i}{2\pi} \int_0^{\ell} \frac{r}{a + r\cos\theta} dr \Rightarrow E = \frac{\mu_0 i \omega}{2\pi\cos\theta}\left[\ell - \frac{a}{\cos\theta}\ell n\left(\frac{a + \ell\cos\theta}{a}\right)\right]$$

Problem 9.



Find the velocity of the moving rod at time t if the initial velocity of the rod is v and a constant force F is applied on the rod. Neglect the resistance of the rod.

Solution : At any time t, let the velocity of the rod be v .

Applying Newtons law: F – ilB = ma ... (1)

Also B l v = $i_1R = \frac{q}{c}$

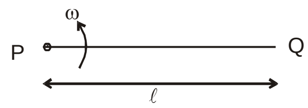
Applying Kcl, $i = i_1 + \frac{dq}{dt} = \frac{B\ell V}{R} + \frac{d}{dt}(BlvC)$ or $i = \frac{B\ell V}{R} + B\ell C\,a$

Putting the value of i in eq (1), $F - \frac{B^2\ell^2 V}{R} = (m + B^2\ell^2 C)a = (m + B^2\ell^2 C)\frac{dv}{dt}$

$(m + B^2\ell^2 C)\dfrac{dv}{F - \frac{B^2 l^2 v}{R}} = dt$

Integrating both sides, and solving we get $v = \frac{FR}{B^2\ell^2}\left(1 - e^{-\frac{tB^2\ell^2}{R(m+CB^2\ell^2)}}\right)$

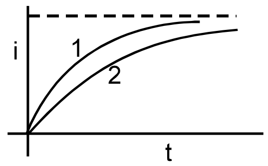
Problem 10. A rod PQ of length ℓ is rotating about end P, with an angular velocity ω. Due to centrifugal forces the free electrons in the rod move towards the end Q and an emf is created. Find the induced emf.



Solution : The accumulation of free electrons will create an electric field which will finally balance the centrifugal forces and a steady state will be reached. In the steady state $m_e\omega^2 x = e\,E$.

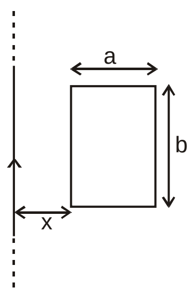
$$V_P - V_Q = \int_{x=0}^{x=\ell} \bar{E}.d\bar{x} = \int_0^{\ell} \frac{m_e\omega^2 x}{e}dx = \frac{m_e\omega^2\ell^2}{2e}$$

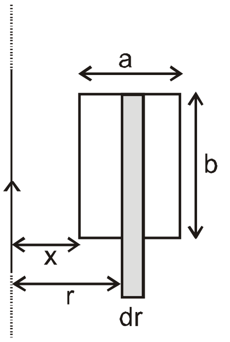
Problem 11. Which of the two curves shown has less time constant.



Solution : Curve1

Problem 12. Find the mutual inductance of a straight long wire and a rectangular loop, as shown in the figure





Solution : $d\phi = \frac{\mu_0 i}{2\pi r} \times bdr$

$$\phi = \int_{x}^{x+a} \frac{\mu_0 i}{2\pi r} \times bdr$$

$M = \phi/i$

$$M = \frac{\mu_0 b}{2\pi} \ln \left(1 + \frac{a}{x}\right)$$