CHAPTER-23
CAPACITANCE

1. INTRODUCTION

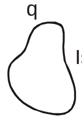
A capacitor can store energy in the form of potential energy in an electric field. In this chapter we'll discuss the capacity of conductors to hold charge and energy.

2. CAPACITANCE OF AN ISOLATED CONDUCTOR

When a conductor is charged its potential increases. It is found that for an isolated conductor (conductor should be of finite dimension, so that potential of infinity can be assumed to be zero) potential of the conductor is proportional to charge given to it.

q = charge on conductor

V = potential of conductor



$q \propto V$

$\Rightarrow$ $q = CV$

Where C is proportionality constant called capacitance of the conductor.

2.1 Definition of capacitance :

Capacitance of conductor is defined as charge required to increase the potential of conductor by one unit.

2.2 Important points about the capacitance of an isolated conductor :

(i) It is a scalar quantity.

(ii) Unit of capacitance is farad in SI units and its dimensional formula is $M^{-1} L^{-2} I^2 T^4$

(iii) 1 Farad : 1 Farad is the capacitance of a conductor for which 1 coulomb charge increases potential by 1 volt.

$$1 \text{ Farad} = \frac{1 \text{ Coulomb}}{1 \text{ Volt}}$$

$1\ \mu F = 10^{-6}$ F, $1 nF = 10^{-9}$ F or $1 pF = 10^{-12}$ F

(iv) Capacitance of an isolated conductor depends on following factors :

(a) Shape and size of the conductor : On increasing the size, capacitance increases.

(b) On surrounding medium : With increase in dielectric constant K, capacitance increases.

(c) Presence of other conductors : When a neutral conductor is placed near a charged conductor, capacitance of conductors increases.

(v) Capacitance of a conductor do not depend on

(a) Charge on the conductor

(b) Potential of the conductor

(c) Potential energy of the conductor.

3. POTENTIAL ENERGY OR SELF ENERGY OF AN ISOLATED CONDUCTOR

Work done in charging the conductor to the charge on it against its own electric field or total energy stored in electric field of conductor is called self energy or self potential energy of conductor.

3.1 Electric potential energy (Self Energy) :

Work done in charging the conductor

$$W = \int_0^q \frac{q}{C} dq = \frac{q^2}{2C}$$

$$W = U = \frac{q^2}{2C} = \frac{1}{2} CV^2 = \frac{qV}{2}.$$

q = Charge on the conductor

V = Potential of the conductor

C = Capacitance of the conductor.

3.2 Self energy is stored in the electric field of the conductor with energy density (Energy per unit volume)

$\frac{dU}{dV} = \frac{1}{2} \in_0 E^2$ [The energy density in a medium is $\frac{1}{2} \in_0 \in_r E^2$]

where E is the electric field at that point.

3.3 In case of charged conductor energy stored is only out side the conductor but in case of charged insulating material it is outside as well as inside the insulator.

4. CAPACITANCE OF AN ISOLATED SPHERICAL CONDUCTOR

The capacitance of an isolated spherical conductor of radius R.

Let there is charge Q on sphere.

$\because$ Potential $V = \frac{KQ}{R}$

Hence by formula : Q = CV

$Q = \frac{CKQ}{R}$

$C = 4\pi\in_0 R$

Capacitance of an isolated spherical conductor

$C = 4\pi\in_0 R$

(i) If the medium around the conductor is vacuum or air.

$C_{Vacuum} = 4\pi\in_0 R$

R = Radius of spherical conductor. (may be solid or hollow.)

(ii) If the medium around the conductor is a dielectric of constant K from surface of sphere to infinity.

$C_{medium} = 4\pi\in_0 KR$

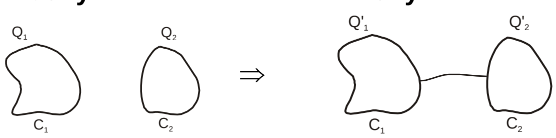
(iii) $\frac{C_{medium}}{C_{air/vaccum}}$ = K = dielectric constant.

Example 1. Find out the capacitance of the earth ? (Radius of the earth = 6400 km)

Solution : $C = 4\pi\in_0 R = \frac{6400 \times 10^3}{9 \times 10^9} = 711\ \mu F$

5. SHARING OF CHARGES ON JOINING TWO CONDUCTORS (BY A CONDUCTING WIRE) :

Initially **Finally**



(i) Whenever there is potential difference, there will be movement of charge.

(ii) If released, charge always have tendency to move from **high potential energy** to **low potential energy .**

(iii) If released, positive charge moves from **high potential** to **low potential** [if only electric force act on charge].

(iv) If released, negative charge moves from **low potential** to **high potential** [if only electric force act on charge].

(v) The movement of charge will continue till there is potential difference between the conductors (finally potential difference = 0).

(vi) Formulae related with redistribution of charges :

Before connecting the conductors		
Parameter	**I^{st} Conductor**	**II^{nd} Conductor**
Capacitance	C_1	C_2
Charge	Q_1	Q_2
Potential	V_1	V_2

After connecting the conductors		
Parameter	**I^{st} Conductor**	**II^{nd} Conductor**
Capacitance	C_1	C_2
Charge	Q_1'	Q_2'
Potential	V	V

$$V = \frac{Q_1'}{C_1} = \frac{Q_2'}{C_2} \quad \Rightarrow \quad \frac{Q_1'}{Q_2'} = \frac{C_1}{C_2}$$

But, $Q_1' + Q_2' = Q_1 + Q_2$

$$\therefore \quad V = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

$\therefore \quad Q_1' = (Q_1 + Q_2)$

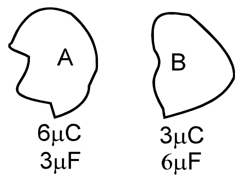
and $Q_2' = \dfrac{C_1}{C_1 + C_2}(Q_1 + Q_2)$

Heat loss during redistribution : $\Delta H = \dfrac{1}{2}\dfrac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$

The loss of energy is in the form of Joule heating in the wire.

Note : Always put Q_1, Q_2, V_1 and V_2 with sign.

Example 2. A and B are two isolated conductors (that means they are placed at a large distance from each other). When they are joined by a conducting wire:



(i) Find out final charges on A and B ?

(ii) Find out heat produced during the process of flow of charges.

(iii) Find out common potential after joining the conductors by conducting wires?

Solution : (i) $Q_A' = \dfrac{3}{3+6}(6 + 3) = 3\mu C$

$Q_B' = \dfrac{6}{3+6}(6 + 3) = 6\mu C$

(ii) $\Delta H = \dfrac{1}{2} \cdot \dfrac{3\mu F.6\mu F}{(3\mu F + 6\mu F)} \cdot \left(2 - \dfrac{1}{2}\right)^2 = \dfrac{1}{2} \cdot (2\mu F) \cdot \left(\dfrac{3}{2}\right)^2 = \dfrac{9}{4}\mu J$

(iii) $V_C = \dfrac{3\mu C + 6\mu C}{3\mu F + 6\mu F} = 1\text{volt}.$

6. CAPACITOR :

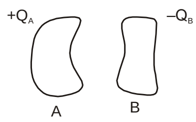
A capacitor or condenser consists of two conductors separated by an insulator or dielectric.

(i) When uncharged conductor is brought near to a charged conductor, the charge on conductors remains same but its potential decreases resulting in the increase of capacitance.

(ii) In capacitor two conductors have equal but opposite charges.

(iii) The conductors are called the plates of the capacitor. The name of the capacitor depends on the shape of the capacitor.

(iv) Formulae related with capacitors



(a) Q = CV

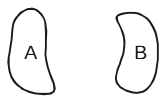
$$\Rightarrow \quad C = \frac{Q}{V} = \frac{Q_A}{V_A - V_B} = \frac{Q_B}{V_B - V_A}$$

Q = Charge of positive plate of capacitor.

V = Potential difference between positive and negative plates of capacitor

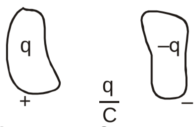
C = Capacitance of capacitor.

(b) Energy stored in the capacitor

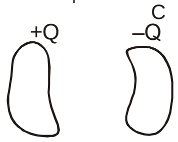


Initially charge = 0 0

Intermediate



Finally,



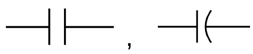
$$W = \int dW = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}$$

$\therefore$ Energy stored in the capacitor = U = $\frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$.

This energy is stored inside the capacitor in its electric field with energy density

$$\frac{dU}{dV} = \frac{1}{2}\in E^2 \text{ or } \frac{1}{2}\in_0 \in_r E^2.$$

(v) The capacitor is represented as following:



(vi) Based on shape and arrangement of capacitor plates there are various types of capacitors.

(a) Parallel plate capacitor. (b) Spherical capacitor. (c) Cylindrical capacitor.

(vii) Capacitance of a capacitor depends on

(a) Area of plates. (b) Distance between the plates.

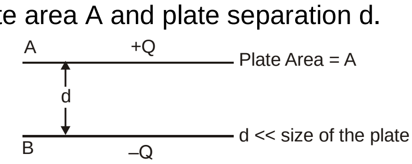
(c) Dielectric medium between the plates.

(viii) Electric field intensity between the plates of capacitors (air filled) $E = \sigma/\in_0 = V/d$

(ix) Force experienced by any plate of capacitor $F = q^2/2A\in_0$

Example 3. Find out the capacitance of parallel plate capacitor of plate area A and plate separation d.

Solution : $mE = \frac{Q}{A\in_0} \Rightarrow V_A - V_B = E.d. = \frac{Qd}{A\in_0} = \frac{Q}{C}$



$\Rightarrow \quad C = \dfrac{\epsilon_0 A}{d}$

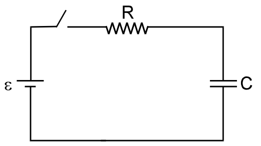
where A = area of the plates.

d = distance between plates.

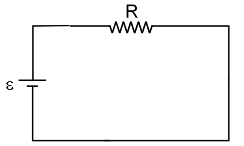
7. CIRCUIT SOLUTION FOR R–C CIRCUIT AT t = 0 (INITIAL STATE) AND AT t = ∞ (FINAL STATE)

Note : (i) Charge on the capacitor does not change instantaneously or suddenly if there is a resistance in the path (series) of the capacitor.

(ii) When an uncharged capacitor is connected with battery then its charge is zero initially hence potential difference across it is zero initially. At this time the capacitor can be treated as a conducting wire

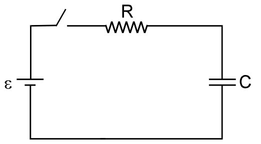


Before connection

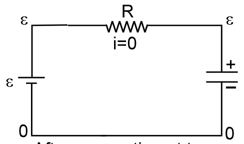


Just after connection at t = 0

(iii) The current will become zero finally (that means in steady state) in the branch which contains capacitor.



Before connection

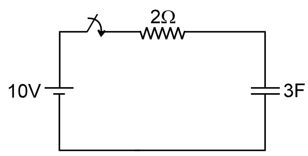


After connection at t = ∞

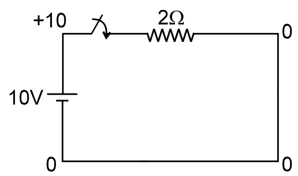
Example 4. Find out current in the circuit and charge on capacitor which is initially uncharged in the following situations.

(a) Just after the switch is closed.

(b) After a long time when switch was closed.



Solution : **(a) For just after closing the switch :** potential difference across capacitor = 0



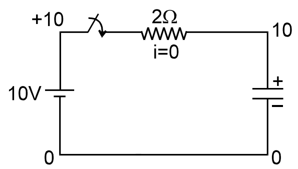
$\therefore \quad Q_C = 0 \qquad \therefore \quad i = \dfrac{10}{2} = 5A$

(b) After a long time

at steady state current i = 0

and potential difference across capacitor = 10 V

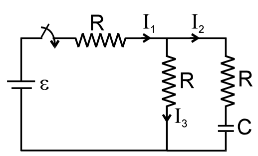
$\therefore \quad Q_C = 3 \times 10 = 30$ C



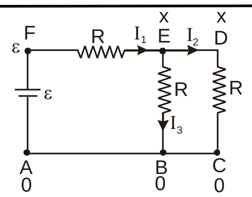
Example 5. Find out current I_1, I_2, I_3, charge on capacitor and $\dfrac{dQ}{dt}$ of capacitor in the circuit which is initially uncharged in the following situations.

(a) Just after the switch is closed

(b) After a long time when switch is closed.



Solution : **(a)** Initially the capacitor is uncharged so its behaviour is like a conductor. Let potential at A is zero so at B and C also zero and at F it is ε. Let potential at E is x so at D also x. Apply Kirchhoff's I[st] law at point E :



$$\frac{x-\varepsilon}{R} + \frac{x-0}{R} + \frac{x-0}{R} = 0 \Rightarrow \frac{3x}{R} = \frac{\varepsilon}{R}$$

$x = \frac{\varepsilon}{3}$; $Q_c = 0$

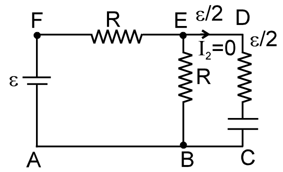
$$\therefore \quad I_1 = \frac{-\varepsilon/3+\varepsilon}{R} = \frac{2\varepsilon}{3R} \Rightarrow I_2 = \frac{dQ}{dt} = \frac{\varepsilon}{3R} \text{ and } I_3 = \frac{\varepsilon}{3R}$$

Alternatively

$$i_1 = \frac{\varepsilon}{R_{eq}} = \frac{\varepsilon}{R+\frac{R}{2}} = \frac{2\varepsilon}{3R} \Rightarrow i_2 = i_3 = \frac{i_1}{2} = \frac{\varepsilon}{3R} \text{ and } \frac{dQ}{dt} = i_2 = \frac{\varepsilon}{3R}$$

(b) at t = ∞ (finally)

capacitor completely charged so their will be no current through it.



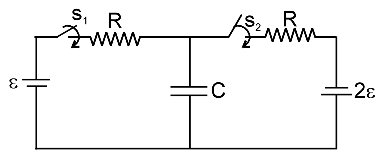
$I_2 = 0$, $I_1 = I_3 = \frac{\varepsilon}{2R}$

$V_E - V_B = V_D - V_C = (\varepsilon/2R)R = \varepsilon/2$

$\Rightarrow Q_C = \frac{\varepsilon C}{2}$, $\frac{dQ}{dt} = I_2 = 0$

Time	I_1	I_2	I_3	Q	dQ /dt
Time t = 0	$\frac{2\varepsilon}{3R}$	$\frac{\varepsilon}{3R}$	$\frac{\varepsilon}{3R}$	0	$\frac{\varepsilon}{3R}$
Finally t = ∞	$\frac{\varepsilon}{2R}$	0	$\frac{\varepsilon}{2R}$	$\frac{\varepsilon C}{2}$	0

Example 6. At t = 0 switch S_1 is closed and remains closed for a long time and S_2 remains open. Now S_1 is opened and S_2 is closed. Find out



(i) The current through the capacitor immediately after that moment

(ii) Charge on the capacitor long after that moment.

(iii) Total charge flown through the cell of emf 2ε after S_2 is closed.

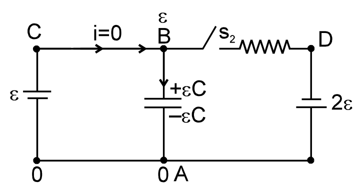
Solution : (i) Let Potential at point A is zero. Then at point B and C it will be ε (because current through the circuit is zero).

$V_B - V_A = (\varepsilon - 0)$

∴ Charge on capacitor = $C(\varepsilon - 0) = C\varepsilon$

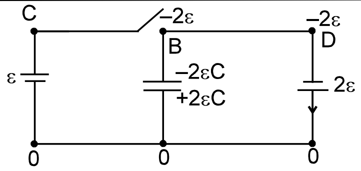
Now S_2 is closed and S_1 is open. (p.d. across capacitor and charge on it will not change suddenly)

Potential at A is zero so at D it is – 2ε.



$$\therefore \quad \text{current through the capacitor} = \frac{\varepsilon-(-2\varepsilon)}{R} = \frac{3\varepsilon}{R} \text{ (B to D)}$$

(ii) after a long time, i = 0



$V_B – V_A = V_D – V_A = – 2\varepsilon$

$\therefore$ Q = C (–2ε – 0) = –2εC

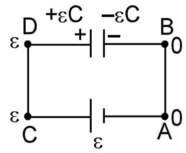
(iii) The charge on the lower plate (which is connected to the battery) changes from –εC to 2εC.

$\therefore$ this charge will come form the battery,

$\therefore$ charge flown from that cell is 3εC downward.

Example 7. A capacitor of capacitance C which is initially uncharged is connected with a battery. Find out heat dissipated in the circuit during the process of charging.

Solution : Final status



Let potential at point A is 0, so at B also 0 and at C and D it is ε. finally, charge on the capacitor

$Q_C = \varepsilon C$

$U_i = 0$

$U_f = \frac{1}{2} CV^2 = \frac{1}{2} C\varepsilon^2$

work done by battery $= \int P dt$

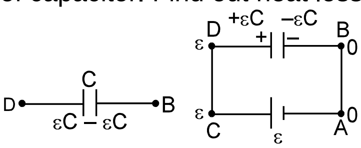
$W = \int \varepsilon i dt = \varepsilon \int i dt = \varepsilon . Q = \varepsilon . \varepsilon C = \varepsilon^2 C$

(Now onwards remember that w.d. by battery = εQ if Q has flown out of the cell from high potential and w.d. on battery is εQ if Q has flown into the cell through high potential)

Heat produced $= W – (U_f – U_i) = \varepsilon^2 C – \frac{1}{2} \varepsilon^2 C = \frac{C\varepsilon^2}{2}$.

Example 8. A capacitor of capacitance C which is initially charged upto a potential difference ε is connected with a battery of emf ε such that the positive terminal of battery is connected with positive plate of capacitor. Find out heat loss in the circuit during the process of charging.

Solution :



Since the initial and final charge on the capacitor is same before and after connection.

Here no charge will flow in the circuit so heat loss = 0

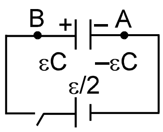
Example 9. A capacitor of capacitance C which is initially charged upto a potential difference ε is connected with a battery of emf ε/2 such that the positive terminal of battery is connected with positive plate of capacitor. After a long time

(i) Find out total charge flow through the battery

(ii) Find out total work done by battery

(iii) Find out heat dissipated in the circuit during the process of charging.

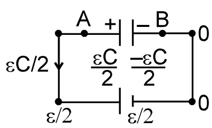
Solution : (i) Let potential of A is 0 so at B it is $\frac{\varepsilon}{2}$. So final charge on capacitor = Cε/2



Charge flow through the capacitor = (Cε/2 – Cε) = –Cε/2

So charge is entering into battery.

(ii) finally, Change in energy of capacitor = $U_{final} – U_{initial}$



$= \frac{1}{2} C\left(\frac{\varepsilon}{2}\right)^2 – \frac{\varepsilon^2 C}{2} = \frac{1}{8}\varepsilon^2 C – \frac{1}{2}\varepsilon^2 C = – \frac{3\ \varepsilon^2 C}{8}$

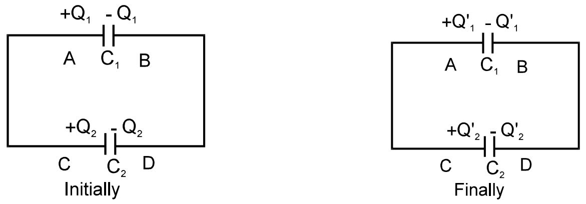
Work done by battery = $\frac{\varepsilon}{2} \times \left(-\frac{\varepsilon C}{2}\right) = -\frac{\varepsilon^2 C}{4}$

(iii) Work done by battery = Change in energy of capacitor + Heat produced

Heat produced = $\frac{3\varepsilon^2 C}{8} - \frac{\varepsilon^2 C}{4} = \frac{\varepsilon^2 C}{8}$

8. DISTRIBUTION OF CHARGES ON CONNECTING TWO CHARGED CAPACITORS:

When two capacitors are C_1 and C_2 are connected as shown in figure



Before connecting the capacitors		
Parameter	**I[st] Capacitor**	**II[nd] Capacitor**
Capacitance	C_1	C_2
Charge	Q_1	Q_2
Potential	V_1	V_2

After connecting the capacitors		
Parameter	**I[st] Capacitor**	**II[nd] Capacitor**
Capacitance	C_1	C_2
Charge	Q'_1	Q'_2
Potential	V	V

(a) Common potential : By charge conservation of plates A and C before and after connection.

$Q_1 + Q_2 = C_1V + C_2V$

$$\Rightarrow \quad V = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{\text{Total charge}}{\text{Total capacitance}}$$

(b) $Q_1' = C_1V = \frac{C_1}{C_1 + C_2}(Q_1 + Q_2) \quad \Rightarrow \quad Q_2' = C_2 V = \frac{C_2}{C_1 + C_2}(Q_1 + Q_2)$

(c) Heat loss during redistribution :

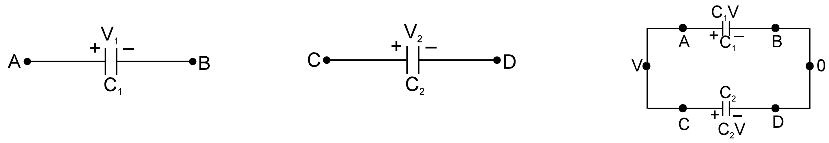
$$\Delta H = U_i - U_f = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$

The loss of energy is in the form of Joule heating in the wire.

Note : (i) When plates of similar charges are connected with each other (+ with + and – with –) then put all values (Q_1, Q_2, V_1, V_2) with positive sign.

(ii) When plates of opposite polarity are connected with each other (+ with –) then take charge and potential of one of the plate to be negative.

Derivation of above formulae :



Let potential of B and D is zero and common potential on capacitors is V, then at A and C it will be V

$C_1V + C_2V = C_1V_1 + C_2V_2$

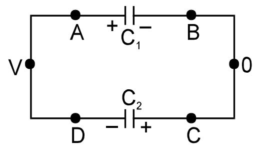
$V = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2}$ $\Rightarrow$ $H = \dfrac{1}{2}C_1V_1^2 + \dfrac{1}{2}C_2V_2^2 - \dfrac{1}{2}(C_1 + C_2)V^2$

$= \dfrac{1}{2}C_1V_1^2 + \dfrac{1}{2}C_2V_2^2 - \dfrac{1}{2}\dfrac{(C_1V_1 + C_2V_2)^2}{(C_1 + C_2)}$

$= \dfrac{1}{2}\left[\dfrac{C_1^2V_1^2 + C_1C_2V_1^2 + C_2C_1V_2^2 + C_2^2V_2^2 - C_1^2V_1^2 - C_2^2V_2^2 - 2C_1C_2V_1V_2}{C_1 + C_2}\right] = \dfrac{1}{2}\dfrac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$

$H = \dfrac{1}{2}\dfrac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$

When oppositely charge terminals are connected then



$\therefore$ $C_1V + C_2V = C_1V_1 - C_2V_2$

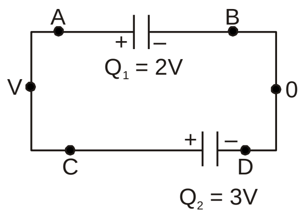
$V = \dfrac{C_1V_1 - C_2V_2}{C_1 + C_2}$ and $H = \dfrac{1}{2}\dfrac{C_1C_2}{C_1 + C_2}(V_1 + V_2)^2$

Example 10 Find out the following if A is connected with C and B is connected with D.

(i) How much charge flows in the circuit.

(ii) How much heat is produced in the circuit.

Solution : **(i)**



Let potential of B and D is zero and common potential on capacitors is V, then at A and C it will be V.

By charge conservation, 3V + 2V = 40 + 30

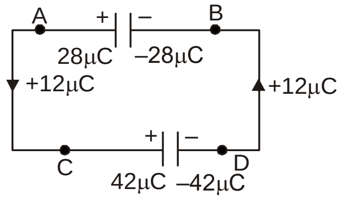
5V = 70

V = 14 volt

Charge flow = 40 – 28 = 12 μC

Now final charges on each plate is shown in the figure

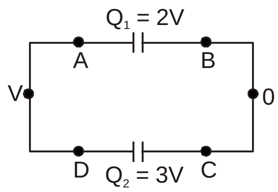
(ii) Heat produced = $\dfrac{1}{2} \times 2 \times (20)^2 + \dfrac{1}{2} \times 3 \times (10)^2 - \dfrac{1}{2} \times 5 \times (14)^2$



$= 400 + 150 – 490 = 550 – 490 = 60\ \mu J$

Note : (i) When capacitor plates are joined then the charge remains conserved.
(ii) We can also use direct formula of redistribution as given above.

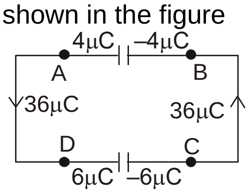
Example 11. Repeat above question if A is connected with D and B is connected with C.



Solution : Let potential of B and C is zero and common potential on capacitors is V, then at A and D it will be V

$2V + 3V = 10 \quad \Rightarrow \quad V = 2$ volt

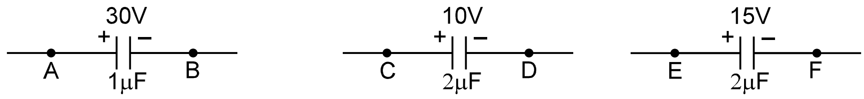
Now charge on each plate is shown in the figure



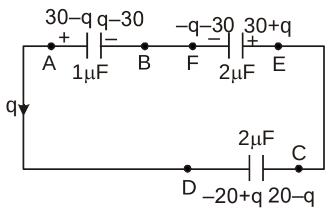
Heat produced $= 400 + 150 - \frac{1}{2} \times 5 \times 4 = 550 - 10 = 540\ \mu J$

Note : Here heat produced is more. Think why?

Example 12. Three capacitors as shown of capacitance 1μF, 2μF and 2μF are charged upto potential difference 30 V, 10 V and 15 V respectively. If terminal A is connected with D, C is connected with E and F is connected with B. Then find out charge flow in the circuit and find the final charges on capacitors.



Solution : Let charge flow is q. Now applying kirchhoff's voltage low

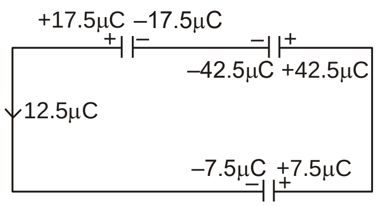


$$-\frac{(q-20)}{2} - \frac{(30+q)}{2} + \frac{30-q}{1} = 0$$

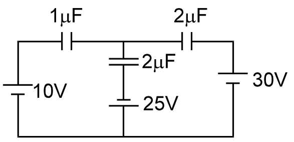
$–2q = – 25$

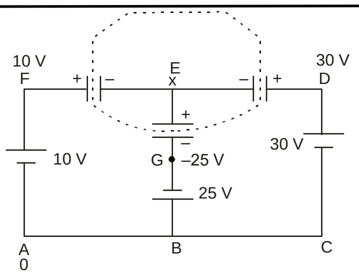
$q = 12.5\ \mu C$

Final charges on plates



Example 13. In the given circuit find out the charge on each capacitor. (Initially they are uncharged)



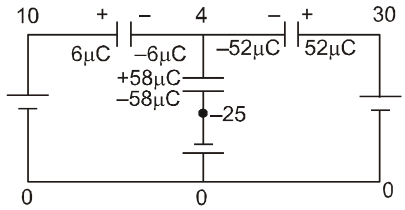


Let potential at A is 0, so at D it is 30 V, at F it is 10 V and at point G potential is –25V and let potential at E is x. Now apply kirchhoff's I[st] law at point E. (Total charge of all the plates connected to 'E' must be same as before i.e. 0)

$\therefore$ $(x - 10) + (x - 30)2 + (x + 25)2 = 0$

$5x = 20$

$x = 4$ V



Final charges :

$Q_{2\mu F} = (30 - 4)2 = 52\mu C$

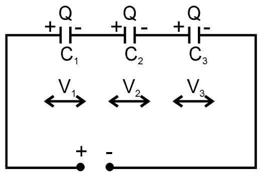
$Q_{1\mu F} = (10 - 4) = 6\mu C$

$Q_{2\mu F} = (4 - (-25))2 = 58\ \mu C$

9. COMBINATION OF CAPACITORS :

9.1 Series Combination :

(i) When initially uncharged capacitors are connected as shown then the combination is called series combination.



(ii) All capacitors will have same charge but different potential difference across them.

(iii) We can say that $V_1 = \frac{Q}{C_1}$

V_1 = potential across C_1

Q = charge on positive plate of C_1

C_1 = capacitance of capacitor similarly

$V_2 = \frac{Q}{C_2}$, $V_3 = \frac{Q}{C_3}$; ....

(iv) $V_1 : V_2 : V_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$

We can say that potential difference across capacitor is inversely proportional to its capacitance in series combination.

$V \propto \frac{1}{C}$

Note : In series combination the smallest capacitor gets maximum potential.

(v) $$V_1 = \frac{\frac{1}{C_1}}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...} V$$

$$V_2 = \frac{\frac{1}{C_2}}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...} V$$

$$V_3 = \frac{\frac{1}{C_3}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+......} V$$

Where $V = V_1 + V_2 + V_3$

(vi) Equivalent Capacitance : Equivalent capacitance of any combination is that capacitance which when connected in place of the combination, stores same charge and energy that of the combination.

In series : $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$

Note : In series combination equivalent capacitance is always less the smallest capacitor of combination.

(vii) Energy stored in the combination $U_{combination} = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3}$

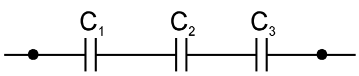
$U_{combination} = \frac{Q^2}{2C_{eq}}$

Energy supplied by the battery in charging the combination $U_{battery} = Q \times V = Q \cdot \frac{Q}{C_{eq}} = \frac{Q^2}{C_{eq}}$

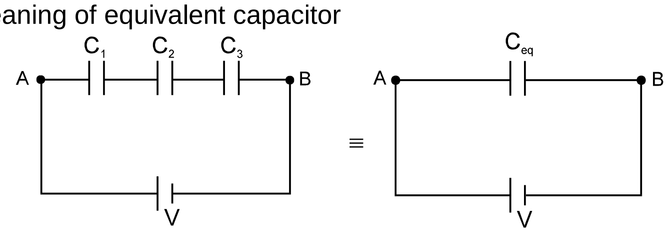
$\frac{U_{combination}}{U_{battery}} = \frac{1}{2}$

Note : Half of the energy supplied by the battery is stored in form of electrostatic energy and half of the energy is converted into heat through resistance. (if capacitors are initially uncharged)

Derivation of Formulae :



meaning of equivalent capacitor



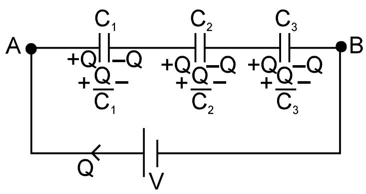
$C_{eq} = \frac{Q}{V}$

Now, initially, the capacitor has no charge. Applying kirchhoff's voltage law

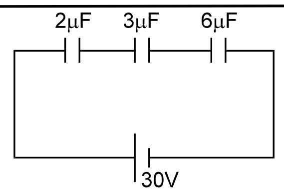
$\frac{-Q}{C_1} + \frac{-Q}{C_2} + \frac{-Q}{C_3} + V = 0.$

$V = Q\left[\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\right]$; $\frac{V}{Q} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$

$\Rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$ in general $\frac{1}{C_{eq}} = \sum_{n=1}^{n}\frac{1}{C_n}$



Example 14. Three initially uncharged capacitors are connected in series as shown in circuit with a battery of emf 30V. Find out following :



(i) charge flow through the battery,

(ii) potential energy in 3 μF capacitor.

(iii) U_{total} in capacitors

(iv) heat produced in the circuit

Solution : $\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{3+2+1}{6} = 1$

$C_{eq} = 1\mu F$.

(i) $Q = C_{eq} V = 30\mu C$.

(ii) charge on 3μF capacitor = 30μC

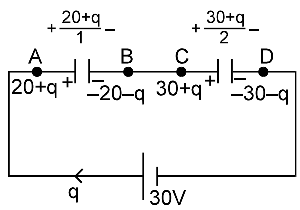
energy = $\frac{Q^2}{2C} = \frac{30 \times 30}{2 \times 3} = 150\mu J$

(iii) $U_{total} = \frac{30 \times 30}{2}$ μJ = 450 μJ

(iv) Heat produced = (30 μC) (30) – 450 μJ = 450 μJ.

Example 15. Two capacitors of capacitance 1 μF and 2μF are charged to potential difference 20V and 15V as shown in figure. If now terminal B and C are connected together terminal A with positive of battery and D with negative terminal of battery then find out final charges on both the capacitor

Solution : Now applying kirchoff voltage law



$\frac{-(20+q)}{1} - \frac{30+q}{2} + 30 = 0$

– 40 – 2q – 30 – q = – 60

3q = –10

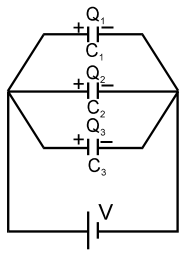
Charge flow = –10/3 μC.

Charge on capacitor of capacitance 1μF = 20 + q = $\frac{50}{3}$ μC

Charge on capacitor of capacitance 2μF = 30 + q = $\frac{80}{3}$ μC

9.2 Parallel Combination :

(i) When one plate of each capacitors (more than one) is connected together and the other plate of each capacitor is connected together, such combination is called parallel combination.



(ii) All capacitors have same potential difference but different charges.

(iii) We can say that :

$Q_1 = C_1V$

Q_1 = Charge on capacitor C_1

C_1 = Capacitance of capacitor C_1

V = Potential across capacitor C_1

(iv) $Q_1 : Q_2 : Q_3 = C_1 : C_2 : C_3$

The charge on the capacitor is proportional to its capacitance

$Q \propto C$

(v) $Q_1 = \dfrac{C_1}{C_1+C_2+C_3}Q \Rightarrow Q_2 = \dfrac{C_2}{C_1+C_2+C_3}Q$

$Q_3 = \dfrac{C_3}{C_1+C_2+C_3}Q$

Where $Q = Q_1 + Q_2 + Q_3$ ......

Note : Maximum charge will flow through the capacitor of largest value.

(vi) Equivalent capacitance of parallel combination $C_{eq} = C_1 + C_2 + C_3$

Note : Equivalent capacitance is always greater than the largest capacitor of combination.

(vii) Energy stored in the combination :

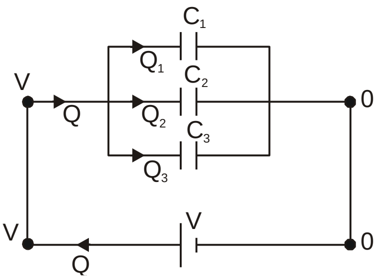
$V_{combination} = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + ... = \frac{1}{2}(C_1 + C_2 + C_3 .....)V^2 = \frac{1}{2}C_{eq}V^2$

$U_{battery} = QV = CV^2$

$\dfrac{U_{combination}}{U_{battery}} = \dfrac{1}{2}$

Note : Half of the energy supplied by the battery is stored in form of electrostatic energy and half of the energy is converted into heat through resistance. (If all capacitors are initially uncharged)

Formulae Derivation for parallel combination :



$Q = Q_1 + Q_2 + Q_3$

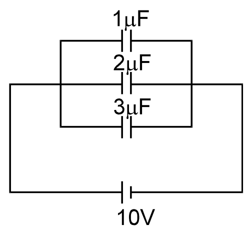
$\dfrac{Q}{V} = C_1V + C_2V + C_3V = V(C_1 + C_2 + C_3)$

$= C_1 + C_2 + C_3$

$C_{eq} = C_1 + C_2 + C_3$

In general $C_{eq} = \sum_{n=1}^{n} C_n$

Example 16. Three initially uncharged capacitors are connected to a battery of 10 V is parallel combination find out following



(i) charge flow from the battery

(ii) total energy stored in the capacitors

(iii) heat produced in the circuit

(iv) potential energy in the 3μF capacitor.

Solution : (i) $Q = (30 + 20 + 10)\mu C = 60\ \mu C$

(ii) $U_{total} = \frac{1}{2} \times 6 \times 10 \times 10 = 300\ \mu J$

(iii) heat produced $= 60 \times 10 - 300 = 300\ \mu J$

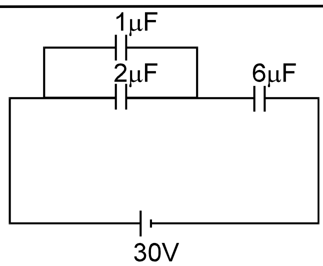
(iv) $U_{3\mu F} = \frac{1}{2} \times 3 \times 10 \times 10 = 150\ \mu J$

9.3 Mixed Combination :

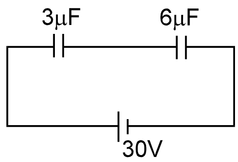
The combination which contains mixing of series parallel combinations or other complex combinations fall in mixed category. There are two types of mixed combinations

(i) Simple (ii) Complex.

Example 17. In the given circuit find out charge on 6μF and 1μF capacitor.



Solution : It can be simplified as $C_{eq} = \frac{18}{9} = 2\mu F$



charge flow through the cell = 30 × 2 μC

Q = 60 μC

Now charge on 3μF = Charge on 6μF= 60 μC

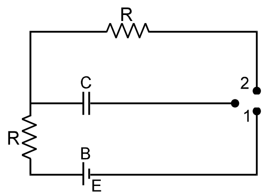
Potential difference across 3μF = 60/ 3= 20 V

∴ Charge on 1μF = 20 μC.

10. CHARGING AND DISCHARGING OF A CAPACITOR

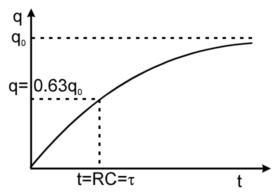
10.1 Charging of a condenser :

(i) In the following circuit. If key 1 is closed then the condenser gets charged. Finite time is taken in the charging process. The quantity of charge at any instant of time t is given by $q = q_0[1 - e^{-(t/RC)}]$



Where q_0 = maximum final value of charge at t = ∞.

According to this equations the quantity of charge on the condenser increases exponentially with increase of time.



(ii) If t = RC = τ then

$$q = q_0 [1 - e^{-(RC/RC)}] = q_0 \left[1 - \frac{1}{e}\right]$$

or $q = q_0 (1 - 0.37) = 0.63\, q_0 = 63\%$ of q_0

(iii) Time t = RC is known as time constant.

i.e. the time constant is that time during which the charge rises on the condenser plates to 63% of its maximum value.

(iv) The potential difference across the condenser plates at any instant of time is given by

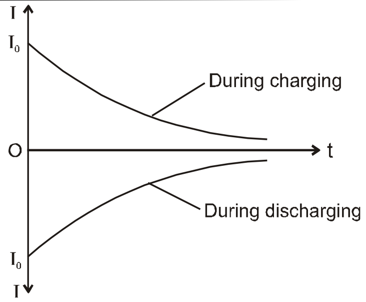
$V = V_0[1 - e^{-(t/RC)}]$ volt

(v) The potential curve is also similar to that of charge. During charging process an electric current flows in the circuit for a small interval of time which is known as the transient current. The value of this current at any instant of time is given by

$I = I_0[e^{-(t/RC)}]$ ampere

According to this equation the current falls in the circuit exponentially (Fig.).

(vi) If t = RC = τ = Time constant

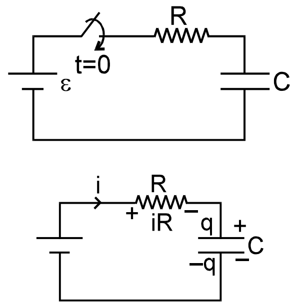


$I = I_0 e^{(-RC/RC)} = \frac{I_0}{e} = 0.37\ I_0$

$= 37\%$ of I_0

i.e. time constant is that time during which current in the circuit falls to 37% of its maximum value.

Derivation of formulae for charging of capacitor



it is given that initially capacitor is uncharged.

let at any time charge on capacitor is q

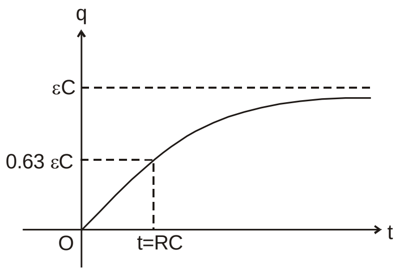
Applying kirchoff voltage law

$\varepsilon - iR - \frac{q}{C} = 0 \quad \Rightarrow \quad iR = \frac{\varepsilon C - q}{C}$

$i = \frac{\varepsilon C - q}{CR} \quad \Rightarrow \quad \frac{dq}{dt} = \frac{\varepsilon C - q}{CR}$

$\frac{dq}{dt} = \frac{\varepsilon C - q}{CR} \quad \Rightarrow \quad \frac{CR}{\varepsilon C - q}\,.\,dq = dt.$

$\int_O^q \frac{dq}{\varepsilon C - q} = \int_O^t \frac{dt}{RC} \quad \Rightarrow \quad -\ln(\varepsilon C - q) + \ln \varepsilon C = \frac{t}{RC}$



$\ln \frac{\varepsilon C}{\varepsilon C - q} = \frac{t}{RC}$; $\varepsilon C - q = \varepsilon C\,.\,e^{-t/RC}$

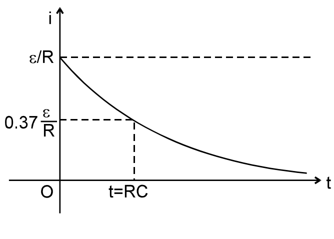
$q = \varepsilon C(1 - e^{-t/RC})$

RC = time constant of the RC series circuit.

After one time constant

$q = \varepsilon C\left(1 - \frac{1}{e}\right) = \varepsilon C\,(1 - 0.37) = 0.63\ \varepsilon C.$

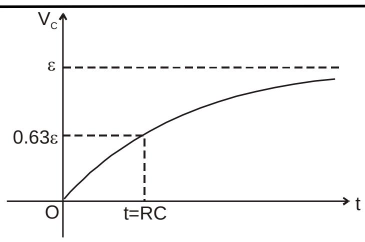
Current at any time t



$i = \frac{dq}{dt} = \varepsilon C \left(-e^{-t/RC}\left(-\frac{1}{RC}\right)\right)$

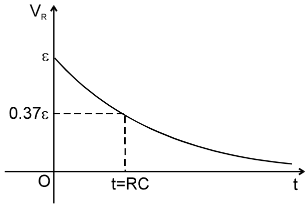
$= \frac{\varepsilon}{R} e^{-t/RC}$

Voltage across capacitor after one time constant V = 0.63 ε



$Q = CV$; $V_C = \varepsilon(1 - e^{-t/RC})$

Voltage across the resistor



$V_R = iR = \varepsilon e^{-t/RC}$

By energy conservation,

Heat dissipated = work done by battery – $\Delta U_{capacitor}$

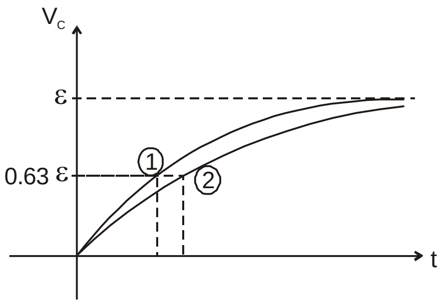
$$= C\varepsilon(\varepsilon) - \left(\frac{1}{2}C\varepsilon^2 - 0\right) = \frac{1}{2}C\varepsilon^2$$

Alternatively : Heat = H = $\int_0^\infty i^2 R dt$

$$= \int_0^\infty \frac{\varepsilon^2}{R^2} e^{-\frac{2t}{RC}} R\,dt = \frac{\varepsilon^2}{R}\int_0^\infty e^{-2t/RC}\,dt = \frac{\varepsilon^2}{R}\left[\frac{e^{-\frac{2t}{RC}}}{-2/RC}\right]_0^\infty$$

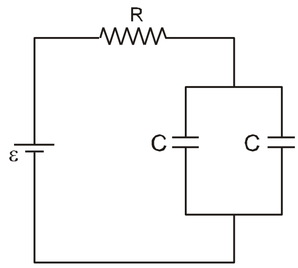
$$= -\frac{\varepsilon^2 RC}{2R}\left[e^{-\frac{2t}{RC}}\right]_0^\infty = \frac{\varepsilon^2 C}{2}$$

Note:



In the figure time constant of (2) is more than (1)

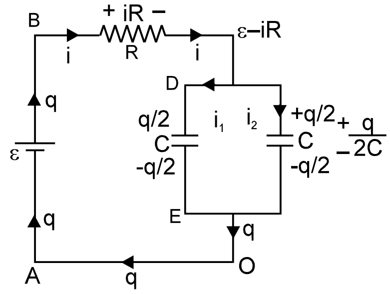
Example 18 Without using the formula of equivalent. Find out charge on capacitor and current in all the branches as a function of time.



Solution : Applying KVL in ABDEA

$$\varepsilon - iR = \frac{q}{2C}$$

$$i = \frac{\varepsilon}{R} - \frac{q}{2CR} = \frac{2C\varepsilon - q}{2CR}$$



$$\frac{dq}{2\varepsilon C - q} = \frac{dt}{2CR}$$

$$\int_0^q \frac{dq}{(2\varepsilon C - q)} = \frac{t}{2CR}$$

$$\frac{2\varepsilon C - q}{2\varepsilon C} = e^{-t/2RC}$$

$$q = 2\varepsilon C\,(1 - e^{-t/2RC})$$

$$q_1 = \frac{q}{2} = \varepsilon C\,(1 - e^{-t/2RC}) \quad \Rightarrow \quad i_1 = \frac{\varepsilon}{2R}\,e^{-t/2RC}$$

$$q_2 = \frac{q}{2} = \varepsilon C\,(1 - e^{-t/2RC}) \quad \Rightarrow \quad i_2 = \frac{\varepsilon}{2R}\,e^{-t/2RC}$$

Alternate solution

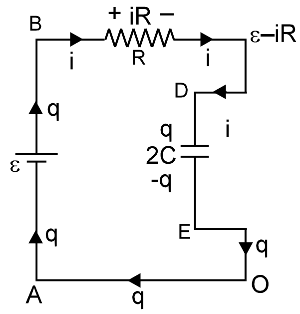
by equivalent

Time constant of circuit = 2C × R = 2RC

maximum charge on capacitor = 2C × ε = 2Cε

Hence equations of charge and current are as given below

$q = 2\varepsilon C\,(1 - e^{-t/2RC})$



$$q_1 = \frac{q}{2} = \varepsilon C\,(1 - e^{-t/2RC}) \quad \Rightarrow \quad i_1 = \frac{\varepsilon}{2R}\,e^{-t/2RC}$$

$$q_2 = \frac{q}{2} = \varepsilon C\,(1 - e^{-t/2RC}) \quad \Rightarrow \quad i_2 = \frac{\varepsilon}{2R}\,e^{-t/2RC}$$

Example 19 A capacitor is connected to a 36 V battery through a resistance of 20Ω. It is found that the potential difference across the capacitor rises to 12.0 V in 2μs. Find the capacitance of the capacitor.

Solution : The charge on the capacitor during charging is given by $Q = Q_0(1 - e^{-t/RC})$.

Hence, the potential difference across the capacitor is $V = Q/C = Q_0/C\,(1 - e^{-t/RC})$.

Here, at t = 2 μs, the potential difference is 12V whereas the steady potential difference is

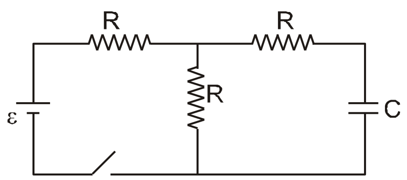
$Q_0/C = 36V$. So, $\Rightarrow$ $12V = 36V(1 - e^{-t/RC})$

or, $1 - e^{-t/RC} = \frac{1}{3}$ or, $e^{-t/RC} = \frac{2}{3}$

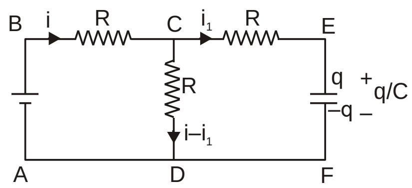
or, $\frac{t}{RC} = \ell n\left(\frac{3}{2}\right) = 0.405$ or, $RC = \frac{t}{0.405} = \frac{2\ \mu s}{0.45} = 4.936\ \mu s$

or, $C = \frac{4.936\mu s}{20\Omega} = 0.25\ \mu F$.

Example 20. Initially the capacitor is uncharged find the charge on capacitor as a function of time, if switch is closed at t = 0.



Solution : Applying KVL in loop ABCDA



$\varepsilon - iR - (i - i_1) R = 0$

$\varepsilon - 2iR + i_1R = 0$ .....(i)

Applying KVL in loop ABCEFDA

$$\varepsilon - iR - i_1R - \frac{q}{C} = 0$$

by eq (i) $\frac{2\varepsilon - \varepsilon - i_1R - 2i_1R}{2} = \frac{q}{C} \Rightarrow \varepsilon C - 3i_1RC = 2q$

$\varepsilon C - 2q = 3\frac{dq}{dt}.RC \quad \Rightarrow \int_0^q \frac{dq}{\varepsilon C - 2q} = \int_0^t \frac{dt}{3RC}$

$-\frac{1}{2}\ln\frac{\varepsilon C - 2q}{\varepsilon C} = \frac{t}{3RC} \Rightarrow q = \frac{\varepsilon C}{2}\left(1 - e^{-2t/3RC}\right)$

Method for objective :

In any circuit when there is only one capacitor then

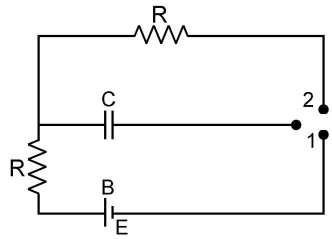
$q = Q_{st}\left(1 - e^{-t/\tau}\right)$; Q_{st} = steady state charge on capacitor (has been found in article 6 in this sheet)

$\tau = R_{eff}.C$

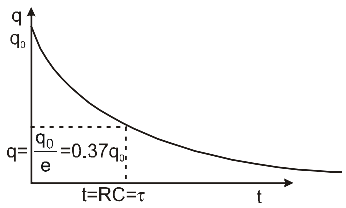
$R_{effective}$ is the resistance between the capacitor when battery is replaced by its internal resistance.

10.2 Discharging of a condenser :

(i) In the above circuit (in article 10.1) if key 1 is opened and key 2 is closed then the condenser gets discharged.



(ii) The quantity of charge on the condenser at any instant of time t is given by $q = q_0\, e^{-(t/RC)}$



i.e. the charge falls exponentially.

here q_0 = initial charge of capacitor

(iii) If $t = RC = \tau$ = time constant, then $q = \frac{q_0}{e} = 0.37q_0$ = 37% of q_0

i.e., the time constant is that time during which the charge on condenser plates in discharge process, falls to 37%

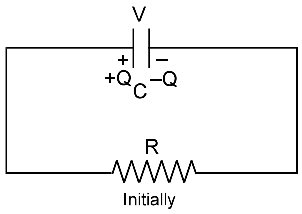
(iv) The dimensions of RC are those of time i.e. $M^0L^0T^1$ and the dimensions of $\frac{1}{RC}$ are those of frequency i.e. $M^0L^0T^{-1}$.

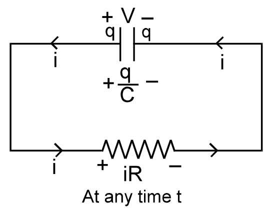
(v) The potential difference across the condenser plates at any instant of time t is given by $V = V_0e^{-(t/RC)}$ Volt.

(vi) The transient current at any instant of time is given by $I = -I_0e^{-(t/RC)}$ ampere.

i.e. the current in the circuit decreases exponentially but its direction is opposite to that of charging current. (– ive only means that direction of current is opposite to that at charging current)

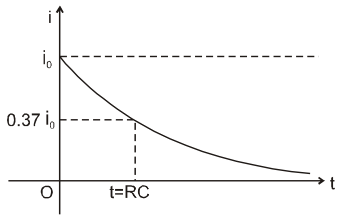
Derivation of equation of discharging circuit :





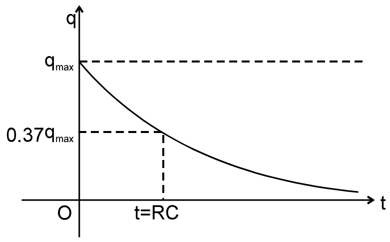
Applying K.V.L.

$$+\frac{q}{C} - iR = 0$$



$$i = \frac{q}{CR}$$

$$\int_Q^q \frac{-dq}{q} = \int_0^t \frac{dt}{CR}$$



$$-\ln\frac{q}{Q} = +\frac{t}{RC}$$

$$q = Q \,.\, e^{-t/RC}$$

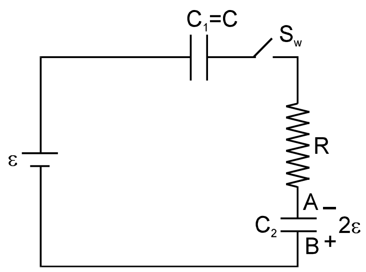
$$i = -\frac{dq}{dt} = \frac{Q}{RC}e^{-t/RC} = i_0 e^{-t/RC}$$

Example 21 At t = 0, switch is closed, if initially C_1 is uncharged and C_2 is charged to a potential difference 2ε then find out following (Given $C_1 = C_2 = C$)

(a) Charge on C_1 and C_2 as a function of time.

(b) Find out current in the circuit as a function of time.

(c) Also plot the graphs for the relations derived in part (a).



Solution : Let q charge flow in time 't' from the battery as shown. The charge on various plates of the capacitor is as shown in the figure. Now applying KVL

$$\varepsilon - \frac{q}{C} - iR - \frac{q-2\varepsilon C}{C} = 0$$

$$\varepsilon - \frac{q}{C} - \frac{q}{C} + 2\varepsilon - iR = 0$$

$$3\varepsilon = \frac{2q}{C} + iR \quad \Rightarrow \quad 3\varepsilon - iR = \frac{2q}{C}$$

$$3\varepsilon C - iRC = 2q \quad \Rightarrow \quad \frac{dq}{dt} RC = 3\varepsilon C - 2q$$

$$\int_0^q \frac{dq}{3\varepsilon C - 2q} = \int_0^t \frac{dt}{RC} \Rightarrow -\frac{1}{2}\ell n\left(\frac{3C\varepsilon - 2q}{3C\varepsilon}\right) = \frac{t}{RC}$$

$$\ln\left(\frac{3\varepsilon C - 2q}{3\varepsilon C}\right) = -\frac{2t}{RC} \Rightarrow 3\varepsilon C - 2q = 3\varepsilon C\ e^{-2t/RC}$$

$$3\varepsilon C\,(1 - e^{-2t/RC}) = 2q \Rightarrow q = \frac{3}{2}\varepsilon C\ (1 - e^{-2t/RC})$$

(charge on C, as function of time) **Ans.**

$$i = \frac{dq}{dt} = \frac{3\varepsilon}{R}e^{-2t/RC}$$ **Ans.**

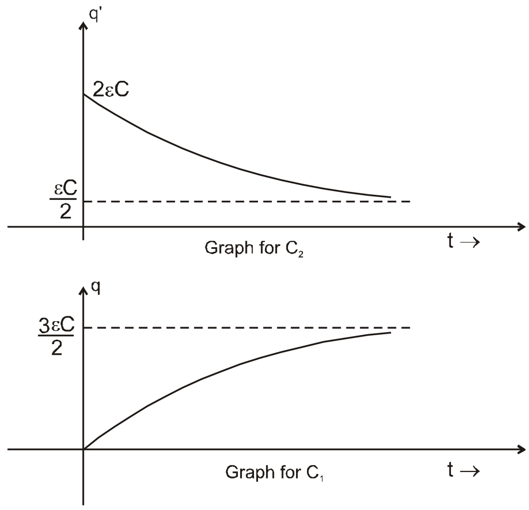
Charge on C_2 as function of time :

$q' = 2\varepsilon C - q$

$$= 2\varepsilon C - \frac{3}{2}\varepsilon C + \frac{3}{2}\varepsilon C\, e^{-2t/RC}$$

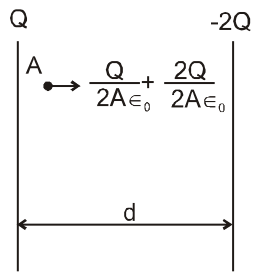
$$= \frac{\varepsilon C}{2} + \frac{3}{2}\varepsilon C e^{-2t/RC}$$

$$= \frac{\varepsilon C}{2}\left[1 + 3e^{-2t/RC}\right]$$



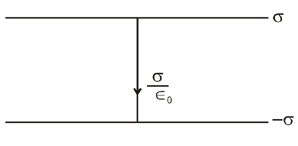
Example 22 Two parallel conducting plates of a capacitor of capacitance C containing charges Q and –2Q at a distance d apart. Find out potential difference between the plates of capacitors.

Solution : Capacitance = C



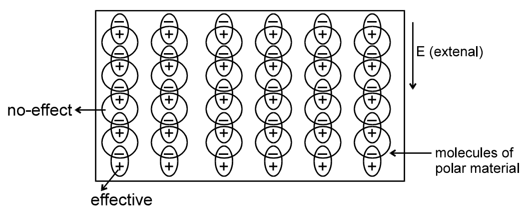
$$\text{Electric field} = \frac{3Q}{2A\in_0}\ ;\ V = \frac{3Qd}{2A\in_0} \Rightarrow V = \frac{3Q}{2C}$$

11. CAPACITORS WITH DIELECTRIC



(i) In absence of dielectric $E = \frac{\sigma}{\in_0}$

(ii) When a dielectric fills the space between the plates then molecules having dipole moment align themselves in the direction of electric field.



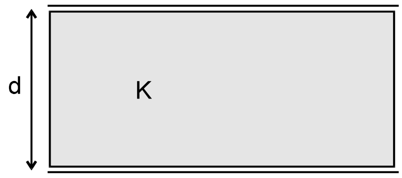
σ_b = induced charge density (called bound charge because it is not due to free electrons).

∗ For polar molecules dipole moment $\neq 0$

∗ For non-polar molecules dipole moment $= 0$

(iii) Capacitance in the presence of dielectric

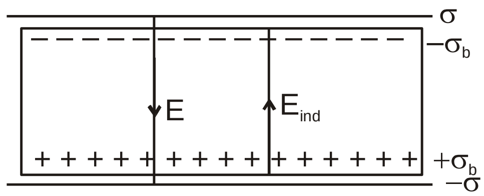
$$C = \frac{\sigma A}{V} = \frac{\sigma \ A}{\frac{\sigma}{K\in_0}.d} = \frac{AK\in_0}{d} = \frac{AK\in_0}{d}$$



Here capacitance is increased by a factor K.

$$C = \frac{AK\in_0}{d}$$

(iv) Polarisation of material : When nonpolar substance is placed in electric field then dipole moment is induced in the molecule. This induction of dipole moment is called polarisation of material. The induced charge also produce electric field.



σ_b = induced (bound) charge density.

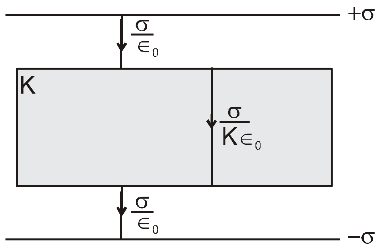
$$E_{in} = E - E_{ind} = \frac{\sigma}{\in_0} - \frac{\sigma_b}{\in_0}$$

It is seen the ratio of electric field between the plates in absence of dielectric and in presence of dielectric is constant for a material of dielectric. This ratio is called 'Dielectric constant' of that material. It is represented by ε_r or k.

$$E_{in} = \frac{\sigma}{K\in_0} \Rightarrow \sigma_b = \sigma\left(1 - \frac{1}{K}\right)$$

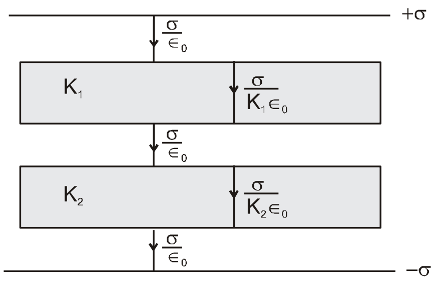
(v) If the medium does not filled between the plates completely then electric field will be as shown in figure

Case : (1) :

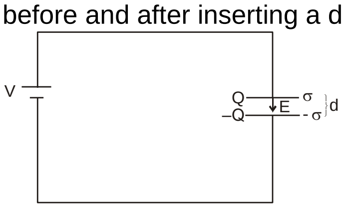


The total electric field produced by bound induced charge on the dielectric outside the slab is zero because they cancel each other.

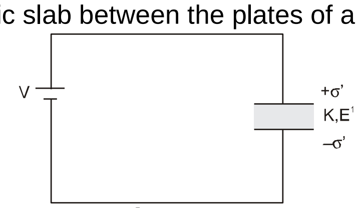
Case : (2)



(vi) Comparison of E (electric field), σ (surface charges density), Q (charge), C (capacitance) and before and after inserting a dielectric slab between the plates of a parallel plate capacitor.



Case I



Case II

$$C = \frac{\in_0 A}{d}$$

$$Q = CV$$

$$E = \frac{\sigma}{\in_0} = \frac{CV}{A\in_0}$$

$$C' = \frac{A\in_0 K}{d}$$

$$Q' = C'V$$

$$E' = \frac{\sigma}{K\in_0} = \frac{CV}{A\in_0}$$

$= \frac{V}{d}$

Here potential difference between the plates,

$Ed = V$

$E = \frac{V}{d}$

$\frac{V}{d} = \frac{\sigma}{\epsilon_0}$

$E' = \frac{V}{d}$

Here potential difference between the plates

$E'd = V$

$E' = \frac{V}{d}$

$\frac{V}{d} = \frac{\sigma'}{K\epsilon_0}$

Equating both

$\frac{\sigma}{\epsilon_0} = \frac{\sigma'}{K\epsilon_0}$

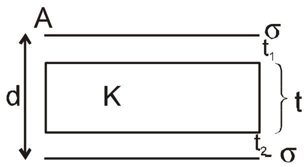
$\sigma' = K\sigma$

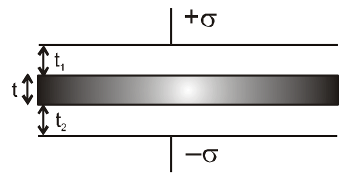
In the presence of dielectric, i.e. in case II capacitance of capacitor is more.

(vii) Energy density in a dielectric = $\frac{1}{2}\epsilon_0\epsilon_r E^2$

Example 23. If a dielectric slab of thickness t and area A is inserted in between the plates of a parallel plate capacitor of plate area A and distance between the plates d (d > t) then find out capacitance of system. What do you predict about the dependence of capacitance on location of slab?

Solution :





$C = \frac{Q}{v} = \frac{\sigma A}{v}$ $\quad V = \frac{\sigma t_1}{\epsilon_0} + \frac{\sigma t}{K\epsilon_0} + \frac{\sigma t_2}{\epsilon_0}$ $\quad (\because t_1 + t_2 = d - t)$

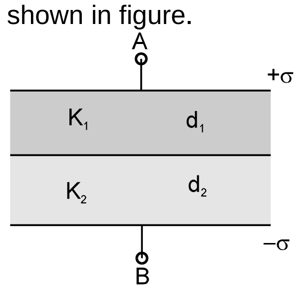
$= \frac{\sigma}{\epsilon_0}\left[t_1 + t_2 + \frac{t}{k}\right] \Rightarrow \quad V = \frac{\sigma}{\epsilon_0}\left[d - t + \frac{t}{k}\right] = \frac{Q}{C} = \frac{\sigma A}{C} \quad \Rightarrow C = \frac{\epsilon_0 A}{d - t + t/K}$

Note :

(i) Capacitance does not depend upon the position of dielectric (it can be shifted up or down still capacitance does not change).

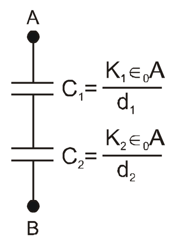
(ii) If the slab is of metal then : $C = \frac{A\epsilon_0}{d - t}$ (for metal $k \to \infty$)

Example 24 Find out capacitance between A and B if two dielectric slabs of dielectric constant K_1 and K_2 of thickness d_1 and d_2 and each of area A are inserted between the plates of parallel plate capacitor of plate area A as shown in figure.



Solution : $C = \frac{\sigma A}{V}$; $V = E_1 d_1 + E_2 d_2 = \frac{\sigma d_1}{K_1\epsilon_0} + \frac{\sigma d_2}{K_2\epsilon_0} = \frac{\sigma}{\epsilon_0}\left(\frac{d_1}{k_1} + \frac{d_2}{k_2}\right)$

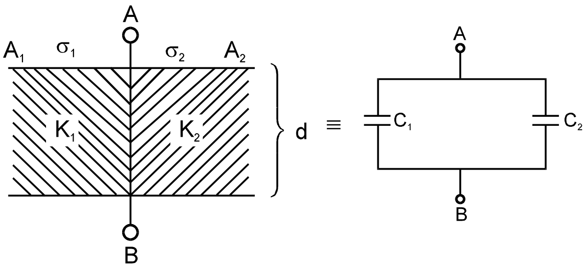
$\therefore \quad C = \frac{A\epsilon_0}{\frac{d_1}{K_1} + \frac{d_2}{K_2}} \quad \Rightarrow \frac{1}{C} = \frac{d_1}{AK_1\epsilon_0} + \frac{d_2}{AK_2\epsilon_0}$



This formula suggests that the system between A and B can be considered as series combination of two capacitors.

Example 25. Find out capacitance between A and B if two dielectric slabs of dielectric constant K_1 and K_2 of area A_1 and A_2 and each of thickness d are inserted between the plates of parallel plate capacitor of plate area A as shown in figure. ($A_1 + A_2 = A$)

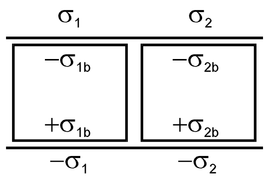
Solution :

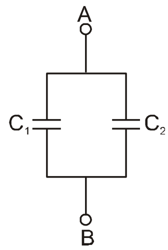


$C_1 = \dfrac{A_1K_1 \in_0}{d}$, $C_2 = \dfrac{A_2K_2 \in_0}{d}$

$E_1 = \dfrac{V}{d} = \dfrac{\sigma_1}{K_1 \in_0}$, $E_2 = \dfrac{V}{d} = \dfrac{\sigma_2}{K_2 \in_0}$

$\sigma_1 = \dfrac{K_1 \in_0 V}{d}$ $\quad \sigma_2 = \dfrac{K_2 \in_0 V}{d}$

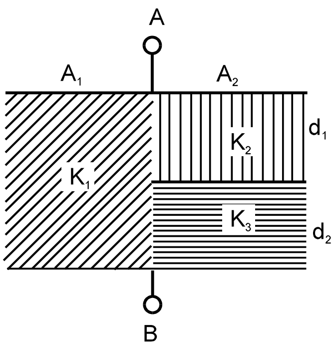




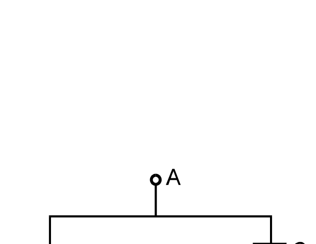
$C = \dfrac{Q_1 + Q_2}{V} = \dfrac{\sigma_1 A_1 + \sigma_2 A_2}{V} = \dfrac{K_1 \in_0 A_1}{d} + \dfrac{K_2 \in_0 A_2}{d}$

The combination is equivalent to : $C = C_1 + C_2$

Example 26. Find out capacitance between A and B if three dielectric slabs of dielectric constant K_1 of area A_1 and thickness d, K_2 of area A_2 and thickness d_1 and K_3 of area A_2 and thickness d_2 are inserted between the plates of parallel plate capacitor of plate area A as shown in figure. (Given distance between the two plates $d = d_1 + d_2$)



Solution : It is equivalent to $C = C_1 + \dfrac{C_2C_3}{C_2 + C_3}$

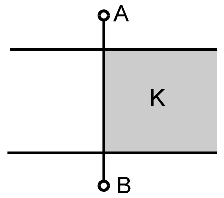


$$C = \frac{A_1K_2 \in_0}{d_1 + d_2} + \frac{\dfrac{A_2K_2 \in_0}{d_1} \cdot \dfrac{A_2K_3 \in_0}{d_1}}{\dfrac{A_2K_2 \in_0}{d_1} + \dfrac{A_2K_3 \in_0}{d_2}}$$

$$= \frac{A_1K_1 \in_0}{d_1 + d_2} + \frac{A_2^2K_2K_3 \in_0}{A_2K_2 \in_0 d_2 + A_2K_3 \in_0 d_1}$$

$$= \frac{A_1K_2 \in_0}{d_1 + d_2} + \frac{A_2^2K_2K_3 \in_0}{K_2d_2 + K_3d_1}$$

Example 27. A dielectric of constant K is slipped between the plates of parallel plate condenser in half of the space as shown in the figure. If the capacity of air condenser is C, then new capacitance between A and B will be-



(A) $\frac{C}{2}$ (B) $\frac{C}{2K}$ (C) $\frac{C}{2}$ [1 + K] (D) $\frac{2[1+K]}{C}$

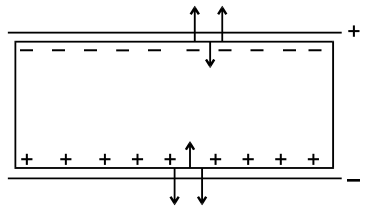
Solution : This system is equivalent to two capacitors in parallel with area of each plate $\frac{A}{2}$.

$$C' = C_1 + C_2 = \frac{\epsilon_0 A/2}{d} + \frac{\epsilon_0 (A/2)K}{d} = \frac{\epsilon_0 A}{2d}[1+K] = \frac{C}{2}[1+K]$$

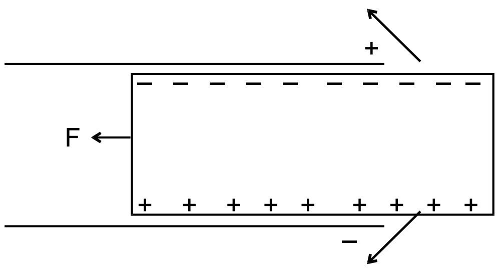
Hence the correct answer will be (C).

(viii) Force on a dielectric due to charged capacitor :

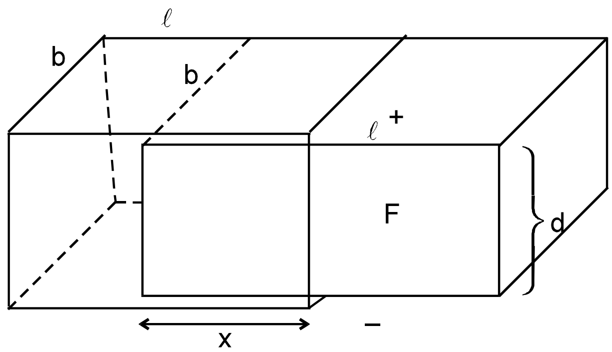
(a) If dielectric is completely inside the capacitor then force is equal to zero.



(b) If dielectric is not completely inside the capacitor.



Case-I : Voltage source remains connected



V = constant.

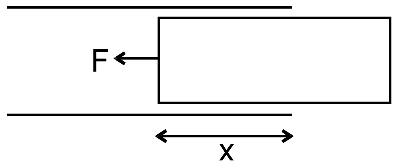
$U = \frac{1}{2} CV^2$

$$F = \left(\frac{dU}{dx}\right) = \frac{V^2}{2}\frac{dC}{dx} \text{ where } C = \frac{xb\epsilon_0 K}{d} + \frac{\epsilon_0(\ell - x)b}{d} \Rightarrow C = \frac{\epsilon_0 b}{d}[Kx + \ell - x]$$

$$\frac{dC}{dx} = \frac{\epsilon_0 b}{d}(K-1)$$

$$\therefore F = \frac{\epsilon_0 b(K-1)V^2}{2d} = \text{constant (does not depend on x)}$$

Case II : When charge on capacitor is constant

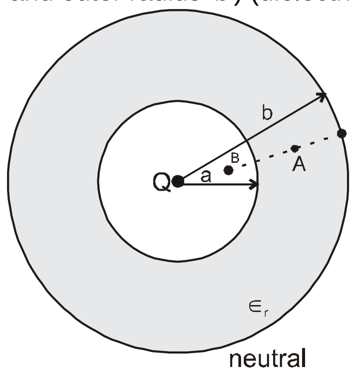


$$C = \frac{xb \epsilon_0 K}{d} + \frac{\epsilon_0 (\ell - x)b}{d}, \; U = \frac{Q^2}{2C}$$

$$F = \left(\frac{dU}{dx}\right) = \frac{Q^2}{2C^2} \cdot \frac{dC}{dx} \qquad \left[\text{where, } \frac{dC}{dx} = \frac{\epsilon_0 b}{d}(K-1)\right]$$

$$= \frac{Q^2}{2C^2} \cdot \frac{dC}{dx} \text{ (here force 'F' depends on x)}$$

Example 28. Find V and E at : (Q is a point charge kept at the centre of the non-conducting neutral thick sphere of inner radius 'a' and outer radius 'b') (dielectric constant = ϵ_r)



(i) $0 < r < a$ (ii) $a \le r < b$ (iii) $r \ge b$

Solution : –q and +q charge will induce on inner and outer surface respectively

$$E(0 < r < a) = \frac{KQ}{r^2}$$

$$E(r \ge b) = \frac{KQ}{r^2}$$

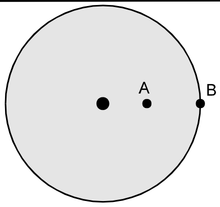
$$E(a \le r < b) = \frac{KQ}{r^2} - \frac{Kq}{r^2} = \frac{KQ}{\epsilon_r r^2} \qquad \textbf{Ans.}$$

$$q = Q \cdot \left(1 - \frac{1}{\epsilon_r}\right) \quad ; \quad V(r \ge b) = \frac{KQ}{r}$$

$$(a \le r \le b) \; V_A = V_P + \int_b^r \frac{KQ}{\epsilon_r r^2}(-dr) = \frac{kQ}{b} + \frac{kQ}{\epsilon_r}\left(\frac{1}{r} - \frac{1}{b}\right)$$

$$V(r \le a) \qquad V_B = V_C + \int_a^r \frac{KQ}{r^2}(-dr) = \frac{kQ}{b} + \frac{kQ}{\epsilon_r}\left(\frac{1}{a} - \frac{1}{b}\right) + kQ\left(\frac{1}{r} - \frac{1}{a}\right)$$

Example 29. What is potential at a distance r (<R) in a dielectric sphere of uniform charge density ρ, radius R and dielectric constant ε_r.



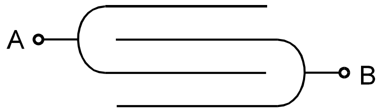
Solution : $V_A = V_B + \frac{W_{B\to A}}{q}$

$$V = \frac{Q}{4\pi \in_0 R} + \int_R^r \frac{\rho r}{3 \in_0 \in_r}(-dr) = \frac{Q}{4\pi \in_0 R} + \frac{\rho(R^2 - r^2)}{3 \in_0 \in_r}$$

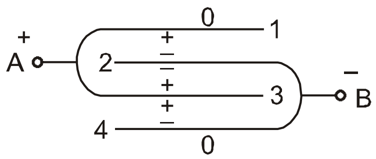
$V_{outiside} = \frac{KQ}{r}$

12. COMBINATION OF PARALLEL PLATES

Example 30. Find out equivalent capacitance between A and B. (take each plate Area = A and distance between two conjugative plates is d)



Solution : Let numbers on the plates The charges will be as shown in the figure.

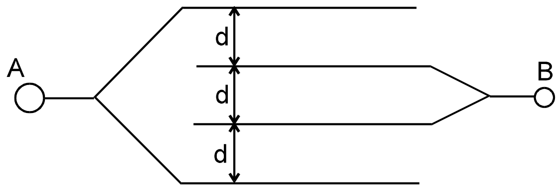


$V_{12} = V_{32} = V_{34}$

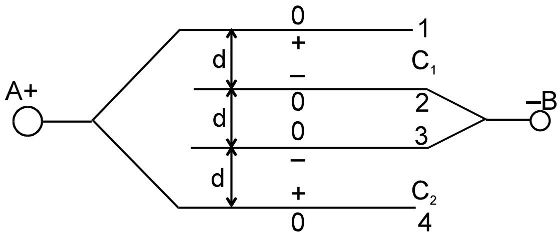
so all the capacitors are in parallel combination.

$C_{eq} = C_1 + C_2 + C_3 = \frac{3A \in_0}{d}$

Example 31. Find out equivalent capacitance between A and B. (take each plate Area = A)

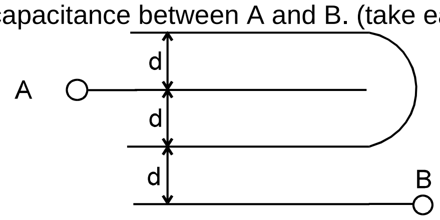


Solution :

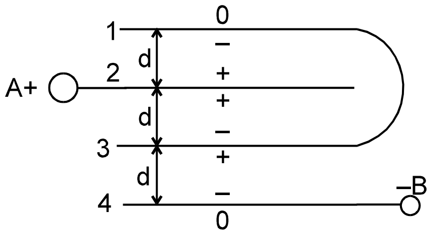


These are only two capacitors $C_{eq} = C_1 + C_2 = \frac{2A \in_0}{d}$

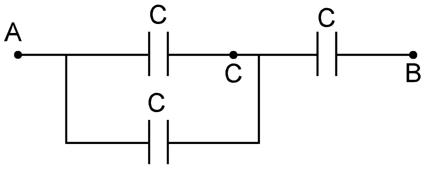
Example 32. Find out equivalent capacitance between A and B. (take each plate Area = A)



Solution :

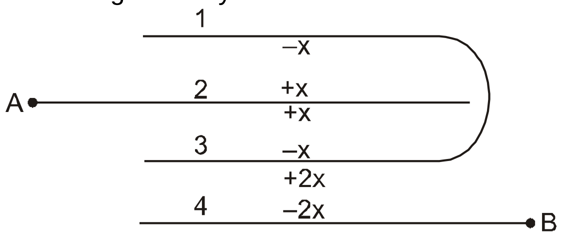


The modified circuit is



$C_{eq} = \frac{2C}{3} = \frac{2A \in_0}{3d}$

Other method : Let charge density as shown

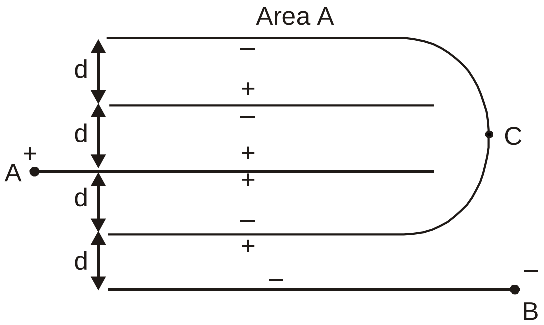


$C_{eq} = \frac{Q}{V} = \frac{2xA}{V}$

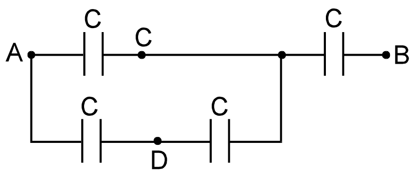
$V = V_2 – V_4 = (V_2 – V_3) + (V_3 – V_4) = \frac{xd}{\in_0} + \frac{2xd}{\in_0} = \frac{3xd}{\in_0}$

$\therefore \quad C_{eq} = \frac{2Ax \in_0}{3xd} = \frac{2A \in_0}{3d} = \frac{2C}{3}$.

Example 33. Find out equivalent capacitance between A and B.



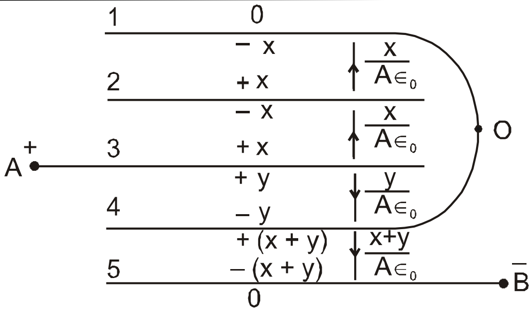
Solution : Let $C = \frac{A \in_0}{d}$ Equivalent circuit :



$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{2}{3C} = \frac{5}{3C}$

$C_{eq} = \frac{3C}{5} = \frac{3A \in_0}{5d}$

Alternative Method : Let charge distribustion on plates as shown :



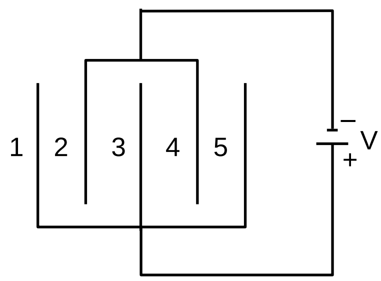
$C = \frac{Q}{V} = \frac{x+y}{V_{AB}}$

Potential of 1 and 4 is same

$\frac{y}{A\epsilon_0} = \frac{2x}{A\epsilon_0}$ y = 2x

$V = \left(\frac{2y+x}{A\varepsilon_0}\right)d \Rightarrow C = \frac{(x+2x)A\epsilon_0}{(5x)d} = \frac{3A\epsilon_0}{5d}$.

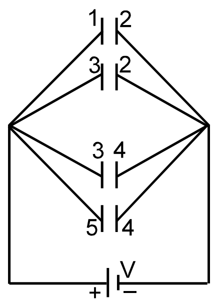
Example 34. Five similar condenser plates, each of area A, are placed at equal distance d apart and are connected to a source of e.m.f. V as shown in the following diagram. The charge on the plates 1 and 4 will be-



(A) $\frac{\epsilon_0 A}{d}, \frac{-2\epsilon_0 A}{d}$ (B) $\frac{\epsilon_0 AV}{d}, \frac{-2\epsilon_0 AV}{d}$

(C) $\frac{-\epsilon_0 AV}{d}, \frac{-3\epsilon_0 AV}{d}$ (D) $\frac{\epsilon_0 AV}{d}, \frac{-4\epsilon_0 AV}{d}$

Solution : by equivalent circuit diagram Charge on first plate Q = CV



$Q = \frac{\epsilon_0 AV}{d}$

Charge on fourth plate Q´ = C(–V) $Q' = \frac{-\epsilon_0 AV}{d}$

As plate 4 is repeated twice, hence charge on 4 will be Q´´ = 2Q´

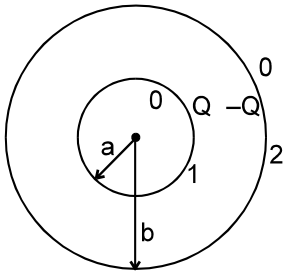
$Q'' = -\frac{2\epsilon_0 AV}{d}$

Hence the correct answer will be (B).

13. OTHER TYPES OF CAPACITORS

Spherical capacitor :

This arrangement is known as spherical capacitor.



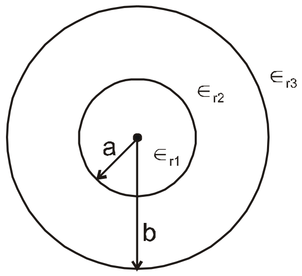
$$V_1 - V_2 = \left[\frac{KQ}{a} - \frac{KQ}{b}\right] - \left[\frac{KQ}{b} - \frac{KQ}{b}\right] = \frac{KQ}{a} - \frac{KQ}{b}$$

$$C = \frac{Q}{V_1 - V_2} = \frac{Q}{\frac{KQ}{a} - \frac{KQ}{b}} = \frac{ac}{K(b-a)} = \frac{4\pi \in_0 ab}{b-a}$$

$$C = \frac{4\pi \in_0 ab}{b-a}$$

If b >> a then

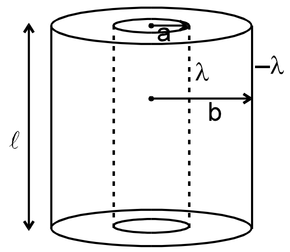
$C = 4\pi\in_0 a$ (Like isolated spherical capacitor)



If dielectric mediums are filled as shown then : $C = \dfrac{4\pi \in_0 \in_{r_2} ab}{b-a}$

Cylindrical capacitor

There are two co-axial conducting cylindrical surfaces where ℓ >> a and ℓ >> b, where a and b is radius of cylinders.



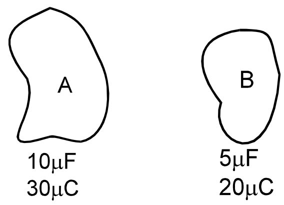
Capacitance per unit length $C = \dfrac{\lambda}{V}$

$$= \frac{\lambda}{2K\lambda \ell n\frac{b}{a}} = \frac{4\pi \in_0}{2\ell n\frac{b}{a}} = \frac{2\pi \in_0}{\ell n\frac{b}{a}}$$

Capacitance per unit length $= \dfrac{2\pi \in_0}{\ell n\frac{b}{a}}$ F/m

Miscellaneous Solved Example

Problem 1. When two isolated conductors A and B are connected by a conducting wire positive charge will flow from.



(A) A to B (B) B to A

(C) will not flow (D) can not say.

Solution : Charge always flows from higher potential body to lower potential body

Hence, $V_A = \dfrac{30}{10} = 3V \Rightarrow V_B = \dfrac{20}{5} = 4$ V As $V_B > V_A$ $\therefore$ (B) is correct Answer.

Problem 2. A conductor of capacitance 10μF connected to other conductor of capacitance 40 μF having equal charges 100 μC initially. Find out final voltage and heat loss during the process?

Answer : (i) V = 4V (ii) H = 225 μJ.

Solution :

$C_1 = 10\mu F$ $C_2 = 40\mu F$

$Q_1 = 100\ \mu C$ $Q_2 = 100\mu C$

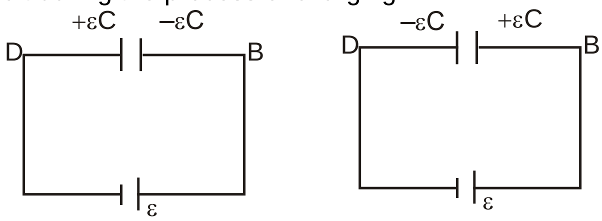
$V_1 = Q_1/C_1 = 10\ V$ $V_2 = Q_2/C_2 = 2.5$

Final voltage (V) = $\frac{C_1V_1 + C_2V_2}{(C_1 + C_2)} = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{200\mu C}{50\mu F} = 4V$

Heat loss during the process = $\frac{1}{2}\left[C_1V_1^2 + C_2V_2^2\right] - \frac{1}{2}V^2(C_1 + C_2)$

$= \frac{1}{2}\left[Q_1V_1 + Q_2V_2\right] - \frac{1}{2}V^2(C_1 + C_2) = \frac{1}{2}\times 100\mu[12.5] - \frac{1}{2}\times 16\ (50)\ \mu = 225\ \mu J$

Problem 3. A capacitor of capacitance C is charged from battery of e.m.f. ε and then disconnected. Now the positive terminal of the battery is connected with negative plate of capacitor. Find out heat loss in the circuit during the process of charging.



Initially finally

Net charge flow through battery = 2εC

Work done by battery = ε × 2εC = $2\varepsilon^2C$

Heat produced = $2\varepsilon^2$ C. **Ans.**

Solution : From figure

Net charge flow through

battery = $Q_{final} - Q_{initial} = \varepsilon C - (-\varepsilon C) = 2\varepsilon C$

$\therefore$ work done by battery (W) = Q × V = 2εC × ε = $2\varepsilon^2C$

or Heat produced = $2\varepsilon^2C$