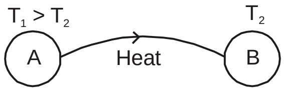
CHAPTER-20
CALORIMETRY AND THERMAL EXPANSION

1. HEAT

The energy that is being transferred between two bodies or between adjacent parts of a body as a result of temperature difference is called heat. Thus, heat is a form of energy. It is energy in transit whenever temperature differences exist. Once it is transferred, it becomes the internal energy of the receiving body. It should be clearly understood that the word "heat" is meaningful only as long as the energy is being transferred. Thus, expressions like "heat in a body" or "heat of a body" are meaningless.



When we say that a body is heated it means that its molecules begin to move with greater kinetic energy.

S.I. unit of heat energy is joule (J). Another common unit of heat energy is calorie (cal).

1 calorie = 4.18 joules.

1 calorie : The amount of heat needed to increase the temperature of 1 gm of water from 14.5 to 15.5 ºC at one atmospheric pressure is 1 calorie.

1.1 Mechanical Equivalent of Heat

In early days heat was not recognized as a form of energy. Heat was supposed to be something needed to raise the temperature of a body or to change its phase. Calorie was defined as the unit of heat. A number of experiments were performed to show that the temperature may also be increased by doing mechanical work on the system. These experiments established that heat is equivalent to mechanical energy and measured how much mechanical energy is equivalent to a calorie. If mechanical work W produces the same temperature change as heat H, we write, **W = JH**

where J is called mechanical equivalent of heat. J is expressed in joule/calorie. The value of J gives how many joules of mechanical work is needed to raise the temperature of 1 g of water by 1°C.

Example 1. What is the change in potential energy (in calories) of a 10 kg mass after 10 m fall ?

Solution : Change in potential energy

$$\Delta U = mgh = 10 \times 10 \times 10 = 1000 \text{ J} = \frac{1000}{4.186} \text{ cal}$$ **Ans.**

2. SPECIFIC HEAT

Specific heat of substance is equal to heat gain or released by that substance to raise or fall its temperature by 1ºC for a unit mass of substance.

When a body is heated, it gains heat. On the other hand, heat is lost when the body is cooled. The gain or loss of heat is directly proportional to:

(a) the mass of the body $\Delta Q \propto m$

(b) rise or fall of temperature of the body $\Delta Q \propto \Delta T$

$$\Delta Q \propto m\,\Delta T \quad \text{or} \quad \Delta Q = m\,s\,\Delta T \quad \text{or} \quad dQ = m\,s\,dT \quad \text{or} \quad Q = \int m\,s\,dT.$$

where s is a constant and is known as the specific heat of the body $s = \frac{Q}{m\Delta T}$. S.I. unit of s is joule/kg-Kelvin and C.G.S. unit is cal./gm °C.

Specific heat of water : S = 4200 J/kgºC = 1000 cal/kgºC = 1 Kcal/kgºC = 1 cal/gmºC

Specific heat of steam = half of specific heat of water = specific heat of ice

Example 2. Heat required to increases the temperate of 1 kg water by 20ºC

Solution : heat required = $\Delta Q = ms\Delta\theta$

$\because$ S = 1 cal/gmºC = 1 Kcal/kgºC = 1 × 20 = 20 Kcal.

2.1 Heat capacity or Thermal capacity :

Heat capacity of a body is defined as the amount of heat required to raise the temperature of that body by 1°. If 'm' is the mass and 's' the specific heat of the body, then **Heat capacity = ms**.

Units of heat capacity in: CGS system is, **cal ºC^{-1}**; SI unit is, **JK^{-1}**

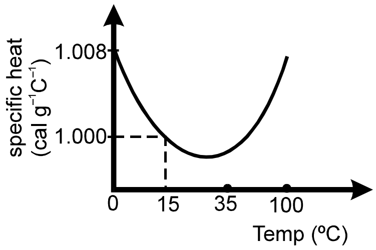
2.2 Important Points:

(a) We know, $s = \frac{Q}{m\Delta T}$, if the substance undergoes the change of state which occurs at constant temperature ($\Delta T = 0$), then $s = Q/0 = \infty$. Thus the specific heat of a substance when it melts or boils at constant temperature is infinite.

(b) If the temperature of the substance changes without the transfer of heat (Q = 0) then $s = \frac{Q}{m\Delta T} = 0$. Thus when liquid in the thermos flask is shaken, its temperature increases without the transfer of heat and hence the specific heat of liquid in the thermos flask is zero.

(c) To raise the temperature of saturated water vapours, heat (Q) is withdrawn. Hence, specific heat of saturated water vapours is negative. (This is for your information only and not in the course)

(d) The slight variation of specific heat of water with temperature is shown in the graph at 1 atmosphere pressure. Its variation is less than 1% over the interval form 0 to 100ºC.



2.3 Relation between Specific heat and Water equivalent :

It is the amount of water which requires the same amount of heat for the same temperature rise as that of the object

$ms\,\Delta T = m_W S_W\,\Delta T \Rightarrow m_W = \frac{ms}{s_W}$

In calorie $s_W = 1$ $\therefore$ $m_W = ms$

m_w is also represent by W

So W = ms.

2.4 Phase change :

Heat required for the change of phase or state,

Q = mL, L = latent heat.

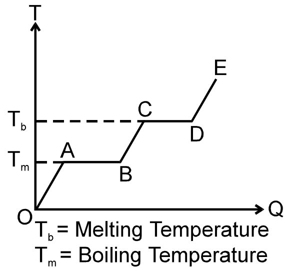
Latent heat (L): The heat supplied to a substance which changes its state at constant temperature is called latent heat of the body.

Latent heat of Fusion (L_f) : The heat supplied to a substance which changes it from solid to liquid state at its melting point and 1 atm. pressure is called latent heat of fusion. Latent heat of fusion of ice is 80 kcal/kg

Latent heat of vaporization (L_v): The heat supplied to a substance which changes it from liquid to vapour state at its boiling point and 1 atm. pressure is called latent heat of vaporization. Latent heat of vaporization of water is 540 kcal kg^{-1}.

Latent heat of ice : L = 80 cal/gm = 80 Kcal/kg = 4200 × 80 J/kg

Latent heat of steam : L = 540 cal/gm = 540 Kcal/kg = 4200 × 540 J/kg



The given figure, represents the change of state by different lines

OA – solid state, AB – solid + liquid state (Phase change)

BC – liquid state, CD – liquid + vapour state (Phase change)

DE – vapour state

$\Delta Q = ms\Delta T$

slope $\frac{\Delta T}{\Delta Q} = \frac{1}{ms} \Rightarrow \frac{\Delta T}{\Delta Q} \propto \frac{1}{S}$

where mass (m) of substance constant slope of T – Q graph is inversely proportional to specific heat, if in given diagram

(slope) OA > (slope) DE

then $(s)_{OA} < (s)_{DE}$

when $\Delta Q = mL$

If (length of AB) > (length of CD)

then (latent heat of AB) > (latent heat of CD)

Example 3. Find the amount of heat released if 1 kg steam at 200ºC is converted into –20ºC ice.

Solution : Heat released ΔQ = heat release to convert steam at 200 ºC into 100ºC steam + heat release to convert 100º C steam into 100ºC water + heat release to convert 100º water into 0ºC water + heat release to convert 0 ºC water into – 20ºC ice.

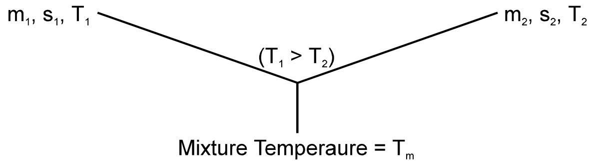
$$\Delta Q = 1 \times \frac{1}{2} \times 100 + 540 \times 1 + 1 \times 1 \times 100 + 1 \times 80 + 1 \times \frac{1}{2} \times 20 = 780 \text{ Kcal.}$$

3. CALORIMETRY

The branch of thermodynamics which deals with the measurement of heat is called calorimetry. A simple calorimeter is a vessel generally made of copper with a stirrer of the same material. The vessel is kept in a wooden box to isolate it thermally from the surrounding. A thermometer is used to measure the temperature of the contents of the calorimeter. Object at different temperatures are made to come in contact with each other in the calorimeter. As a result, heat is exchanged between the object as well as with the calorimeter. Neglecting any heat exchange with the surrounding.

3.1 Law of Mixture:

When two substances at different temperatures are mixed together, then exchange of heat continues to take place till their temperatures become equal. This temperature is then called final temperature of mixture. Here, **Heat taken by one substance = Heat given by another substance**



$\Rightarrow m_1 s_1 (T_1 - T_m) = m_2 s_2 (T_m - T_2)$

Example 4. An iron block of mass 2 kg, fall from a height 10 m. After colliding with the ground it loses 25% energy to surroundings. Then find the temperature rise of the block. (Take sp. heat of iron 470 J/kg ºC)

Solution : $mS\Delta\theta = \frac{1}{4} mgh \Rightarrow \Delta\theta = \frac{10 \times 10}{4 \times 470}$

Zeroth law of thermodynamics :

If objects A and B are separately in thermal equilibrium with a third object C, then objects A and B are in thermal equilibrium with each other.

Example 5. The temperature of equal masses of three different liquids A, B, and C are 10ºC 15ºC and 20ºC respectively. The temperature when A and B are mixed is 13ºC and when B and C are mixed, it is 16ºC. What will be the temperature when A and C are mixed?

Solution : When A and B are mixed $mS_1 \times (13 - 10) = m \times S_2 \times (15 - 13)$

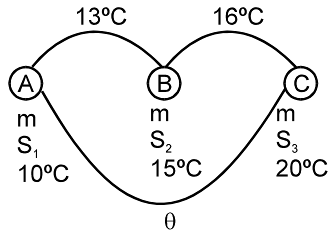
$3S_1 = 2S_2$ .....(1)

when B and C are mixed

$S_2 \times 1 = S_3 \times 4$ .....(2)

when C and A are mixed

$S_1(\theta - 10) = S_3 \times (20 - \theta)$ .....(3)



by using equation (1), (2) and (3)

we get $\theta = \frac{140}{11}$ ºC

Example 6. If three different liquid of different masses specific heats and temperature are mixed with each other and then what is the temperature mixture at thermal equilibrium.

$m_1, s_1, T_1 \rightarrow$ specification for liquid

$m_2, s_2, T_2 \rightarrow$ specification for liquid

$m_3, s_3, T_3 \rightarrow$ specification for liquid.

Solution : Total heat lost or gain by all substance is equal to zero

$\Delta Q = 0$

$m_1s_1(T - T_1) + m_2s_2(T - T_2) + m_3s_3(T - T_3) = 0$

So $T = \frac{m_1s_1T_1 + m_2s_2T_2 + m_3s_3T_3}{m_1s_1 + m_2s_2 + m_3s_3}$

Example 7. In following equation calculate value of H 1 kg ice at –20ºC = H + 1 Kg water at 100ºC, here H means heat required to change the state of substance.

Solution : Heat required to convert 1 kg ice at – 20ºC into 1 kg water at 100ºC

= 1 kg ice at – 20ºC to 1 kg ice at 0ºC ice at 0ºC + 1 kg water

at 0ºC + 1 kg water at 0ºC to 1 kg water at 100ºC

$= 1 \times \frac{1}{2} \times 20 + 1 \times 80 + 1 \times 100 = 190$ Kcal. So H = – 190 Kcal

Negative sign indicate that 190 Kcal heat is with drawn from 1 kg water at 100ºC to convert it into 1 kg ice at – 20ºC

Example 8. 1kg ice at –20ºC is mixed with 1 kg steam at 200ºC. Then find equilibrium temperature and mixture content.

Solution : Let equilibrium temperature is 100 ºC heat required to convert 1 kg ice at –20ºC to 1 kg water at 100ºC is equal to

$H_1 = 1 \times \frac{1}{2} \times 20 + 1 \times 80 + 1 \times 1 \times 100 = 190$ Kcal

heat release by steam to convert 1 kg steam at 200ºC to 1 kg water at 100ºC is equal to

$H_2 = 1 \times \frac{1}{2} \times 100 + 1 \times 540 = 590$ Kcal

1 kg ice at – 20ºC = H_1 + 1kg water at 100ºC .....(1)

1 kg steam at 200ºC = H_2 + 1kg water at 100ºC .....(2)

by adding equation (1) and (2)

1 kg ice at –20ºC + 1 kg steam at 200ºC = $H_1 + H_2$ + 2 kg water at 100ºC.

Here heat required to ice is less than heat supplied by steam so mixture equilibrium temperature is 100ºC then steam is not completely converted into water.

So mixture has water and steam which is possible only at 100ºC mass of steam which converted into water is equal to

$$m = \frac{190 - 1\times\frac{1}{2}\times 100}{540} = \frac{7}{27}\ \text{kg}$$

so mixture content

mass of steam = $1 - \frac{7}{27} = \frac{20}{27}$ kg

mass of water = $1 + \frac{7}{27} = \frac{34}{27}$ kg

4. THERMAL EXPANSION

Most materials expand when their temperature is increased. Rails roads tracks, bridges all have some means of compensating for thermal expansion. When a homogeneous object expands, the distance between any two points on the object increases. Figure shows a block of metal with a hole in it. **The expanded object is like a photographic enlargement.** That in the hole expands in the same proportion as the metal, it does not get smaller

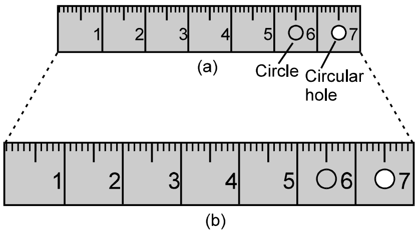
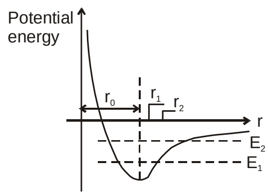


Fig. The same steel ruler two different temperatures. When it expands, the scale, the numbers, the thickness, and the diameters of the circle and circular hole are all incrased by the same factor. (The expansion has been exaggerated for clarity.)

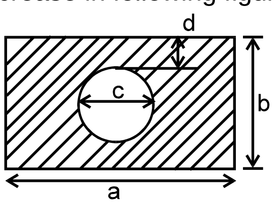


Thermal expansion arises because the well is not symmetrical about the equilibrium position r_0. As the temperature rise the energy of the atom increases. The average position when the energy is E_2 is not the same as that when the energy is E_1.

At the atomic level, thermal expansion may be understood by considering how the potential energy of the atoms varies with distance. The equilibrium position of an atom will be at the minimum of the potential energy well if the well is symmetric. At a given temperature each atom vibrates about its equilibrium position and its average remains at the minimum point. If the shape of the well is not symmetrical the average position of an atom will not be at the minimum point. When the temperature is raised the amplitude of the vibrations increases and the average position is located at a greater inter atomic separation. This increased separation is manifested as expansion of the material.

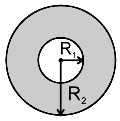
Almost all solids and liquids expand as their temperature increases. Gases also expand if allowed. Solids can change in length, area or volume, while liquids change in their volumes.

Example 9. A rectangular plate has a circular cavity as shown in the figure. If we increase its temperature then which dimension will increase in following figure.



Solution : Distance between any two point on an object increases with increase in temperature. So, all dimension a, b, c and d will increase

Example 10. In the given figure, when temperature is increased then which of the following increases

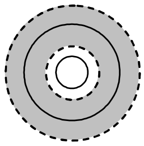


(A) R_1 (B) R_2 (C) $R_2 - R_1$

Solution : All of the above

- - - - - represents expanded Boundary

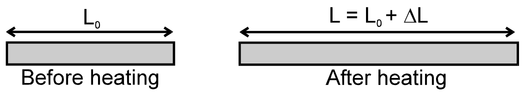
--------- represents original Boundary



As the intermolecular distance between atoms increases on heating hence the inner and outer perimeter increases. Also if the atomic arrangement in radial direction is observed then we can say that it also increases hence all A, B, C are true.

5. LINEAR EXPANSION

When the rod is heated, its increase in length ΔL is proportional to its original length L_0 and change in temperature ΔT where ΔT is in ºC or K.



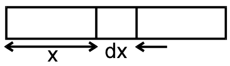
$dL = \alpha L_0\, dT \quad \Rightarrow \quad \Delta L = \alpha\, L_0\, \Delta T \quad$ If $\alpha\, \Delta T << 1$

$\alpha = \dfrac{\Delta L}{L_0 \Delta T}$ where α is called the coefficient of linear expansion whose unit is $^{o}C^{-1}$ or K^{-1}.

$L = L_0\,(1 + \alpha\, \Delta\, T)$. Where L is the length after heating the rod.

Variation of α with temperature and distance

(a) If α varies with distance, $\alpha = ax + b$.



Then total expansion = $\int (ax + b)\, \Delta T\, dx$.

(b) If α varies with temperature, $\alpha = f(T)$. Then $\Delta L = \int \alpha\, L_0\, dT$

Note : Actually thermal expansion is always 3-D expansion. When other two dimensions of object are negligible with respect to one, then observations are significant only in one dimension and it is known as linear expansion.

Example 11. What is the percentage change in length of 1m iron rod if its temperature changes by 100ºC. α for iron is $2 \times 10^{-5}/^{o}C$.

Solution : percentage change in length due to temperature change $\%\ell = \times 100 = \alpha \Delta \theta \times 100$

$= 2 \times 10^{-5} \times 100 \times 100 = 0.2\%$ **Ans.**

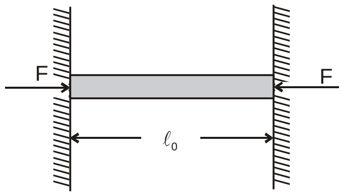
5.1 Thermal stress of a material :

If the rod is free to expand then there will be no stress and strain. Stress and strain is produced only when an object is restricted to expand or contract according to change in temperature. When the temperature of the rod is decreased or increased under constrained condition, compressive or tensile stresses are developed in the rod. These stresses are known as thermal stresses.

Strain = $\dfrac{\Delta L}{L_0} = \dfrac{\text{final length} - \text{original length}}{\text{original length}} = \alpha\, \Delta T$

Note : Original and final length should be at same temperature.

Consider a rod of length ℓ_0 which is fixed between to rigid end separated at a distance ℓ_0 now if the temperature of the rod is increased by $\Delta\theta$ then the strain produced in the rod will be :



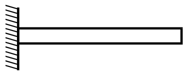
$$\text{strain} = \frac{\text{length of the rod at new temperature-natural length of the rod at new temperature}}{\text{natural length of the rod at new temperature}}$$

$$= \frac{\ell_0 - \ell_0(1+\alpha\Delta\theta)}{\ell_0(1+\alpha\Delta\theta)} = \frac{-\ell_0\alpha\Delta\theta}{\ell_0(1+\alpha\Delta\theta)}$$

$\because$ α is very small so

strain = – $\alpha\Delta\theta$ (negative sign in the answer represents that the length of the rod is less than the natural length that means is compressed by the ends.)

Example 12. In the given figure a rod is free at one end and other end is fixed. When we change the temperature of rod by $\Delta\theta$, then strain produced in the rod will be



(A) $\alpha\Delta\theta$ (B) $\frac{1}{2}\alpha\Delta\theta$ (C) zero (D) information incomplete

Solution : Here rod is free to expand from one side by so by changing temperature no strain will be produced in the rod. Hence ans. is (C)

Example 13. An iron ring measuring 15.00 cm in diameter is to be shrunk on a pulley which is 15.05 cm in diameter. All measurements refer to the room temperature 20°C. To what minimum temperature should the ring be heated to make the job possible? Calculate the strain developed in the ring when it comes to the room temperature. Coefficient of linear expansion of iron = 12×10^{-6}/°C.

Solution : The ring should be heated to increase its diameter from 15.00 cm to 15.05 cm.

Using $\ell_2 = \ell_1 (1 + \alpha\, \Delta\theta)$,

$$= \frac{0.05\,\text{cm}}{15.00\,\text{cm} \times 12 \times 10^{-6}\,/^\circ\text{C}} = 278^\circ\text{C}$$

The temperature = 20°C + 278°C = 298°C.

The strain developed = $\frac{\ell_2 - \ell_1}{\ell_1} = 3.33 \times 10^{-3}$.

Example 14. A steel rod of length 1m rests on a smooth horizontal base. If it is heated from 0°C to 100°C, what is the longitudinal strain developed?

Solution : in absence of external force no strain or stress will be created hear rod is free to move.

Example 15. A steel rod is clamped at its two ends and rests on a fixed horizontal base. The rod is in natural length at 20°C. Find the longitudinal strain developed in the rod if the temperature rises to 50°C. Coefficient of linear expansion of steel = 1.2×10^{-5}/°C.

Solution : as we known that strain

$$\text{strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta\ell}{\ell_0} \qquad \therefore \text{ Strain} = \alpha\Delta\theta = 1.2 \times 10^{-5} \times (50 - 20) = 3.6 \times 10^{-4}$$

here strain is compressive strain because final length is smaller than initial length.

Example 16. A steel wire of cross-sectional area 0.5 mm^2 is held between two fixed supports. If the wire is just taut at 20°C, determine the tension when the temperature falls to 0°C. Coefficient of linear expansion of steel is 1.2×10^{-5}/°C and its Young's modulus is 2.0×10^{11} N/m^2.

Solution : here final length is more than original length so that strain is tensile and tensile force is given by
$F = AY \alpha \Delta t = 0.5 \times 10^{-6} \times 2 \times 10^{11} \times 1.2 \times 10^{-5} \times 20$ **=** 24 N

5.2 Variation of time period of pendulum clocks :

The time represented by the clock hands of a pendulum clock depends on the number of oscillation performed by pendulum every time it reaches to its extreme position the second hand of the clock advances by one second that means second hand moves by two seconds when one oscillation in complete

Let $T = 2\pi\sqrt{\frac{L_0}{g}}$ at temperature θ_0 and $T' = 2\pi\sqrt{\frac{L}{g}}$ at temperature θ.

$$\frac{T'}{T} = \sqrt{\frac{L'}{L}} = \sqrt{\frac{L[1+\alpha\Delta\theta]}{L}} = 1 + \frac{1}{2}\alpha\,\Delta\,\theta$$

Therefore change (loss or gain) in time per unit time lapsed is $\frac{T'-T}{T} = \frac{1}{2}\alpha\Delta\theta$

gain or loss in time in duration of 't' in

$\boldsymbol{\Delta t = \frac{1}{2}\alpha\Delta\theta\, t,}$ if T is the correct time then

(a) $\theta < \theta_0$, $T' < T$ clock becomes fast and gain time

(b) $\theta > \theta_0$, $T' > T$ clock becomes slow and loose time

Example 17. A pendulum clock consists of an iron rod connected to a small, heavy bob. If it is designed to keep correct time at 20°C, how fast or slow will it go in 24 hours at 40°C? Coefficient of linear expansion of iron = 1.2×10^{-6}/°C.

Solution : The time difference occurred in 24 hours (86400 seconds) is given by

$\Delta t = \frac{1}{2}\alpha\,\Delta\theta\, t = \frac{1}{2} \times 1.2 \times 10^{-6} \times 20 \times 86400 = 1.04$ sec. **Ans.**

This is loss of time as θ is greater than θ_0. As the temperature increases, the time period also increases. Thus, the clock goes slow.

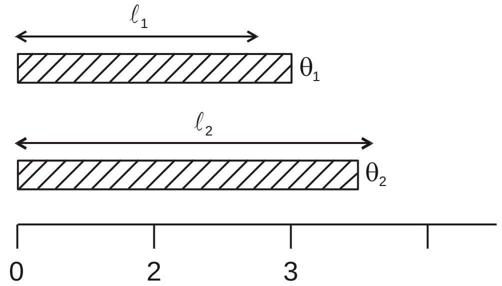
5.3 Measurement of length by metallic scale :

Case (i) : When object is expanded only $\ell_2 = \ell_1 \{1 + \alpha_0(\theta_2 - \theta_1)$

ℓ_1 = actual length of object at θ_1ºC = measure length of object at θ_1ºC.

ℓ_2 = actual length of object at θ_2ºC = measure length of object at θ_2ºC.

α_0 = linear expansion coefficient of object.



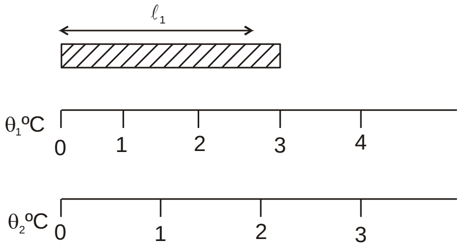
Case (ii) : When only measurement instrument is expanded actual length of object will not change but measured value (MV) decreases.

$MV = \ell_1 \{1 - \alpha_S (\theta_2 - \theta_1)\}$

α_S = linear expansion coefficient of measuring instrument.

at θ_1 °C MV = 3

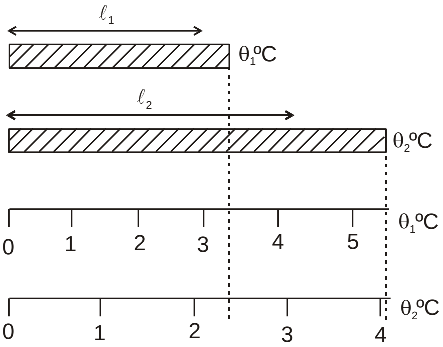
at θ_2 °C MV = 2.2



Case (iii) : If both expanded simultaneously $MV = \{1 + (\alpha_0 - \alpha_s) (\theta_2 - \theta_1)$

(i) If $\alpha_0 > \alpha_s$, then measured value is more then the actual value at θ_1ºC

(ii) If $\alpha_0 < \alpha_s$, then measured value is less then the actual value at θ_1ºC



at θ_1ºC MV = 3.4

θ_2ºC MV = 4.1

Measured value = calibrated value × $\{1 + \alpha\, \Delta\, \theta\}$

where $\alpha = \alpha_0 - \alpha_s$

α_o = coefficient of linear expansion of object material, α_s = coefficient of linear expansion of scale material

$\Delta\theta = \theta - \theta_C$

θ = temperature at the time of measurement θ_C = temperature at the time of calibration.

For scale, **true measurement = scale reading $[1 + \alpha\, (\theta - \theta_0)]$**

If **$\theta > \theta_0$ true measurement > scale reading**

$\theta < \theta_0$ true measurement < scale reading

Example 18. A bar measured with a Vernier caliper is found to be 180mm long. The temperature during the measurement is 10ºC. The measurement error will be if the scale of the Vernier caliper has been graduated at a temperature of 20ºC : ($\alpha = 1.1 \times 10^{-5}$ °C^{-1}. Assume that the length of the bar does not change.)

(A) 1.98×10^{-1} mm (B) 1.98×10^{-2} mm (C) 1.98×10^{-3} mm (D) 1.98×10^{-4} mm

Answer : **(B)**

Solution : True measurement = scale reading $[1 + \alpha\, (\theta - \theta_0)] = 180\ [1 - 10 \times 1.1 \times 10^{-5}]$

error $= 180 - 180\ [1 - 1.1 \times 10^{-4}] = 1.98 \times 10^{-2}$ mm

6. SUPERFICIAL OR AREAL EXPANSION

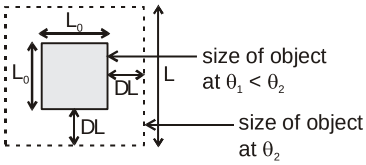
When a solid is heated and its area increases, then the thermal expansion is called superficial or areal expansion. Consider a solid plate of area A_0. When it is heated, the change in area of the plate is directly proportional to the original area A_0 and the change in temperature ΔT.

$dA = \beta A_0\, dT$ or $\mathbf{\Delta A = \beta\, A_0\, \Delta\, T}$ $\beta =$

$\frac{\Delta A}{A_0 \Delta T}$ Unit of β is $^oC^{-1}$ or K^{-1}.

$A = A_0 (1 + \beta\Delta T)$

where A is area of the plate after heating,



Example 19. A plane lamina has area $2m^2$ at 10^oC then what is its area at 110^oC It's superficial expansion is 2×10^{-5}/C

Solution : $A = A_0(1 + \beta\Delta\theta) = 2\{1 + 2 \times 10^5 \times (110 - 10)\}$

$= 2 \times \{1 + 2 \times 10^{-3}\}$ **Ans.**

7. VOLUME OR CUBICAL EXPANSION

When a solid is heated and its volume increases, then the expansion is called volume expansion or cubical expansion. Let us consider a solid or liquid whose original volume is V_0. When it is heated to a new volume, then the change ΔV

$dV = \gamma V_0\, dT$ or $\mathbf{\Delta V = \gamma V_0 \Delta T}$

γ = Unit of γ is $^oC^{-1}$ or K^{-1}.

$\mathbf{V = V_0 (1 + \gamma\Delta T)}$

where V is the volume of the body after heating

Example 20. The volume of glass vessel is 1000 cc at 20°C. What volume of mercury should be poured into it at this temperature so that the volume of the remaining space does not change with temperature? Coefficient of cubical expansion of mercury and glass are 1.8×10^{-4}/°C and 9.0×10^{-6}/°C respectively.

Solution : Let volume of glass vessel at 20^oC is V_g and volume of mercury at 20^oC is V_m

so volume of remaining space is $= V_g - V_m$

It is given constant so that

$V_g - V_m = V_g' - V'_m$

where V_o' and V_m' are final volumes.

$V_g - V_m = V_g \{1 + \gamma_g \Delta\theta\} - V_m\{1 + \gamma_{Hg} \Delta\theta\}$ $\Rightarrow$ $V_g \gamma_g = V_m \gamma_{Hg}$

$\Rightarrow$ $V_m = \frac{100 \times 9 \times 10^{-6}}{1.8 \times 10^{-4}}$ $\Rightarrow$ $V_m = 50$ cc.

8. RELATION BETWEEN α, β AND γ

(i) For isotropic solids: $\boldsymbol{\alpha : \beta : \gamma = 1 : 2 : 3}$ **or** $\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3}$

(ii) For non-isotropic solid $\beta = \alpha_1 + \alpha_2$ and $\gamma = \alpha_1 + \alpha_2 + \alpha_3$. Here α_1, α_2 and α_3 are coefficient of linear expansion in X, Y and Z direction.

Example 21. If percentage change in length is 1% with change in temperature of a cuboid object ($\ell \times 2\ell \times 3\ell$) then what is percentage change in its area and volume.

Solution : percentage change in length with change in temperature = % ℓ

$\frac{\Delta\ell}{\ell} \times 100 = \alpha\Delta\theta \times 100 = 1$

change in area

$\Rightarrow \% A = \frac{\Delta A}{A} \times 100 = \beta\Delta\theta \times 100 \Rightarrow 2(\alpha\Delta\theta \times 100)$

% A = 2 % **Ans.**

change in volume

$\% V = \frac{\Delta V}{V} \times 100 = V \Delta \theta \times 100 = 3 (\alpha \Delta \theta \times 100)$

% V = 3 % **Ans.**

9. VARIATION OF DENSITY WITH TEMPERATURE

As we known that mass = volume × density.

Mass of substance does not change with change in temperature so with increase of temperature, volume increases so density decreases and vice-versa.

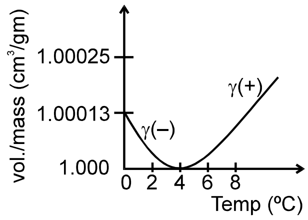
$$d = \frac{d_0}{(1+\gamma\Delta T)}.$$

For solids values of γ are generally small so we can write $d = d_0 (1 - \gamma \Delta T)$ (using binomial expansion).

Note : (i) γ for liquids are in order of 10^{-3}.

(ii) **Anamolous expansion of water :**

For water density increases from 0 ºC to 4 ºC so γ is negative and for 4 ºC to higher temperature γ is positive. At 4 ºC density is maximum. This anamolous behaviour of water is due to presence of three types of molecules i.e. H_2O, $(H_2O)_2$ and $(H_2O)_3$ having different volume/mass at different temperatures.



This anomalous behaviour of water causes ice to form first at the surface of a lake in cold weather. As winter approaches, the water temperature decreases initially at the surface. The water there sinks because of its increase density. Consequently, the surface reaches 0^0C first and the lake becomes covered with ice. Aquatic life is able to survive the cold winter as the lake bottom remains unfrozen at a temperature of about 4^0C.

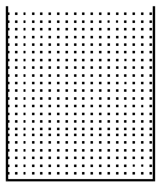
Example 22. The densities of wood and benzene at 0°C are 880 kg/m^3 and 900 kg/m^3 respectively. The coefficients of volume expansion are 1.2×10^{-3}/°C for wood and 1.5×10^{-3}/°C for benzene. At what temperature will a piece of wood just sink in benzene?

Solution : At just sink gravitation force = up thrust force

$\Rightarrow$ $mg = F_B$ $\Rightarrow$ $V\rho_1 g = V\rho_2 g$ $\Rightarrow$ $\rho_1 = \rho_2$

$\Rightarrow$ $\frac{880}{1+1.2\times10^{-3}\theta} = \frac{900}{1+1.5\times10^{-3}\theta}$ $\Rightarrow$ $\theta = 83^o$ C

10. APPARENT EXPANSION OF A LIQUID IN A CONTAINER



Initially container was full. When temperature change by ΔT,

volume of liquid $V_L = V_0 (1 + \gamma_L \Delta T)$

volume of container $V_C = V_0 (1 + \gamma_C \Delta T)$

So overflow volume of liquid relative to container $\Delta V = V_L - V_C$

$\mathbf{\Delta V = V_0 (\gamma_L - \gamma_C) \Delta T}$

So, coefficient of apparent expansion of liquid w.r.t. container

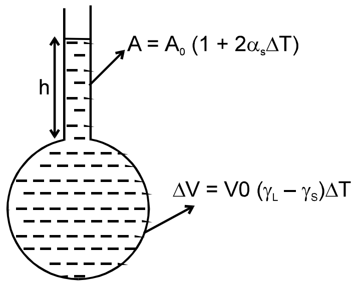
$\gamma_{apparent} = \gamma_L - \gamma_C$.

In case of expansion of liquid + container system:

if $\gamma_L > \gamma_C \longrightarrow$ level of liquid rise

if $\gamma_L < \gamma_C \longrightarrow$ level of liquid fall

Increase in height of liquid level in tube when bulb was initially not completely filled



$$h = \frac{\text{volume of liquid}}{\text{area of tube}} = \frac{V_0(1+\gamma_L\Delta T)}{A_0(1+2\alpha_S\Delta T)} = h_0 \{1 + (\gamma_L - 2\alpha_S) \Delta T\}$$

$\mathbf{h = h_0 \{1 + (\gamma_L - 2\alpha_S) \Delta T\}}$

where h_0 = original height of liquid in container

α_S = linear coefficient of expansion of container.

Example 23. A glass vessel of volume 100 cm^3 is filled with mercury and is heated from 25°C to 75°C. What volume of mercury will overflow? Coefficient of linear expansion of glass = 1.8×10^{-6}/°C and coefficient of volume expansion of mercury is 1.8×10^{-4}/°C.

Solution : $\Delta V = V_0(\gamma_L - \gamma_C)\,\Delta T = 100 \times \{1.8 \times 10^{-4} - 3 \times 1.8 \times 10^{-6}\} \times 50$

$\Delta V = 0.87\ cm^3$ Ans.

11. VARIATION OF FORCE OF BUOYANCY WITH TEMPERATURE

If body is submerged completely inside the liquid

For solid, Buoyancy force

$F_B = V_0\, d_L\, g$

V_0 = Volume of the solid inside liquid,

d_L = density of liquid

Volume of body after increase its temperature $V = V_0\,[1 + \gamma_S\,\Delta\theta]$,

Density of body after increase its temperature $d'_L = \dfrac{d_L}{[1+\gamma_L\Delta\theta]}$.

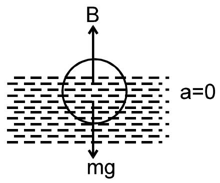
Buoyancy force of body after increase its temperature, $F'_B = V\, d'_L\, g$, $\dfrac{F'_B}{F_B} = \dfrac{[1+\gamma_S\Delta\theta]}{[1+\gamma_L\Delta\theta]}$,

if $\gamma_S < \gamma_L$ then $F'_B < F_B$

(Buoyant force decreases) or apparent weight of body in liquid gets increased

$[W - F'_B > W - F_B]$.

Example 24. A body is float inside liquid if we increases temperature then what changes occur in Buoyancy force. (Assume body is always in floating condition)



Solution : Body is in equilibrium

So mg = B

and gravitational force does not change with change in temperature. So Buoyancy force remains constant.

By increasing temperature density of liquid decreases so volume of body inside the liquid increases to kept the Buoyance force constant for equal to gravitational force)

Example 25. In previous question discuss the case when body move downward, upwards and remains at same position when we increases temperature.

Solution : Let f = fraction of volume of body submerged in liquid.

$$f = \frac{\text{volume of body submerged in liquid}}{\text{total volume of body}}$$

$f_1 = \dfrac{v_1}{v_0}$ at θ_1°C

$f_2 = \dfrac{v_2}{v_0(1+3\alpha_S\Delta\theta)}$ at θ_2°C

for equilibrium $mg = B = v_1 d_1 g = v_2 d_2 g$.

so $v_2 = \dfrac{v_1 d_1}{d_2}$ $\because$ $d_2 = \dfrac{d_1}{1+\gamma_L\Delta\theta}$ $= v_1(1 + \gamma_L\,\Delta\theta)$ $\therefore$ $f_2 = \dfrac{v_1(1+\gamma_L\Delta\theta)}{v_0(1+3\alpha_s\Delta\theta)}$

where $\Delta\theta = \theta_2 - \theta_1$

Case I : Body move downward if $f_2 > f_1$
means $\gamma_L > 3\alpha_S$

Case II : Body move upwards if $f_2 < f_1$
means $\gamma_L < 3\alpha_S$

Case III : Body remains at same position
if $f_2 = f_1$
means $\gamma_L = 3\alpha_S$

12. BIMETALLIC STRIP

It two strip of different metals are welded together to form a bimetallic strip, when heated uniformly it bends in form of an arc, the metal with greater coefficient of linear expansion lies on convex side. The radius of arc thus formed by bimetal is :

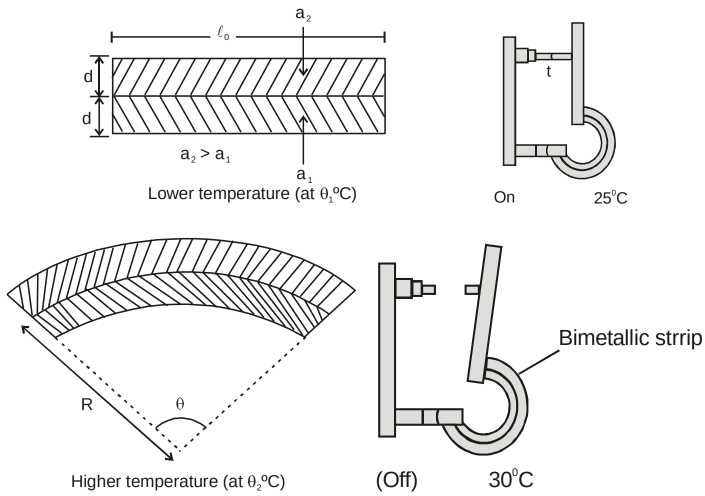
$$\ell_0 (1 + \alpha_1\Delta\theta) = \left(R - \frac{d}{2}\right)\theta$$

$$\ell_0 (1 + \alpha_2\Delta\theta) = \left(R + \frac{d}{2}\right)\theta$$

$$\Rightarrow \frac{1+\alpha_2\Delta\theta}{1+\alpha_1\Delta\theta} = \frac{R + \frac{d}{2}}{R - \frac{d}{2}}$$

$$\Rightarrow R = \frac{d}{(\alpha_2 - \alpha_1)\Delta\theta}$$

$\Delta\theta$ = change in temperature
$= \theta_2 - \theta_1$



A bimetallic strip, consisting of a strip of brass and a strip of steel welded together, at temperature T_0 in figure (a) and figure (b). The strip bends as shown at temperatures above the reference temperature. Below the reference temperature the strip bends the other way. Many thermostats operate on this principle, making and breaking an electrical circuit as the temperature rises and falls.

13. APPLICATIONS OF THERMAL EXPANSION

(a) A small gap is left between two iron rails of the railway.
(b) Iron rings are slipped on the wooden wheels by heating the iron rings
(c) Stopper of a glass bottle jammed in its neck can be taken out by heating the neck.
(d) The pendulum of a clock is made of invar [an alloy of zinc and copper].

14. TEMPERATURE

Temperature may be defined as the **degree of hotness or coldness** of a body. Heat energy flows from a body at higher temperature to that at lower temperature until their temperatures become equal. At this stage, the bodies are said to be in thermal equilibrium.

14.1 Measurement of Temperature

The branch of thermodynamics which deals with the measurement of temperature is called thermometry. A thermometer is a device used to measure the temperature of a body. The substances like liquids and gases which are used in the thermometer are called thermometric substances.

14.2 Different Scales of Temperature

A thermometer can be graduated into following scales.

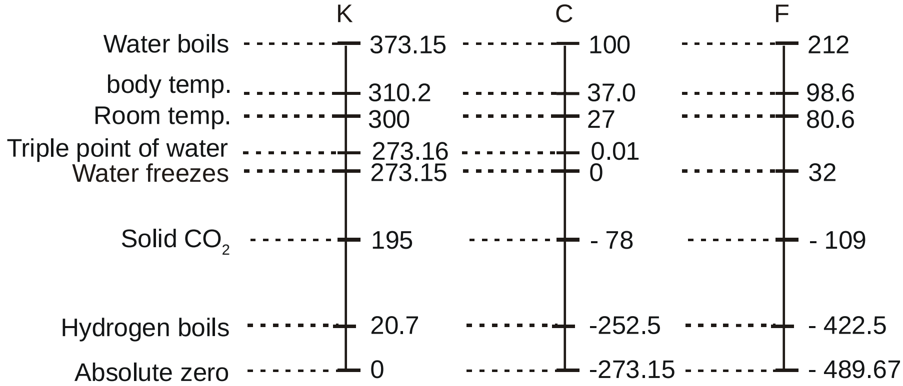
(a) The Centigrade or Celsius scale (ºC)

(b) The Fahrenheit scale (ºF)

(c) The Reaumer scale (ºR)

(d) Kelvin scale of temperature (K)

14.3 Comparison between Different Temperature Scales



The formula for the conversion between different temperature scales is:

$$\frac{K-273}{100} = \frac{C}{100} = \frac{F-32}{180} = \frac{R}{80}$$

General formula for the conversion of temperature from one scale to another:

$$\frac{\text{Temp on one scale}(S_1)\text{-Lower fixed point }(S_1)}{\text{Upper fixed point }(S_2)-\text{Lower fixed point }(S_1)}$$

$$= \frac{\text{Temp. on other scale}(S_2)\text{-Lower fixed point }(S_2)}{\text{Upper fixed point }(S_2)-\text{Lower fixed point }(S_2)}$$

14.4 Thermometers

Thermometers are device that are used to measure temperatures. All thermometers are based on the principle that some physical property of a system changes as the system temperature changes.

Required properties of good thermometric substance.

(1) Non-sticky (absence of adhesive force)

(2) Low melting point (in comparison with room temperature)

(3) High boiling temperature

(4) Coefficient of volumetric expansion should be high (to increase accuracy in measurement).

(5) Heat capacity should be low.

(6) Conductivity should be high

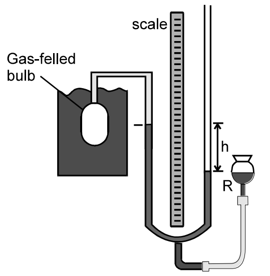
Mercury (Hg) suitably exhibits above properties.

14.5 Types of Thermometers

Type of thermometer and its range	Thermometric property	Advantages	Disadvantages	Particular Uses
Mercury-in-glass – 39ºC to 450ºC	Length of column of mercury in capillary tube	(i) Quick and easy to (direct reading) (ii) Easily portable	(i) Fragile (ii) Small size limits (iii) Limited range	(i) Every laboratory use where high accuracy is not required. (ii) Can be calibrated against constant-volume gas thermometer for more accurate work
Constant-volume gas thermometer –270º to 1500ºC	Pressure of a fixed mass of gas at constant volume	(i) Very accurate (ii) Very sensitive (iii) Wide range (iv) Easily reproducible	(i) Very large volume of bulb (ii) Slow to use and inconvenient	(i) Standard against which others calibrated (ii) He, H_2 or N_2 used depending on range (iii) can be corrected to the ideal gas scale (iv) Used as standard below-183ºC
Platinum resistance –180º to 1150ºC	Electrical resistance of a platinum coil	(i) Accurate (ii) Wide range	Not suitable for varying temperature (i.e., is slow to respond to changes)	(i) Best thermometer for small steady temperature differences (ii) Used as standard between 183ºC and 630ºC.
Thermocouple –250ºC to 1150ºC	Emf produced between junctions of dissimilar metals at different temperatures for measurement of emfs	(i) Fast response because of low heat capacity. (ii) wide range (iii) can be employed for remote readings using long leads.	Accuracy is lost if emf is measured using a moving-coil voltmeter (as may be necessary for rapid changes when potentiometer is unsuitable)	(i) Best thermometer for small steady temperature differences (ii) Can be made direct reading by calibrating galvanometer (iii) Used as standard between 630ºC and 1063ºC
Radiation pyrometer above 1000ºC	Colour of radiation emitted by a hot body	Does not come into contact when temperature is measured	(i) Cumbersome (ii) Not direct reading (needs a trained observer)	(i) Only thermometer possible for very high temperatures (ii) Used as standard above 1063ºC.

14.6 The constant-volume gas thermometer

The standard thermometer, against which all other thermometers are calibrated, is based on the pressure of a gas in a fixed volume. Figure shows such a constant volume gas thermometer; it consists of a gas-filled bulb connected by a tube to a mercury monometer.



A constant volume gas thermometer, its bulb immersed in a liquid whose temperature T is to be measured.

$$T = (273.16\ K)\left(\lim_{gas\to 0}\frac{P}{P_3}\right)$$

P = Pressure at the temperature being measured, P_3 = pressure when bulb in a triple point cell.

Example 26. The readings of a thermometer at 0ºC and 100ºC are 50 cm and 75 cm of mercury column respectively. Find the temperature at which its reading is 80 cm of mercury column?

Solution : By using formula $\frac{80-50}{75-50} = \frac{T-0}{100-0}$ $\Rightarrow$ T = 120ºC

Problem 1. A bullet of mass 10 gm in moving with speed 400m/s. Find its kinetic energy in calories ?

Solution : $\Delta k = \frac{1}{2} \times \frac{10}{1000} \times 400 \times 400 = 800$ $\Rightarrow$ $\frac{800}{4.2} = 191.11$ Cal.

Problem 2. Calculate amount of heat required to convert 1 kg steam from 100ºC to 200ºC steam

Solution : Heat required = $1 \times \frac{1}{2} \times 100 = 50$ kcal

Problem 3. Calculate heat required to raise the temperature of 1 g of water through 1ºC ?

Solution : heat required = $1 \times 10^{-3} \times 1 \times 1 = 1 \times 10^{-3}$ kcal = 1 cal

Problem 4. 420 J of energy supplied to 10 g of water will raise its temperature by

Solution : $\frac{420 \times 10^{-3}}{4.20} = 10 \times 10^{-3} \times 1 \times \Delta t = 10^{o}$ C

Problem 5. The ratio of the densities of the two bodies is 3 : 4 and the ratio of specific heats is 4 : 3. Find the ratio of their thermal capacities for unit volume ?

Solution : $\frac{\rho_1}{\rho_2} = \frac{3}{4}, \frac{s_1}{s_2} = \frac{4}{3}$

ratio = $\frac{m \times s}{m/\rho}$ $\Rightarrow$ $\frac{\theta_1}{\theta_2} = \frac{s_1}{s_2} \times \frac{\rho_1}{\rho_2} = 1 : 1$.

Problem 6. Heat releases by 1 kg steam at 150ºC if it convert into 1 kg water at 50ºC.

Solution : $H = 1 \times \frac{1}{2} \times 50 + 1 \times 540 + 1 \times 1 \times 50 = 540 + 75 = 615$ Kcal

Heat release = 615 Kcal.

Problem 7. 200 gm water is filled in a calorimetry of negligible heat capacity. It is heated till its temperature is increase by 20ºC. Find the heat supplied to the water.

Solution : $H = 200 \times 10^{-3} \times 1 \times 20 = 4$ Kcal.

Heat supplied = 4000 cal

Problem 8. A bullet of mass 5 gm is moving with speed 400 m/s. strike a target and energy. Then calculate rise of temperature of bullet. Assuming all the lose in kinetic energy is converted into heat energy of bullet if its specific heat is. 500J/kgºC.

Solution : Kinetic energy = $\frac{1}{2} \times 5 \times 10^{-3} \times 400 \times 400$

$ms\,\Delta T = 5 \times 10^{-3} \times 500 \times \Delta T$

$\Delta T = 160^{o}$ C

Rise in temperature is 160 ºC

Problem 9. 1 kg ice at –10ºC is mixed with 1 kg water at 100ºC. Then find equilibrium temperature and mixture content.

Solution : Heat taken by 1 kg Ice = Heat given by 1 kg water

$1 \times \frac{1}{2} \times 10 + 1 \times 80 + 1 \times T = 1 \times (100 - T)$

$85 = 100 - 2T \Rightarrow 2T = 15$

$\theta = \frac{15}{2} = 7.5\ ^{o}C$, water

Problem 10. 1kg ice at –10º is mixed with 1kg water at 50ºC. Then find equilibrium temperature and mixture content.

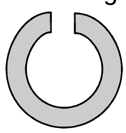
Solution : Heat taken by ice = 5 Kcal + 80 Kcal = 85 Kcal ; Heat given by water = 1 × 1 × 50 = 50 Kcal
Heat taken > Heat given so, ice will not complete melt let m g ice melt then

$1 \times \frac{1}{2} \times 10 + 80\ m = 50$

$80\ m = 45 \Rightarrow m = \frac{45}{80}$

Content of mixture $\left\{\begin{matrix} \text{water} & \left(1+\frac{45}{80}\right) & \text{kg} \\ \text{ice} & \left(1-\frac{45}{80}\right) & \text{kg} \end{matrix}\right\}$ and temperature is 0ºC

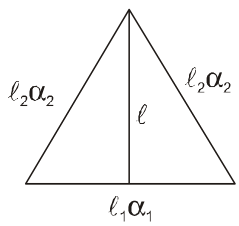
Problem 11. A small ring having small gap is shown in figure on heating what will happen to size of gap.



Solution : Gap will also increase. The reason is same as in above example.

Problem 12. An isosceles triangle is formed with a thin rod of length ℓ_1 and coefficient of linear expansion α_1, as the base and two thin rods each of length ℓ_2 and coefficient of linear expansion α_2 as the two sides. If the distance between the apex and the midpoint of the base remain unchanged as the temperature is varied show that $\frac{\ell_1}{\ell_2} = 2\sqrt{\frac{\alpha_2}{\alpha_1}}$.

Solution : $\ell = \sqrt{\left(\frac{\ell_1}{2}\right)^2 + (\ell_2)^2}$



$\ell^2 = \left(\frac{\ell_1}{2}\right)^2 + (\ell_2)^2$

$0 = \frac{2\ell_1}{2}\frac{1}{2}\frac{d\ell_1}{dT} + 2\ell_2 \times \frac{d\ell_2}{dT} = 2\ell_2 \times \ell_2\ \alpha_2\Delta T$

$\Rightarrow \frac{\ell_1^2}{\ell_2^2} = 4\ \frac{\alpha_2}{\alpha_1} \Rightarrow \frac{\ell_1}{\ell_2} = 2\sqrt{\frac{\alpha_2}{\alpha_1}}$.

Problem 13. A concrete slab has a length of 10 m on a winter night when the temperature is 0°C. Find the length of the slab on a summer day when the temperature is 35°C. The coefficient of linear expansion of concrete is 1.0×10^{-5} /°C.

Solution : $\ell_t = 10(1 + 1 \times 10^{-5} \times 35) = 10.0035$ m

Problem 14. A steel rod is clamped at its two ends and rests on a fixed horizontal base. The rod is unstrained at 20°C. Find the longitudinal strain developed in the rod if the temperature rises to 50°C. Coefficient of linear expansion of steel = 1.2×10^{-5}/°C.

Solution : $\frac{\Delta\ell}{\ell} = \frac{\ell_0\alpha\Delta\theta}{\ell_0}$ = 3.6 × 10^{-4}

Problem 15. If rod is initially compressed by $\Delta\ell$ length then what is the strain on the rod when the temperature

(a) is increased by $\Delta\theta$ (b) is decreased by $\Delta\theta$.

Solution: (a) Strain = $\frac{\Delta\ell}{\ell} + \alpha\Delta\theta$ (b) Strain = $\left|\frac{\Delta\ell}{\ell} - \alpha\Delta\theta\right|$

Problem 16. A pendulum clock having copper rod keeps correct time at 20°C. It gains 15 seconds per day if cooled to 0°C. Calculate the coefficient of linear expansion of copper.

Solution : $\frac{15}{24\times60\times60} = \frac{1}{2}\alpha \times 20$ ⇒ $\alpha = \frac{1}{16\times3600}$ = 1.7 × 10^{-5}/°C

Problem 17. A meter scale made of steel is calibrated at 20°C to give correct reading. Find the distance between 50 cm mark and 51 cm mark if the scale is used at 10°C. Coefficient of linear expansion of steel is 1.1 × 10^{-5}/°C.

Solution : $\ell_t = 1\,(1 - 1.1 \times 10^{-5} \times 10) = 0.99989$ cm

Problem 18. A uniform solid brass sphere is rotating with angular speed ω_0 about a diameter. If its temperature is now increased by 100ºC. What will be its new angular speed. (Given $\alpha_B = 2.0 \times 10^{-5}$ per-ºC)

(A) $\frac{\omega_0}{1-0.002}$ (B) $\frac{\omega_0}{1+0.002}$ (C*) $\frac{\omega_0}{1+0.004}$ (D) $\frac{\omega_0}{1-0.004}$

Solution : $I_0\,\omega_0 = I_t\,\omega t$

$Mr_0^2\,\omega_0 = Mr_0^2\,(1 + 2\alpha\Delta T)\omega_t$; $\omega_t = \frac{\omega_0}{1+0.004}$.

Problem 19. The volume occupied by a thin - wall brass vessel and the volume of a solid brass sphere are the same and equal to 1,000 cm^3 at 0ºC. How much will the volume of the vessel and that of the sphere change upon heating to 20ºC? The coefficient of linear expansion of brass is $\alpha = 1.9 \times 10^{-5}$.

Solution : $\Delta V = V_0\,3\alpha\,\Delta T = 1.14\ cm^3$

1.14 cm^3 for both

Problem 20. A thin copper wire of length L increases in length by 1%, when heated from temperature T_1 to T_2. What is the percentage change in area when a thin copper plate having dimensions 2L × L is heated from T_1 to T_2 ?

(A) 1% (B) 3% (C) 4% (D*) 2%

Solution : $L_f = L\,(1 + \alpha\Delta t) \Rightarrow \frac{L_f}{L} \times 100 = (1 + \alpha\Delta t) \times 100 = 1\%$

$A_f = 2L \times L\,(1 + 2\,\alpha\Delta t) \Rightarrow \frac{A_f}{2L\times L} \times 100 = (1 + 2\alpha\,\Delta t) \times 100 = 2\%$

Problem 21. The density of water at 0°C is 0.998 g/cm^3 and at 4°C is 1.000 g/cm^3. Calculate the average coefficient of volume expansion of water in the temperature range 0 to 4°C.

Solution : $d_t = \frac{d_0}{1+\gamma\Delta t} \Rightarrow 1 = \frac{0.998}{1+\gamma\times4} \Rightarrow \gamma = -5 \times 10^{-4}\ /\ ^0C$

Problem 22. A glass vessel measures exactly 10 cm × 10 cm × 10 cm at 0°C. it is filled completely with mercury at this temperature. When the temperature is raised to 10°C, 1.6 cm^3 of mercury overflows. Calculate the coefficient of volume expansion of mercury. Coefficient of linear expansion of glass = 6.5 × 10^{-6}/°C

Solution : $\Delta V = V_{Hg} - V_V$

$1.6 = 10^3 \gamma_\ell \times 10 - 10^3 \times 3 \times 6.5 \times 10^{-6} \times 10$

$\gamma_L = (1.6 + 0.195) \times 10^{-4} = 1.795 \times 10^{-4} /^0C$

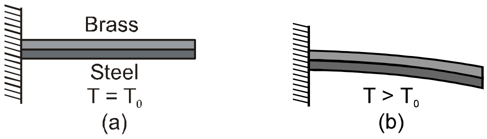
Problem 23. A metal ball immersed in alcohol weighs W_1 at 0^0C and W_2 at 50^0C. The coefficient of cubical expansion of the metal is less than alcohol. Assuming that density of the metal is large compared to that of the alcohol, find which of W_1 and W_2 is greater?

Solution : $\gamma_M < \gamma_\ell$ so, $\frac{F'_B}{F_B} = \frac{[1+\gamma_S\Delta\theta]}{[1+\gamma_\ell\Delta\theta]} \Rightarrow F'_B < F_B$

so Apparent weight increases

so, $W_2 > W_1$

Problem 24. In figure which strip brass or steel have higher coefficient of linear expansion.



Solution : Brass Strip

Problem 25. The upper and lower fixed points of a faulty thermometer are 5ºC and 105ºC. If the thermometer reads 25ºC, what is the actual temperature ?

Solution : $\frac{25-5}{100} = \frac{C-0}{100}$

C = 20º C

Problem 26. At what temperature is the Fahrenheit scale reading equal to twice of Celsius ?

Solution : $\frac{F-32}{180} = \frac{C-0}{100}$

$\frac{2x-32}{180} = \frac{x-0}{100}$

10x – 160 = 9x

x = 160º C

Problem 27. Temperature of a patient is 40º C. Find the temperature on Fahrenheit scale ?

Solution : $\frac{F-32}{180} = \frac{40-0}{100} \Rightarrow F = 104^o F$