CHAPTER-15
HEAT TRANSFER

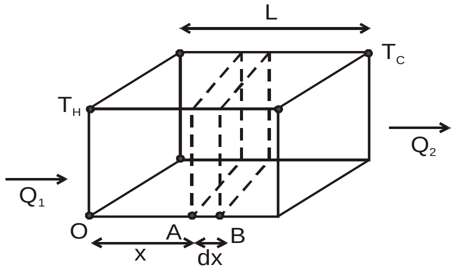
1. INTRODUCTION

Heat is energy in transit which flows due to temperature difference; from a body at higher temperature to a body at lower temperature. This transfer of heat from one body to the other takes place through three routes.

(i) Conduction (ii) Convection (iii) Radiation

2. CONDUCTION

The process of transmission of heat energy in which heat is transferred from one particle of the medium to the other, but each particle of the medium stays at its own position is called conduction, for example if you hold an iron rod with one of its end on a fire for some time, the handle will get hot. The heat is transferred from the fire to the handle by conduction along the length of iron rod. The vibrational amplitude of atoms and electrons of the iron rod at the hot end takes on relatively higher values due to the higher temperature of their environment. These increased vibrational amplitude are transferred along the rod, from atom to atom during collision between adjacent atoms. In this way a region of rising temperature extends itself along the rod to your hand.



Consider a slab of face area A, Lateral thickness L, whose faces have temperatures T_H and T_C ($T_H > T_C$). Now consider two cross sections in the slab at positions A and B separated by a lateral distance of dx. Let temperature of face A be T and that of face B be T + ΔT. Then experiments show that Q, the amount of heat crossing the area A of the slab at position x in time t is given by

$$\frac{Q}{t} = -KA\frac{dT}{dx} \quad \text{... (2.1)}$$

Here K is a constant depending on the material of the slab and is named thermal conductivity of the material, and the quantity $\left(\frac{dT}{dx}\right)$ is called temperature gradient. The (–) sign in equation (2.1) shows heat flows from high to low temperature (ΔT is a –ve quantity)

3. STEADY STATE

If the temperature of a cross-section at any position x in the above slab remains constant with time (remember, it does vary with position x), the slab is said to be in steady state.

Remember steady-state is distinct from thermal equilibrium for which temperature at any position (x) in the slab must be same.

For a conductor in steady state there is no absorption or emission of heat at any cross-section. (As temperature at each point remains constant with time). The left and right face are maintained at constant temperatures T_H and T_C respectively, and all other faces must be covered with adiabatic walls so that no heat escapes through them and same amount of heat flows through each cross-section in a given Interval of time. Hence $Q_1 = Q = Q_2$. Consequently the temperature gradient is constant throughout the slab.

Hence, $$\frac{dT}{dx} = \frac{\Delta T}{L} = \frac{T_f - T_i}{L} = \frac{T_C - T_H}{L}$$

and $$\frac{Q}{t} = -KA\frac{\Delta T}{L}$$

$$\Rightarrow \quad \frac{Q}{t} = KA\left(\frac{T_H - T_C}{L}\right) \quad .... (3.1)$$

Here Q is the amount of heat flowing through a cross-section of slab at any position in a time interval of t.

Example 1. One face of an aluminium cube of edge 2 metre is maintained at 100ºC and the other end is maintained at 0ºC. All other surfaces are covered by adiabatic walls. Find the amount of heat flowing through the cube in 5 seconds. (Thermal conductivity of aluminium is 209 W/m–ºC)

Solution : Heat will flow from the end at 100ºC to the end at 0ºC. Area of cross-section perpendicular to direction of heat flow,

$A = 4m^2$

then $$\frac{Q}{t} = KA\frac{(T_H - T_C)}{L}$$

$$Q = \frac{(209W/m^o C)(4m^2)(100^o C - 0^o C)(5 sec)}{2m} = 209 \text{ KJ}$$ **Ans.**

4. THERMAL RESISTANCE TO CONDUCTION

If you are interested in insulating your house from cold weather or for that matter keeping the meal hot in your tiffin-box, you are more interested in poor heat conductors, rather than good conductors. For this reason, the concept of thermal resistance R has been introduced.

For a slab of cross-section A, Lateral thickness L and thermal conductivity K,

$$R = \frac{L}{KA} \quad ... (4.1)$$

In terms of R, the amount of heat flowing though a slab in steady-state (in time t)

$$\frac{Q}{t} = \frac{(T_H - T_C)}{R}$$

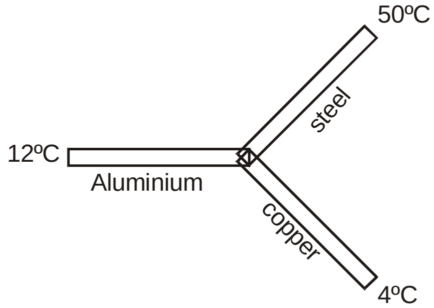
If we name $\frac{Q}{t}$ as thermal current i_T

then, $$i_T = \frac{T_H - T_C}{R} \quad (4.2)$$

This is mathematically equivalent to OHM's law, with temperature playing the role of electric potential. Hence results derived from OHM's law are also valid for thermal conduction.

More over, for a slab in steady state we have seen earlier that the thermal current i_L remains same at each cross-section. This is analogous to kirchoff's current law in electricity, which can now be very conveniently applied to thermal conduction.

Example 2. Three identical rods of length 1m each, having cross-section area of $1cm^2$ each and made of Aluminium, copper and steel respectively are maintained at temperatures of 12ºC, 4ºC and 50ºC respectively at their separate ends.



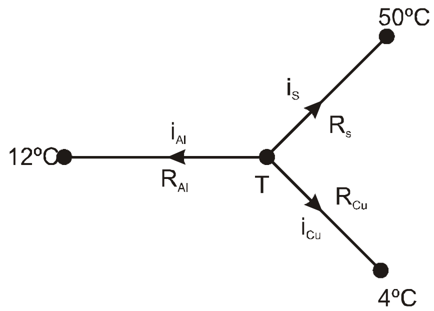
Find the temperature of their common junction. [K_{Cu} = 400 W/m-K, K_{Al} = 200 W/m-K, K_{steel} = 50 W/m-K]

Solution : $R_{Al} = \dfrac{L}{KA} = \dfrac{1}{10^{-4} \times 200} = \dfrac{10^4}{200}$

Similarly $R_{steel} = \dfrac{10^4}{50}$ and $R_{copper} = \dfrac{10^4}{400}$

Let temperature of common junction = T

then from Kirchoff;s current laws,



$i_{Al} + i_{steel} + i_{Cu} = 0$

$\Rightarrow \dfrac{T-12}{R_{Al}} + \dfrac{T-50}{R_{steel}} + \dfrac{T-4}{R_{Cu}} = 0$

$\Rightarrow$ (T – 12) 200 + (T – 50) 50 + (T – 4) 400

$\Rightarrow$ 4(T – 12) + (T – 50) + 8 (T – 4) = 0

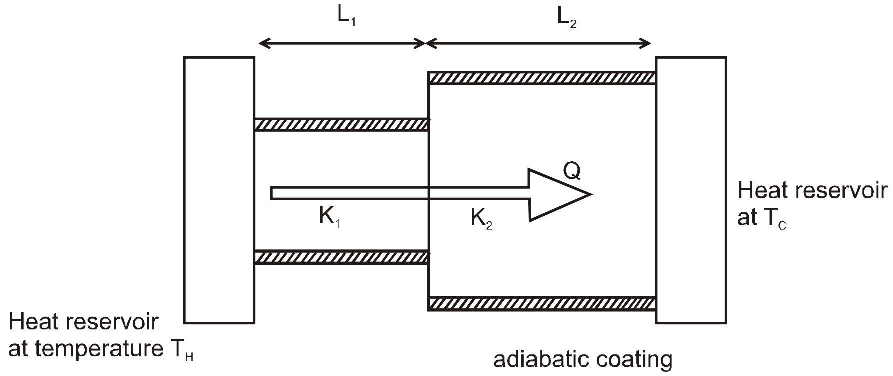
$\Rightarrow$ 13T = 48 + 50 + 32 = 130

$\Rightarrow$ T = 10ºC **Ans.**

5. SLABS IN PARALLEL AND SERIES

5.1 Slabs in series (in steady state)

Consider a composite slab consisting of two materials having different thicknesses L_1 and L_2 different cross-sectional areas A_1 and A_2 and different thermal conductivities K_1 and K_2. The temperature at the outer surface of the slabs are maintained at T_H and T_C, and all lateral surfaces are covered by an adiabatic coating.



Let temperature at the junction be T, since steady state has been achieved thermal current through each slab will be equal. Then thermal current through the first slab.

$$i = \frac{Q}{t} = \frac{T_H - T}{R_1} \quad \text{or } T_H - T = iR_1 \qquad \text{... (5.1)}$$

and that through the second slab,

$$i = \frac{Q}{t} = \frac{T - T_C}{R_2} \qquad \text{or} \qquad T - T_C = iR_2 \qquad \text{....(5.2)}$$

adding eqn. 5.1 and eqn 5.2

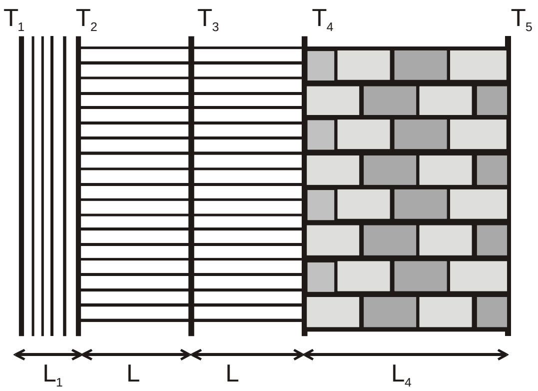
$$T_H - T_L = (R_1 + R_2)\, i \qquad \text{or} \qquad i = \frac{T_H - T_C}{R_1 + R_2}$$

Thus these two slabs are equivalent to a single slab of thermal resistance $R_1 + R_2$.

If more than two slabs are joined in series and are allowed to attain steady state, then equivalent thermal resistance is given by

$$R = R_1 + R_2 + R_3 + ... \qquad \text{...(5.3)}$$

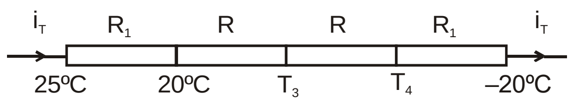
Example 3 The figure shows the cross-section of the outer wall of a house built in a hill-resort to keep the house insulated from the freezing temperature of outside. The wall consists of teak wood of thickness L_1 and brick of thickness ($L_2 = 5L_1$), sandwitching two layers of an unknown material with identical thermal conductivities and thickness. The thermal conductivity of teak wood is K_1 and that of brick is ($K_2 = 5K$). Heat conduction through the wall has reached a steady state with the temperature of three surfaces being known. (T_1 = 25ºC, T_2 = 20ºC and T_5 = –20ºC). Find the interface temperature T_4 and T_3.



Solution : Let interface area be A. then thermal resistance of wood, $R_1 = \frac{L_1}{K_1 A}$

and that of brick wall $R_2 = \frac{L_2}{K_2 A} = \frac{5L_1}{5K_1 A} = R_1$

Let thermal resistance of the each sand witch layer = R. Then the above wall can be visualised as a circuit



thermal current through each wall is same. Hence $\frac{25 - 20}{R_1} = \frac{20 - T_3}{R} = \frac{T_3 - T_4}{R} = \frac{T_4 + 20}{R_1}$

$\Rightarrow$ 25 – 20 = T_4 + 20 $\Rightarrow$ T_4 = –15ºC **Ans.**

also, 20 – T_3 = T_3 – T_4 $\Rightarrow$ $T_3 = \frac{20 + T_4}{2}$ = 2.5ºC **Ans.**

Example 4 In example 3, K_1 = 0.125 W/m–ºC, $K_2 = 5K_1$ = 0.625 W/m–ºC and thermal conductivity of the unknown material is K = 0.25 W/mºC. L_1 = 4cm, $L_2 = 5L_1$ = 20cm. If the house consists of a single room of total wall area of 100 m^2, then find the power of the electric heater being used in the room.

Solution : **Ist method** $R_1 = R_2 = \frac{(4\times10^{-2}m)}{(0.125 w/m–^oC)(100m^2)} = 32 \times 10^{-4}$ ºC/w

$\because \frac{25-20}{R_1} = \frac{20-T_3}{R} \Rightarrow \quad L = \frac{17.5}{5} \times \frac{K}{K_1} L_1 = 28$ cm

$R = \frac{L}{KA} = 112 \times 10^{-4}$ ºC/W

the equivalent thermal resistance of the entire wall = $R_1 + R_2 + 2R = 288 \times 10^{-4}$ ºC/W

$\therefore$ Net heat current, i.e. amount of heat flowing out of the house per second = $\frac{T_H - T_C}{R}$

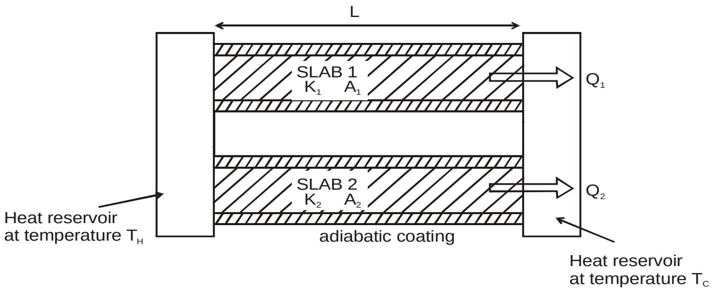
$= \frac{25^oC - (-20^oC)}{288\times10^{-4}\,^oC/w} = \frac{45\times10^4}{288}$ watt = 1.56 Kwatt

Hence the heater must supply 1.56 kW to compensate for the outflow of heat. **Ans.**

IInd method

$i = \frac{T_1 - T_2}{R_1} = \frac{25-20}{32\times10^{-4}} = 1.56$ Kwatt

5.2 Slabs in parallel :



Consider two slabs held between the same heat reservoirs, their thermal conductivities K_1 and K_2 and cross-sectional areas A_1 and A_2

then $R_1 = \frac{L}{K_1A_1}$, $R_2 = \frac{L}{K_2A_2}$

thermal current through slab 1

$$i_1 = \frac{T_H - T_C}{R_1}$$

and that through slab 2

$$i_2 = \frac{T_H - T_C}{R_2}$$

Net heat current from the hot to cold reservoir

$$i = i_1 + i_2 = (T_H - T_C)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

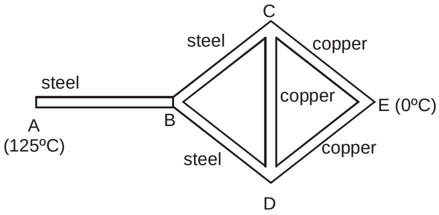
Comparing with $i = \frac{T_H - T_C}{R_{eq}}$, we get,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

If more than two rods are joined in parallel, the equivalent thermal resistance is given by

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ... \qquad .... (5.4)$$

Example 5 Three copper rods and three steel rods each of length ℓ = 10 cm and area of cross-section 1 cm^2 are connected as shown



If ends A and E are maintained at temperatures 125ºC and 0ºC respectively, calculate the amount of heat flowing per second from the hot to cold function. [K_{Cu} = 400 W/m-K, K_{steel} = 50 W/m-K]

Solution : $R_{steel} = \frac{L}{KA} = \frac{10^{-1}m}{50(W/m\text{–}^oC)\times 10^{-4}m^2} = \frac{1000}{50}$ ºC/W.

Similarly $R_{Cu} = \frac{1000}{400}$ ºC/W

Junction C and D are identical in every respect and both will have same temperature. Consequently, the rod CD is in thermal equilibrium and no heat will flow through it. Hence it can be neglected in further analysis.

Now rod BC and CE are in series their equivalent resistance is $R_1 = R_S + R_{Cu}$ similarly rods BD and DE are in series with same equivalent resistance $R_1 = R_S + R_{Cu}$ these two are in parallel giving an equivalent resistance of

$$\frac{R_1}{2} = \frac{R_S + R_{Cu}}{2}$$

This resistance is connected in series with rod AB. Hence the net equivalent of the combination is

$$R = R_{steel} + \frac{R_1}{2} = \frac{3R_{steel} + R_{Cu}}{2} = 500\left(\frac{3}{50} + \frac{1}{400}\right)^oC/W$$

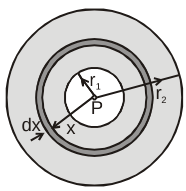
Now $i = \frac{T_H - T_C}{R} = \frac{125\ ^oC}{500\left(\frac{3}{50}+\frac{1}{400}\right)^oC/W}$ = 4 watt. **Ans.**

Example 6. Two thin concentric shells made of copper with radius r_1 and r_2 ($r_2 > r_1$) have a material of thermal conductivity K filled between them. The inner and outer spheres are maintained at temperatures T_H and T_C respectively by keeping a heater of power P at the centre of the two spheres. Find the value of P.

Solution : Heat flowing per second through each cross-section of the sphere = P = i.

Thermal resistance of the spherical shell of radius x and thickness dx,

$$dR = \frac{dx}{K.4\pi x^2}$$



$$\Rightarrow \quad R = \int_{r_1}^{r_2} \frac{dx}{4\pi x^2.K} = \frac{1}{4\pi K}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

thermal current $i = P = \frac{T_H - T_C}{R} = \frac{4\pi K(T_H - T_C)r_1 r_2}{(r_2 - r_1)}$. **Ans.**

Example 7. A container of negligible heat capacity contains 1 kg of water. It is connected by a steel rod of length 10 m and area of cross-section 10cm^2 to a large steam chamber which is maintained at 100ºC. If initial temperature of water is 0ºC, find the time after which it becomes 50ºC.

(Neglect heat capacity of steel rod and assume no loss of heat to surroundings) (Use table 3.1, take specific heat of water = 4180 J/kg ºC)

Solution : Let temperature of water at time t be T, then thermal current at time t,

$$i = \left(\frac{100 - T}{R}\right)$$

This increases the temperature of water from T to T + dT

$$\Rightarrow \quad i = \frac{dH}{dt} = ms\frac{dT}{dt}$$

$$\Rightarrow \quad \frac{100 - T}{R} = ms\frac{dT}{dt} \quad \Rightarrow \quad \int_0^{50} \frac{dT}{100 - T} = \int_0^t \frac{dT}{Rms}$$

$$\Rightarrow \quad -\ell n\left(\frac{1}{2}\right) = \frac{t}{Rms}$$

or $t = Rms\ \ell n2 \text{ sec} = \frac{L}{KA} ms\ \ell n2 \text{ sec}$

$$= \frac{(10m)(1kg)(4180J/kg\text{–}^oC)}{46(w/m^oC)\times(10\times10^{-4}m^2)}\ell n2$$

$$= \frac{418}{46}(0.69)\times10^5 = 6.27\times10^5 \text{ sec} = 174.16 \text{ hours}$$ **Ans.**

Can you now see how the following facts can be explained by thermal conduction ?

(a) In winter, iron chairs appear to be colder than the wooden chairs.
(b) Ice is covered in gunny bags to prevent melting.
(c) Woolen clothes are warmer.
(d) We feel warmer in a fur coat.
(e) Two thin blankets are warmer than a single blanket of double the thickness.
(f) Birds often swell their feathers in winter.
(g) A new quilt is warmer than old one.
(h) Kettles are provided with wooden handles.
(i) Eskimo's make double walled ice houses.
(j) Thermos flask is made double walled.

6. CONVECTION *(not in JEE Syllabus)

When heat is transferred from one point to the other through actual movement of heated particles, the process of heat transfer is called convection. In liquids and gases, some heat may be transported through conduction. But most of the transfer of heat in them occurs through the process of convection. Convection occurs through the aid of earth’s gravity. Normally the portion of fluid at greater temperature is less dense, while that at lower temperature is denser. Hence hot fluid rises up while colder fluid sink down, accounting for convection. In the absence of gravity convection would not be possible.
Also, the anomalous behaviour of water (its density increases with temperature in the range 0-4ºC) give rise to interesting consequences vis-a-vis the process of convection. One of these interesting consequences is the presence of aquatic life in temperate and polar waters. The other is the rain cycle.

Can you now see how the following facts can be explained by thermal convection ?

(a) Oceans freeze top to down and not bottom to up. (this fact is singularly responsible for presence of aquatic life is temperate and polar waters.)
(b) The temperature in the bottom of deep oceans is invariably 4ºC, whether it is winter or summer.
(c) You cannot illuminate the interior of a lift in free fall or an artificial satellite of earth with a candle.
(d) You can Illuminate your room with a candle.

7. RADIATION :

The process of the transfer of heat from one place to another place without heating the intervening medium is called radiation. The term radiation used here is another word for electromagnetic waves. These waves are formed due to the superposition of electric and magnetic fields perpendicular to each other and carry energy.

Properties of Radiation:

(a) All objects emit radiations simply because their temperature is above absolute zero, and all objects absorb some of the radiation that falls on them from other objects.

(b) Maxwell on the basis of his electromagnetic theory proved that all radiations are electromagnetic waves and their sources are vibrations of charged particles in atoms and molecules.

(c) More radiations are emitted at higher temperature of a body and lesser at lower temperature.

(d) The wavelength corresponding to maximum emission of radiations shifts from longer wavelength to shorter wavelength as the temperature increases. Due to this the colour of a body appears to be changing. Radiations from a body at NTP has predominantly infrared waves.

(e) Thermal radiations travels with the speed of light and move in a straight line.

(f) Radiations are electromagnetic waves and can also travel through vacuum.

(g) Similar to light, thermal radiations can be reflected, refracted, diffracted and polarized.

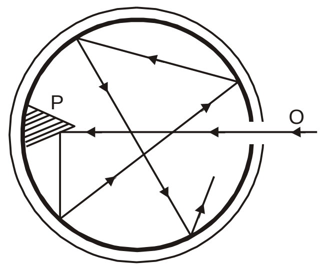
(h) Radiation from a point source obeys inverse square law (intensity $\alpha \frac{1}{r^2}$).

8. PREVOST THEORY OF EXCHANGE :

According to this theory, all bodies radiate thermal radiation at all temperatures. The amount of thermal radiation radiated per unit time depends on the nature of the emitting surface, its area and its temperature. The rate is faster at higher temperatures. Besides, a body also absorbs part of the thermal radiation emitted by the surrounding bodies when this radiation falls on it. If a body radiates more than what it absorbs, its temperature falls. If a body radiates less than what it absorbs, its temperature rises. And if the temperature of a body is equal to the temperature of its surroundings it radiates at the same rate as it absorbs.

9. PERFECTLY BLACK BODY AND BLACK BODY RADIATION

(FERY'S BLACK BODY)

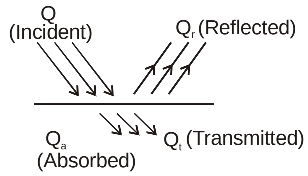


A perfectly black body is one which absorbs all the heat radiations of whatever wavelength, incident on it. It neither reflects nor transmits any of the incident radiation and therefore appears black whatever be the colour of the incident radiation.

In actual practice, no natural object possesses strictly the properties of a perfectly black body. But the lamp-black and platinum black are good approximation of black body. They absorb about 99 % of the incident radiation. The most simple and commonly used black body was designed by Ferry.

It consists of an enclosure with a small opening which is painted black from inside. The opening acts as a perfect black body. Any radiation that falls on the opening goes inside and has very little chance of escaping the enclosure before getting absorbed through multiple reflections. The cone opposite to the opening ensures that no radiation is reflected back directly.

10. ABSORPTION, REFLECTION AND EMISSION OF RADIATIONS



$Q = Q_r + Q_t + Q_a$

$$1 = \frac{Q_r}{Q} + \frac{Q_t}{Q} + \frac{Q_a}{Q} \quad ; \quad 1 = r + t + a$$

where r = reflecting power, a = absorptive power and t = transmission power.

(i) r = 0, t = 0, a = 1, perfect black body

(ii) r = 1, t = 0, a = 0, perfect reflector

(iii) r = 0, t = 1, a = 0, perfect transmitter

10.1 Absorptive power :

In particular absorptive power of a body can be defined as the fraction of incident radiation that is absorbed by the body. $a = \frac{\text{Energy absorbed}}{\text{Energy incident}}$

As all the radiations incident on a black body are absorbed, a = 1 for a black body.

10.2 Emissive power:

Energy radiated per unit time per unit area along the normal to the area is known as emissive power.

$E = \frac{Q}{\Delta A\, \Delta t}$ (Notice that unlike absorptive power, emissive power is not a dimensionless quantity).

10.3 Spectral Emissive power (E_λ) :

Emissive power per unit wavelength range at wavelength λ is known as spectral emissive power, E_λ. If E is the total emissive power and E_λ is spectral emissive power, they are related as follows,

$$E = \int_0^\infty E_\lambda \, d\lambda \text{ and } \frac{dE}{d\lambda} = E_\lambda$$

10.4 Emissivity:

$$e = \frac{\text{Emissive power of a body at temperature T}}{\text{Emissive power of a black body at same temperature T}} = \frac{E}{E_0}.$$

11. KIRCHOFF'S LAW:

The ratio of the emissive power to the absorptive power for the radiation of a given wavelength is same for all substances at the same temperature and is equal to the emissive power of a perfectly black body for the same wavelength and temperature.

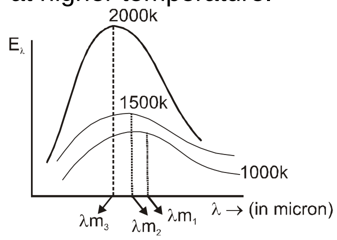
$$\frac{E(\text{body})}{a(\text{body})} = E(\text{black body})$$

Hence we can conclude that good emitters are also good absorbers.

12. NATURE OF THERMAL RADIATIONS : (WIEN'S DISPLACEMENT LAW)

From the energy distribution curve of black body radiation, the following conclusions can be drawn :

(a) The higher the temperature of a body, the higher is the area under the curve i.e. more amount of energy is emitted by the body at higher temperature.



(b) The energy emitted by the body at different temperatures is not uniform. For both long and short wavelengths, the energy emitted is very small.

(c) For a given temperature, there is a particular wavelength (λ_m) for which the energy emitted (E_λ) is maximum.

(d) With an increase in the temperature of the black body, the maxima of the curves shift towards shorter wavelengths.

From the study of energy distribution of black body radiation discussed as above, it was established experimentally that the wavelength (λ_m) corresponding to maximum intensity of emission decreases inversely with increase in the temperature of the black body.

i.e. $\lambda_m \propto \frac{1}{T}$ or $\lambda_m T = b$

This is called Wien's displacement law. Here b = 0.282 cm-K, is the Wien's constant.

Example 8. Solar radiation is found to have an intensity maximum near the wavelength range of 470 nm. Assuming the surface of sun to be perfectly absorbing (a = 1), calculate the temperature of solar surface.

Solution : Since a =1, sun can be assumed to be emitting as a black body from Wien's law for a black body

$\lambda_m \cdot T = b$

$\Rightarrow \quad T = \dfrac{b}{\lambda_m} = \dfrac{0.282(cm-K)}{(470 \times 10^{-7} cm)} \simeq 6000$ K. **Ans.**

13. STEFAN-BOLTZMANN'S LAW :

According to this law, the amount of radiation emitted per unit time from an area A of a black body at absolute temperature T is directly proportional to the fourth power of the temperature.

$u = \sigma A T^4$ ..... (13.1)

where σ is Stefan's constant = 5.67×10^{-8} W/m^2 K^4

A body which in not a black body absorbs and hence emits less radiation then

For such a body, $u = e\sigma AT^4$ .....(13.2)

where e = emissivity (which is equal to absorptive power) which lies between 0 to 1

With the surroundings of temperature T_0, net energy radiated by an area A per unit time.

$\Delta u = u - u_0 = e\sigma A(T^4 - T_0^4)$ ....(13.3)

Example 9. A body of emissivity (e = 0.75), surface area of 300 cm^2 and temperature 227ºC is kept in a room at temperature 27ºC. Calculate the initial value of net power emitted by the body.

Solution: Using equation. (13.3) $P = e\,\sigma\,A\,(T^4 - T_0^4)$

$= (0.75)\,(5.67 \times 10^{-8}\ W/m^2 - K^4)\,(300 \times 10^{-4}\ m^2) \times \{(500\ K)^4 - (300\ K)^4\} = 69.4$ Watt. **Ans.**

Example 10. A hot black body emits the energy at the rate of 16 J m^{-2} s^{-1} and its most intense radiation corresponds to 20,000 Å. When the temperature of this body is further increased and its most intense radiation corresponds to 10,000 Å, then find the value of energy radiated in Jm^{-2} s^{-1}.

Solution : Wein's displacement law is : $\lambda_m.T = b$

i.e. $T \propto \dfrac{1}{\lambda_m}$; Here, λ_m becomes half.

$\therefore$ Temperature doubles. Also $e = \sigma T^4$

$\Rightarrow \quad \dfrac{e_1}{e_2} = \left(\dfrac{T_1}{T_2}\right)^4 \Rightarrow e_2 = \left(\dfrac{T_2}{T_1}\right)^4 .e_1 = (2)^4 \cdot 16 = 16.16 = 256$ J m^{-2} s^{-1} **Ans.**

14. NEWTON'S LAW OF COOLING :

For small temperature difference between a body and its surrounding, the rate of cooling of the body is directly proportional to the temperature difference and the surface area exposed.

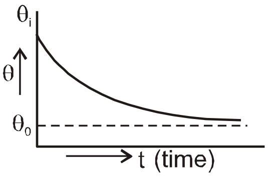
$\dfrac{d\theta}{dt} \propto (\theta - \theta_0)$, where θ and θ_0 are temperature corresponding to object and surroundings.

From above expression, $\dfrac{d\theta}{dt} = -k(\theta - \theta_0)$ .....(14.1)

This expression represents Newton's law of cooling. It can be derived directly from stefan's law, which gives,

$k = \dfrac{4e\sigma\theta_0^3}{mc} A$ .....(14.2)

Now $\dfrac{d\theta}{dt} = -k[\theta - \theta_0] \Rightarrow \int_{\theta_i}^{\theta_f} \dfrac{d\theta}{(\theta - \theta_0)} = \int_0^t -kdt$



where θ_i = initial temperature of object and

θ_f = final temperature of object.

$\Rightarrow \quad \ell n \dfrac{(\theta_f - \theta_0)}{(\theta_i - \theta_0)} = -kt \Rightarrow (\theta_f - \theta_0) = (\theta_i - \theta_0)\,e^{-kt}$

$\Rightarrow \quad \theta_f = \theta_0 + (\theta_i - \theta_0)\,e^{-kt}$ .....(14.3)

14.1 Limitations of Newton's Law of Cooling:

(a) The difference in temperature between the body and surroundings must be small

(b) The loss of heat from the body should be by radiation only.

(c) The temperature of surroundings must remain constant during the cooling of the body.

14.2 Approximate method for applying Newton's law of cooling

Sometime when we need only approximate values from Newton's law, we can assume a constant rate of cooling, which is equal to the rate of cooling corresponding to the average temperature of the body during the interval.

$$\left\langle \frac{d\theta}{dt} \right\rangle = -k(\langle\theta\rangle - \theta_0) \qquad .....(14.4)$$

If θ_i & θ_f be initial and final temperature of the body then,

$$\langle\theta\rangle = \frac{\theta_i + \theta_f}{2} \qquad .....(14.5)$$

Remember equation (14.5) is only an approximation and equation (14.1) must be used for exact values.

Example 11. A body at temperature 40ºC is kept in a surrounding of constant temperature 20ºC. It is observed that its temperature falls to 35ºC in 10 minutes. Find how much more time will it take for the body to attain a temperature of 30ºC.

Solution : from equation (14.3)

$$\Delta\theta_f = \Delta\theta_i\, e^{-kt}$$

for the interval in which temperature falls from 40 to 35ºC.

$$(35 - 20) = (40 - 20)\, e^{-k.\,10}$$

$$\Rightarrow \quad e^{-10k} = \frac{3}{4} \qquad \Rightarrow \qquad k = \frac{\ell n\frac{4}{3}}{10}$$

for the next interval

$$(30 - 20) = (35 - 20)e^{-kt}$$

$$\Rightarrow \quad e^{-kt} = \frac{2}{3} \qquad \Rightarrow \qquad kt = \ell n\frac{3}{2}$$

$$\Rightarrow \quad \frac{\left(\ell n\frac{4}{3}\right)t}{10} = \ell n\frac{3}{2} \qquad \Rightarrow \qquad t = 10\,\frac{\left(\ell n\frac{3}{2}\right)}{\left(\ell n\frac{4}{3}\right)} \text{ minute} = 14.096 \text{ min} \qquad \textbf{Ans.}$$

Aliter : (by approximate method) for the interval in which temperature falls from 40 to 35ºC

$$\langle\theta\rangle = \frac{40+35}{2} = 37.5^\circ C$$

from equation (14.4) $\left\langle \frac{d\theta}{dt} \right\rangle = -k(\langle\theta\rangle - \theta_0)$

$$\Rightarrow \quad \frac{(35^\circ C - 40^\circ C)}{10(\text{min})} = -K(37.5^\circ C - 20^\circ C) \qquad \Rightarrow \qquad K = \frac{1}{35}(\text{min}^{-1})$$

for the interval in which temperature falls from 35ºC to 30ºC

$$\langle\theta\rangle = \frac{35+30}{2} = 32.5^\circ C$$

from equation (14.4)

$$\frac{(30^\circ C - 35^\circ C)}{t} = -(32.5^\circ C - 20^\circ C)\, K$$

$$\Rightarrow \quad \text{required time,} \quad t = \frac{5}{12.5} \times 35 \text{min} = 14 \text{ min} \qquad \textbf{Ans.}$$