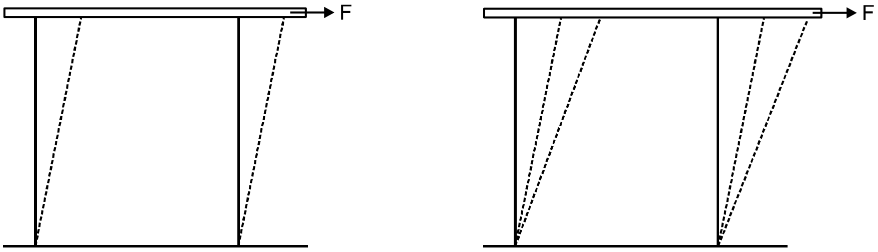
CHAPTER-13
FLUID MECHANICS

Fluid mechanics deals with the behaviour of fluids at rest and in motion. A fluid is a substance that deforms continuously under the application of shear (tangential) stress no matter how small the shear stress may be: Thus, fluid comprise the liquid and gas (or vapour) phases of the physical forms in which matter exists.

We may alternatively define a fluid as a substance that cannot sustain a shear stress when at rest.



(a) Solid (b) Fluid

1. Density of a Liquid

Density (ρ) of any substance is defined as the mass per unit volume or

$$\rho = \frac{\text{mass}}{\text{volume}} \qquad \rho = \frac{m}{V}$$

2. Relative Density (RD)

In case of a liquid, sometimes an another term relative density (RD) is defined. It is the ratio of density of the substance to the density of water at 4°C. Hence,

$$RD = \frac{\text{Density of substance}}{\text{Density of water at } 4°C}$$

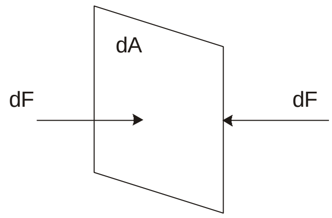
RD is a pure ratio. So, it has no units. It is also sometimes referred as specific gravity.

Density of water at 4°C in CGS is 1g/cm^3. Therefore, numerically the RD and density of substance (in CGS) are equal. In SI units the density of water at 4°C is 1000 kg/m^3.

Example 1. Relative density of an oil is 0.8. Find the absolute density of oil in CGS and SI units.

Solution : Density of oil (in CGS) = (RD)g/cm^3 = 0.8 g/cm^3 = 800 kg/m^3

3. Pressure in a Fluid



When a fluid (either liquid or gas) is at rest, it exerts a force perpendicular to any surface in contact with it, such as a container wall or a body immersed in the fluid.

While the fluid as a whole is at rest, the molecules that makes up the fluid are in motion, the force exerted by the fluid is due to molecules colliding with their surrounding.

If we think of an imaginary surface within the fluid, the fluid on the two sides of the surface exerts equal and opposite forces on the surface, otherwise the surface would accelerate and the fluid would not remain at rest.

Consider a small surface of area dA centered on a point on the fluid, the normal force exerted by the fluid on each side is $dF_{\perp}$. The pressure P is defined at that point as the normal force per unit area, i.e.,

$$P = \frac{dF_{\perp}}{dA}$$

If the pressure is the same at all points of a finite plane surface with area A, then

$$P = \frac{F_{\perp}}{A}$$

where $F_{\perp}$ is the normal force on one side of the surface. The SI unit of pressure is pascal

where 1 pascal = 1Pa = 1.0 N/m^2

One unit used principally in meterology is the Bar which is equal to 10^5 Pa

1 Bar = 10^5 Pa

Note : Fluid pressure acts perpendicular to any surface in the fluid no matter how that surface is oriented. Hence, pressure has no intrinsic direction of its own, its a **scalar**. By contrast, force is a vector with a definite direction.

Atmospheric Pressure (P_0)

It is pressure of the earth's atmosphere. This changes with weather and elevation. Normal atmospheric pressure at sea level (an average value) is 1.013×10^5 Pa

Absolute pressure and Gauge Pressure

The excess pressure above atmospheric pressure is usually called gauge pressure and the total pressure is called absolute pressure. Thus,

Gauge pressure = absolute pressure – atmospheric pressure

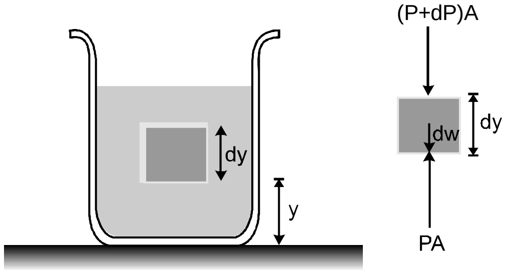
Absolute pressure is always greater than or equal to zero. While gauge pressure can be negative also.

Variation in pressure with depth

If the weight of the fluid can be neglected, the pressure in a fluid is the same throughout its volume. But often the fluid's weight is not negligible and under such condition pressure increases with increasing depth below the surface.

Let us now derive a general relation between the pressure P at any point in a fluid at rest and the elevation y of that point. We will assume that the density ρ and the acceleration due to gravity g are the same throughout the fluid. If the fluid is in equilibrium, every volume element is in equilibrium.

Consider a thin element of fluid with height dy. The bottom and top surfaces each have area A, and they are at elevations y and y + dy above some reference level where y = 0. The weight of the fluid element is



$$dW = (\text{volume})\ (\text{density})\ (g) = (A\ dy)\ (\rho)\ (g)$$

or $$dW = \rho g A dy$$

What are the other forces in y-direction on this fluid element ? Call the pressure at the bottom surface P, the total y component of upward force is PA. The pressure at the top surface is P + dP and the total y-component of downward force on the top surface is (P + dP)A. The fluid element is in equilibrium, so the total y component of force including the weight and the forces at the bottom and top surfaces must be zero.

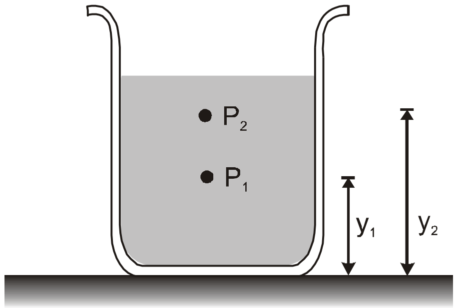
$$\Sigma F_y = 0$$

$$\therefore \quad PA - (P + dP)A - \rho g A\, dy = 0 \quad \text{or} \quad \frac{dP}{dy} = -\rho g$$

This equation shows that when y increases, P decreases, i.e., as we move upward in the fluid pressure decreases.

If P_1 and P_2 be the pressures at elevations y_1 and y_2 and if ρ and g are constant, then integration Equation (i), we get

or $P_2 - P_1 = -\rho g (y_2 - y_1)$ ....(ii)



It's often convenient to express Equation (ii) in terms of the depth below the surface of a fluid. Take point 1 at depth h below the surface of fluid and let P represents pressure at this point. Take point 2 at the surface of the fluid, where the pressure is P_0 (subscript for zero depth). The depth of point 1 below the surface is,

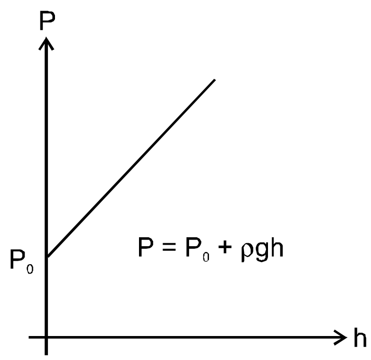
$h = y_2 - y_1$

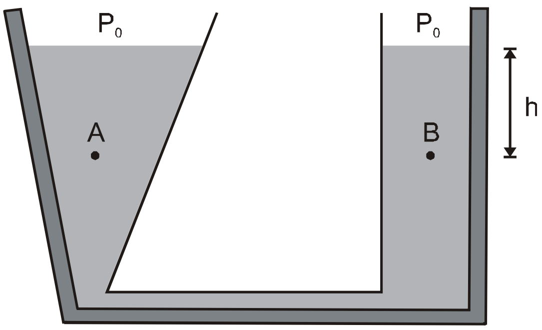
and equation (ii) becomes

$P_0 - P = -\rho g (y_2 - y_1) = -\rho gh$

$\therefore$ $P = P_0 + \rho gh$ ....(iii)

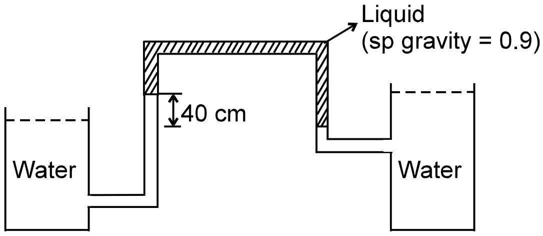
Thus pressure increases linearly with depth, if ρ and g are uniform. A graph between P and h is shown below.





Further, the pressure is the same at any two points at the same level in the fluid. The shape of the container does not matter.

Example 2. The manometer shown below is used to measure the difference in water level between the two tanks. Calculate this difference for the conditions indicated.



Solution :

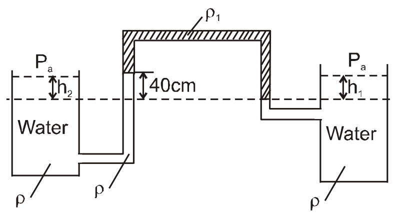
$P_a + h_1 \rho g - 40\rho_1 g + 40\rho g = P_a + h_2 \rho g$

$h_2 \rho g - h_1 \rho g = 40 \rho g - 40 \rho_1 g$

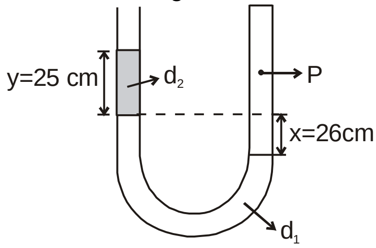
as $\rho_1 = 0.9\rho$

$(h_2 - h_1) \rho g = 40\rho g - 36\rho g$

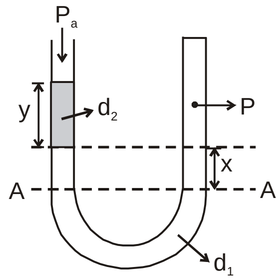
$h_2 - h_1 = 4$ cm



Example 3. In a given U-tube (open at one-end) find out relation between P and P_a.
Given $d_2 = 2 \times 13.6$ gm/cm^3 $d_1 = 13.6$ gm/cm^3



Solution : Pressure in a liquid at same level is same i.e. at A – A–,



$P_a + d_2 y g + x d_1 g = P$

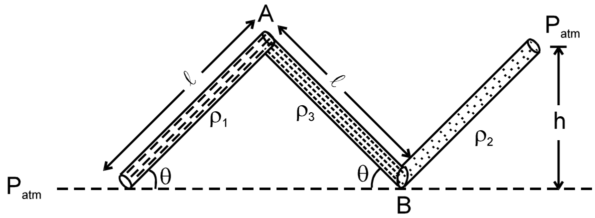
In C.G.S.

$P_a + 13.6 \times 2 \times 25 \times g + 13.6 \times 26 \times g = P$

$P_a + 13.6 \times g\,[50 + 26] = P$

$2P_a = P$ $\qquad [P_a = 13.6 \times g \times 76]$

Example 4. Find out pressure at points A and B. Also find angle 'θ'.

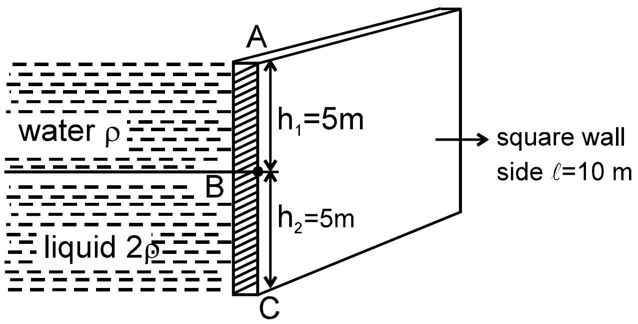


Solution :
Pressure at A – $\qquad P_A = P_{atm} - \rho_1 gl \sin\theta$
Pressure at B $\qquad P_B = P_{atm} + \rho_2 gh\,\theta$
But P_B is also equal to $\qquad P_B = P_A + \rho_3 gl \sin\theta$
Hence - $\qquad P_{atm} + \rho_2 gh = P_A + \rho_3 gl \sin\theta$

$P_{atm} + \rho_2 gh = P_{atm} - \rho_1 gl \sin\theta + \rho_3 gl \sin\theta$

$$\sin\theta = \frac{\rho_2 h}{(\rho_3 - \rho_1)\ell}.$$

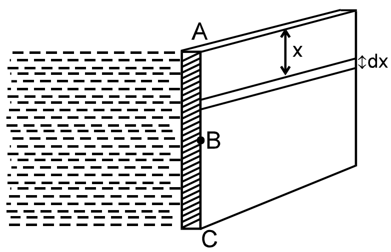
Example 5. Water and liquid is filled up behind a square wall of side ℓ. Find out



(a) Pressures at A, B and C $\qquad$ (b) Forces in part AB and BC

Solution :
(a) As there is no liquid above 'A',
So pressure at A, $P_A = 0$
Pressure at B, $P_B = \rho g h_1$
Pressure at C, $P_C = \rho g h_1 + 2\rho g h_2$

(b) Force at A = 0 Take a strip of width 'dx' at a depth 'x' in part AB.
Pressure is equal to $\rho g x$.

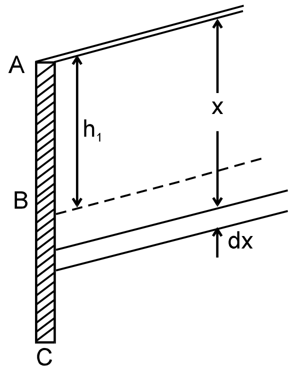


Force on strip = pressure × area

$dF = \rho g x \,\ell dx$

Total force upto B

$F = \int_0^{h_1} \rho g x \ell dx = \frac{\rho g x \ell h_1^2}{2} = \frac{1000 \times 10 \times 10 \times 5 \times 5}{2}$



$= 1.25 \times 10^6$ N

In part BC for force take a elementary strip of width dx in portion BC. Pressure is equal to

$= \rho g h_1 + 2\rho g(x - h_1)$

Force on elementary strip = pressure × area

$dF = [\rho g h_1 + 2\rho g(x - h_1)]\,\ell\, dx$

Total force on part BC

$F = \int_{h_1}^{\ell} [\rho g h_1 + 2\rho g(x - h_1)]\ell dx = \left[\rho g h_1 x + 2\rho g\left[\frac{x^2}{2} - h_1 x\right]\right]_{h_1}^{\ell} \ell$

$= \rho g h_1 h_2 \ell + 2\rho g \ell \left[\frac{\ell^2 - h_1^2}{2} - \ h_1\ell + h_1^2\right]$

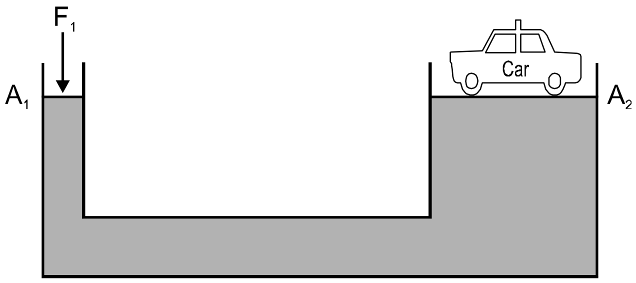
$= \rho g h_1 h_2 \ell + \frac{2\rho g \ell}{2} [\ell^2 + h_1^2 - 2h_1\ell] = \rho g h_1 h_2 \ell + \rho g \ell (\ell - h_1)^2$

$= \rho g h_2 \ell [h_1 + h_2] = \rho g h_2 \ell^2 = 1000 \times 10 \times 5 \times 10 \times 10 = 5 \times 10^6$ N

Pascal's Law

It states that "pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel".

A well known application of Pascal's law is the hydraulic lift used to support or lift heavy objects. It is schematically illustrated in figure.

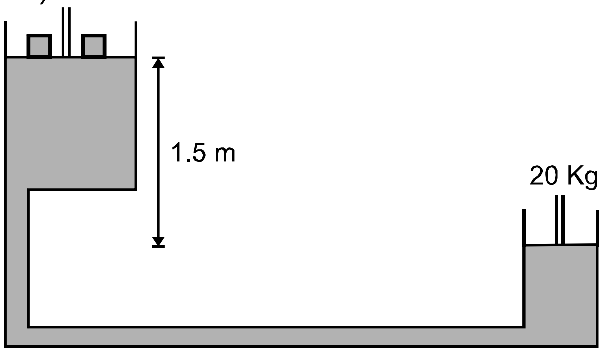


A piston with small cross section area A_1 exerts a force F_1 on the surface of a liquid such as oil. The applied pressure $P = \frac{F_1}{A_1}$ is transmitted through the connection pipe to a larger piston of area A_2. The applied pressure is the same in both cylinders, so

$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \text{or} \quad F_2 = \frac{A_2}{A_1}.F_1$$

Now, since $A_2 > A_1$, therefore, $F_2 > F_1$. Thus hydraulic lift is a force multiplying device with a multiplication factor equal to the ratio of the areas of the two pistons. Dentist's chairs, car lifts and jacks, elevators and hydraulic brakes all are based on this principle.

Example 6. Figure shows a hydraulic press with the larger piston of diameter 35 cm at a height of 1.5 m relative to the smaller piston of diameter 10 cm. The mass on the smaller piston is 20 kg. What is the force exerted on the load by the larger piston ? The density of oil in the press is 750 kg/ m^3. (Take g = 9.8m/s^2)



Solution : Pressure on the smaller piston $= \dfrac{20 \times 9.8}{\pi \times (5 \times 10^{-2})^2} \, N/m^2$

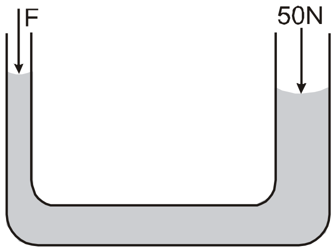
Pressure on the larger piston = $\dfrac{F}{\pi \times (17.5 \times 10^{-2})^2} \, N/m^2$

The difference between the two pressures = $h\rho g$
where h = 1.5 m and ρ = 750 kg/m^3

Thus, $\dfrac{20 \times 9.8}{\pi \times (5 \times 10^{-2})^2} - \dfrac{F}{\pi \times (17.5 \times 10^{-2})^2} = 1.5 \times 750 \times 9.8 = 11025 \Rightarrow F = 1.3 \times 10^3$ N

Note : atmospheric pressure is common to both pistons and has been ignored.

Example 7. The area of cross-section of the two arms of a hydraulic press are 1 cm^2 and 10 cm^2 respectively (figure). A force of 50 N is applied on the water in the thicker arm. What force should be applied on the water in the thinner arm so that the water may remain in equilibrium?

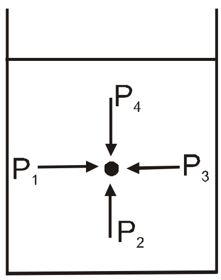


Solution : In equilibrium, the pressures at the two surfaces should be equal as they lie in the same horizontal level. If the atmospheric pressure is P and a force F is applied to maintain the equilibrium, the pressures are $P_0 + \dfrac{50N}{10cm^2}$ and $P_0 + \dfrac{F}{1\ cm^2}$ respectively.

This givens F = 5 N.

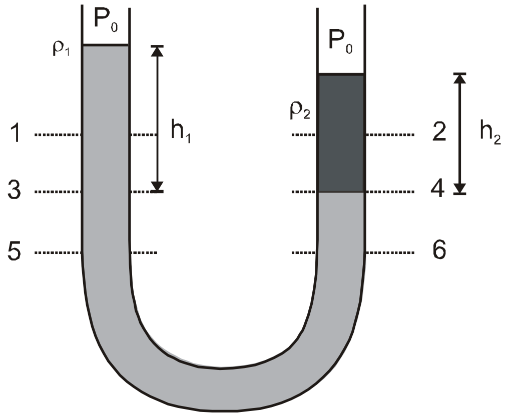
Important points in Pressure

1. At same point on a fluid pressure is same in all direction. In the figure,
$P_1 = P_2 = P_3 = P_4$



2. Forces acting on a fluid in equilibrium have to be perpendicular to its surface. Because it cannot sustain the shear stress.

3. In the **same liquid** pressure will be same at all points at the same level. For example, in the figure:



$P_1 \neq P_2$

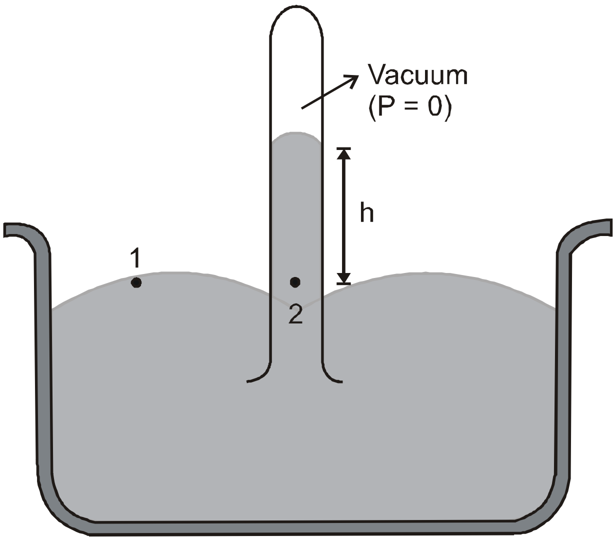
$P_3 = P_4$ and $P_5 = P_6$

Further $P_3 = P_4$

$\therefore \quad P_0 + \rho_1 gh_1 = P_0 + \rho_2 gh_2 \quad$ or $\quad \rho_1 h_1 = \rho_2 h_2 \quad$ or $\quad h \propto \frac{1}{\rho}$

4. **Torricelli Experiment (Barometer) :**

It is a device used to measure atmospheric pressure .In principle any liquid can be used to fill the barometer, but mercury is the substance of choice because its great density makes possible an instrument of reasonable size.



$P_1 = P_2$

Here, P_1 = atmospheric pressure (P_0)

and $P_2 = 0 + \rho gh = \rho gh$

$P_0 = \rho gh$

Here ρ = density of mercury

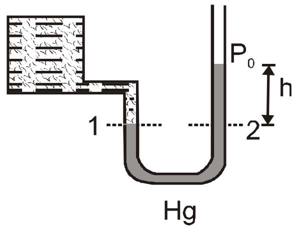
Thus, the mercury barometer reads the atmosheric pressure (P_0) directly from the height of the mercury column.

For example if the height of mercury in a barometer is 760 mm. then atmospheric pressure will be ,

$P_0 = \rho gh = (13.6 \times 10^3)(9.8)(0.760) = 1.01 \times 10^5\ N/m^2$

5. **Manometer** :

It is a device used to measure the pressure of a gas inside a container. The U- shaped tube often contains mercury



$P_1 = P_2$

Here P_1 = pressure of the gas in the container (P)

and P_2 = atmospheric pressure (P_0) + ρgh

$P = P_0 + h\rho g$

This can also be written as

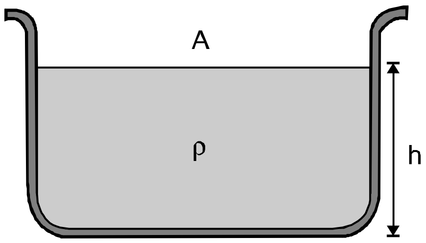
$P - P_0$ = gauge pressure = ρgh

Here, ρ is the density of the liquid used in U - tube.

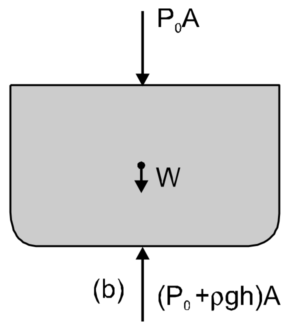
Thus by measuring h we can find absolute (or gauge) pressure in the vessel.

6. Free body diagram of a liquid :

The free body diagram of the liquid(showing the vertical forces only) is shown in fig (b) For the equilibrium of liquid .



(a)



(b)

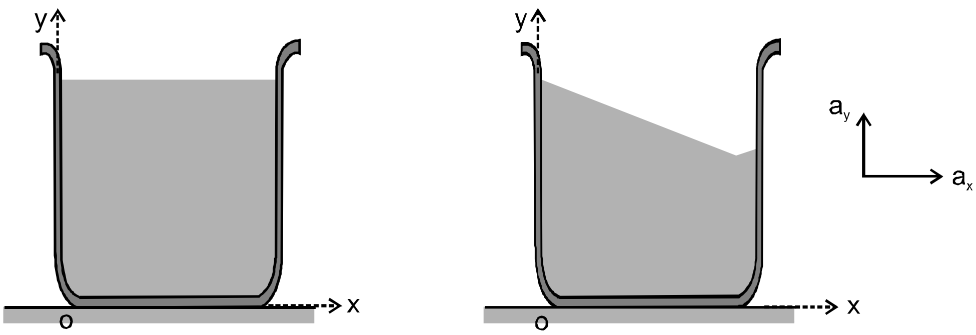
Net downward force = net upward force

$\therefore \quad P_0A + W = (P_0 + \rho gh)A$ or $W = \rho ghA$

7. Pressure Difference in Accelerating Fluids

Consider a liquid kept at rest in a beaker as shown in figure (a). In this case we know that pressure do not change in horizontal direction (x-direction), it decreases upwards along y-direction So, we can write the equations,

$\frac{dP}{dx} = 0$ and $\frac{dP}{dy} = \rho g$



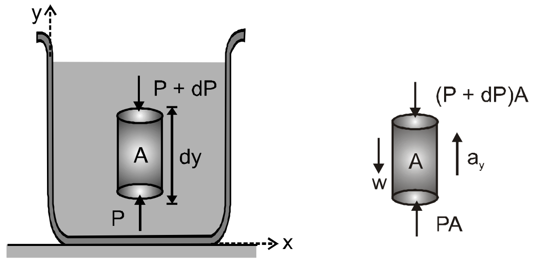
(a) (b)

But, suppose the beaker is accelerated and it has components of acceleration a_x and a_y in x and y-directions respectively, then the pressure decreases along both x and y directions. The above equation in that case reduces to

$$\frac{dP}{dx} = -\rho a_x \text{ and } \frac{dP}{dy} = -\rho(g + a_y)$$

These equations can be derived as under. Consider a beaker filled with some liquid of density ρ accelerating upwards with an acceleration a_y along positive y-direction, Let us draw the free body diagram of a small element of fluid of area A and length dy as shown in figure . Equation of motion for this element is,

$PA - W - (P + dP)A = (\text{mass})(a_y)$ or $-W - (dP)A = (A\rho\, dy)(a_y)$

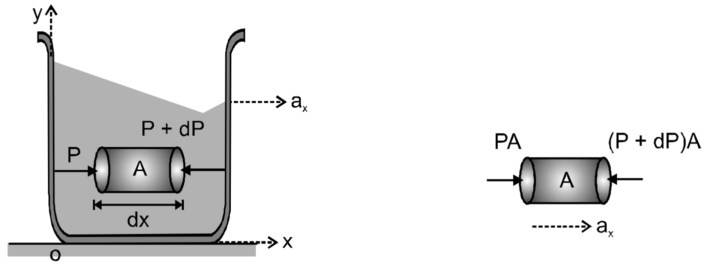


(a)

or $-(A\rho g\ dy) - (dP)\ A = (A\rho\ dy)(a_y)$ or $\frac{dP}{dy} = -\rho(g + a_y)$

Similarly, if the beaker moves along positive x-direction with acceleration a_x, the equation of motion for the fluid element shown in figure is

$PA - (P + dP)\ A = (\text{mass})\ (a_x)$ or $-(dP)\ A = (A\rho\ dx)\ a_x$ or $\frac{dP}{dx} = -\rho a_x$

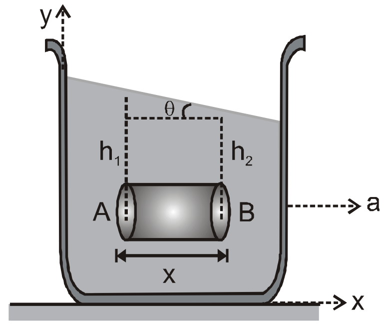


(a)

8. Free Surface of a Liquid Accelerated in Horizontal Direction

Consider a liquid placed in a beaker which is accelerating horizontally with an acceleration 'a'. Let A and B be two points in the liquid at a separation x in the same horizontal line. As we have seen in this case

$dp = \rho a\ dx$ or $\frac{dP}{dx} = -\rho a$



Integrating this with proper limits, we get $P_A - P_B = \rho ax$

Furter $P_A = P_0 + \rho gh_1$

and $P_B = P_0 + \rho gh_2$

substituting in Eq. (iii) we get

$\rho g\ (h_1 - h_2) = \rho ax$

$\frac{h_1 - h_2}{x} = \frac{a}{g} = \tan\theta$

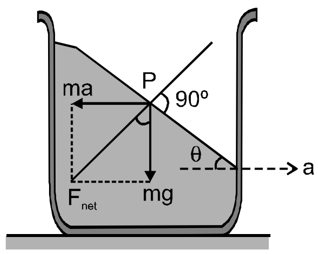
$\tan\theta = \frac{a}{g}$

Alternate Method

Consider a fluid particle of mass m at point P on the surface of liquid. From the accelerating frame of reference, two forces are acting on it,

(i) pseudo force(ma) (ii) Weight (mg)

As we said earlier also, net force in equilibrium should be perpendicular to the surface.



$\therefore\ \tan\theta = \frac{ma}{mg}$ or $\tan\theta = \frac{a}{g}$

Example 8. An open rectangular tank 1.5 m wide 2m deep and 2m long is half filled with water. It is accelerated horizontally at 3.27 m/sec^2 in the direction of its length. Determine the depth of water at each end of tank. [g = 9.81 m/sec^2]

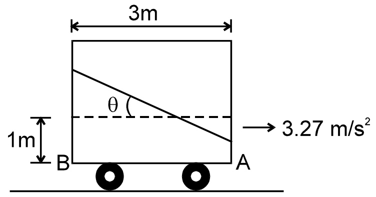
Solution : $\tan\theta = \frac{a}{g} = \frac{1}{3}$

Depth at corner 'A'

$= 1 - 1.5 \tan\theta = 0.5$ m **Ans.**

Depth at corner 'B'

$= 1 + 1.5 \tan\theta = 1.5$ m **Ans.**



9. Archimedes 'Principle

If a heavy object is immersed in water, it seems to weigh less than when it is in air. This is because the water exerts an upward force called buoyant force. It is equal to the weight of the fluid displaced by the body **A body wholly or partially submerged in a fluid is buoyed up by a force equal to the weight of the displaced fluid.**

This result known as **Archimedes 'principle.**

Thus, the magnitude of buoyant force (F) is given by, $F = V_i \rho_L g$

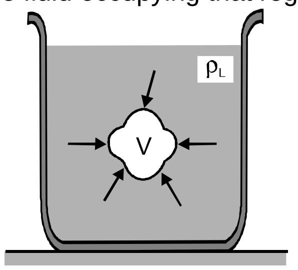
Here, V_i = immersed volume of solid

ρ_L = density of liquid

and g = acceleration due to gravity

Proof

Consider an arbitrarily shaped body of volume V placed in a container filled with a fluid of density ρ_L. The body is shown completely immersed, but complete immersion is not essential to the proof. To begin with, imagine the situation before the body was immersed. The region now occupied by the body was filled with fluid, whose weight was $V\rho_L g$. Because the fluid as a whole was in hydrostatic equilibrium, the net upwards force (due to difference in pressure at different depths) on the fluid in region was equal to the weight of the fluid occupying that region.



Now, consider what happens when the body has displaced the fluid. The pressure at every point on the surface of the body is unchanged from the value at the same location when the body was every point on. This is because the pressure at any point depends only on the depth of that point the surface. Hence, the net force exerted by the surrounding fluid on the body is exactly the same as that exerted on the region before the body was present. But we now latter to be $V\rho_L g$, the weight of the displaced fluid Hence, this must also be the buoyant force exerted of the body. Archimedes' principle is thus proved.

10. Law of Floatation

Consider an object of volume V and density ρ_s floating in a liquid of density ρ_L. Let V_i be the object immersed in the liquid.

For equilibrium of object,

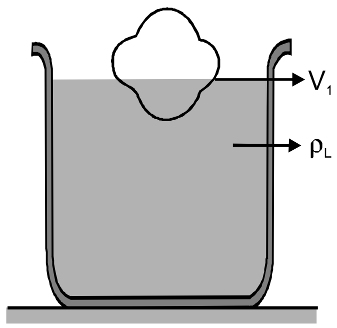
Weight = Upthrust

$\therefore \quad V\rho_s g = V_i \rho_L g$

$\therefore \quad \frac{V_i}{V} = \frac{\rho_s}{\rho_L}$

This is the fraction of volume immersed in liquid.

Percentage of volume immersed in liquid $= \frac{V_i}{V} \times 100 = \frac{\rho_s}{\rho_L} \times 100$



Three possibilities may now arise.

(i) If $\rho_s < \rho_L$, only fraction of body will be immersed in the liquid. This fraction will be given by the above equation.

(ii) If $\rho_s = \rho_L$, the whole of the rigid body will be immersed in the liquid. Hence the body remains floating in the liquid wherever it is left.

(iii) If $\rho_s > \rho_L$, the body will sink.

Apparent Weight of a Body inside a Liquid

If a body is completely immersed in a liquid its effective weight gets decreased. The decrease in its weight is equal to the upthrust on the body. Hence,

$W_{app} = W_{actual}$ – Upthrust

or $W_{app} = V\rho_s g - V\rho_L g$

Here, V = total volume of the body

ρ_s = density of body

and ρ_L = density of liquid

Thus, $W_{app} = Vg(\rho_s - \rho_L)$

If the liquid in which body is immersed, is water, then

$$\frac{\text{Weight in air}}{\text{Decrease in weight}} = \text{Relative density of body (R.D)}$$

This can be shown as under :

$$\frac{\text{Weight in air}}{\text{Decrease in weight}} = \frac{\text{Weight in air}}{\text{Upthrust}} = \frac{V\rho_s g}{V\rho_w g} = \frac{\rho_s}{\rho_w} = \text{RD}$$

Buoyant Force in Accelerating Fluids

Suppose a body is dipped inside a liquid of density ρ_L placed in an elevator moving with an acceleration $\vec{a}$.The buoyant force F in this case becomes,

$F = V\rho_L g_{eff}$

Here , $g_{eff} = |\vec{g} - \vec{a}|$

For example, if the lift is moving upwards with an acceleration a, value of g_{eff} is g + a and if it is moving downwards with acceleration a, the g_{eff} is g – a In a freely falling lift g_{eff} is zero (as a = g) and hence, net buoyant force is zero. This is why, in a freely falled with some liquid, the air bubbles do not rise up (which otherwise move up due to buoyant force).

Example 9. Density of ice is 900kg/m^3. A piece of ice is floating in water of density 1000 kg/m^3. Find the fraction of volume of the piece of ice out side the water.

Solution : Let V be the total volume and V_i the volume of ice piece immersed in water. For equilibrium of ice piece,

weight = upthrust

$\therefore \quad V\rho_i g = V_i\rho_w g$

Here ρ_i = density of ice = 900kg/m^3

and ρ_w= density of water = 1000kg/m^3

Substituting in above equation,

$$\frac{V_i}{V} = \frac{900}{1000} = 0.9$$

i.e, the fraction of volume outside the water,

f = 1– 0.9 = 0.1

Example 10. A piece of ice is floating in a glass vessel filled with water. How the level of water in the vessel change when the ice melts ?

Solution : Let m be the mass of ice piece floating in water.

In equilibrium, weight of ice piece = upthrust

$mg = V_i\rho_w g$

or $V_i = \frac{m}{\rho_w}$

Here , V_i is the volume of ice piece immersed in water

When the ice melt, let V be the volume of water formed by m mass of ice. Then,

$V_i = \frac{m}{\rho_w}$

From Eqs. (i) and (ii) we see that

$V_i = V$

Hence, the level will not change.

Example 11. A piece of ice having a stone frozen in it floats in a glass vessel filled with water. How will the level of water in the vessel change when the ice melts ?

Solution : Let, m_1 = mass of ice ,

m_2 = mass of stone

ρ_s = density of stone

and ρ_w = density of water

In equilibrium, when the piece of ice floats in water , weight of (ice + stone) = upthrust

$(m_1 + m_2)g = V_i \rho_w g$ $\therefore V_i = \frac{m_1}{\rho_w} + \frac{m_2}{\rho_w}$

Here, V_i = Volume of ice immersed

when the ice melts m_1 mass of ice converts into water and stone of mass m_2 is completely submerged .

Volume of water formed by m_1 mass of ice,

$V_1 = \frac{m_1}{\rho_w}$

Volume of stone (which is also equal to the volume of water displaced)

$V_2 = \frac{m_2}{\rho_s}$

Since, $\rho_s > \rho_w$ Therefore, $V_1 + V_2 < V_i$

or, the level of water will decrease .

Example 12. An ornament weighing 50 g in air weighs only 46 g is water. Assuming that some copper is mixed with gold to prepare the ornament. Find the amount of copper in it. Specific gravity of gold is 20 and that of copper is 10.

Solution : Let m be the mass of the copper in ornament . Then mass of gold in it is (50 – m).

Volume of copper $V_1 = \frac{m}{10}$ $\left(\text{volume} = \frac{\text{mass}}{\text{density}}\right)$

and volume of gold $V_2 = \frac{50 - m}{20}$

when immersed in water ($\rho_w = 1 g/cm^3$)

Decrease in weight = upthrust

$\therefore$ $(50 - 46)g = (V_1 + V_2)\rho_w g$

or $4 = \frac{m}{10} + \frac{50 - m}{20}$ or $80 = 2m + 50 - m$

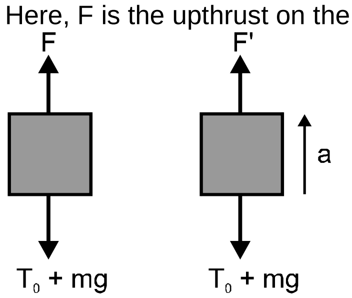
$\therefore$ $m = 30g$

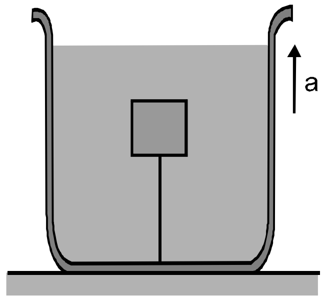
Example 13. The tension in a string holding a solid block below the surface of a liquid (of density greater then that of solid) as shown in figure is T_0 when the system is at rest. What will be the tension in the string if the system has an upward acceleration a ?

Solution : Let m be the mass of block. Initially for the equilibrium of block,

$F = T_0 + mg$

Here, F is the upthrust on the block .





Where the lift is accelerated upwards, g_{eff} becomes g + a instead of g.

Hence, $F' = F\left(\frac{g+a}{g}\right)$

From Newton's second law, $F' - T - mg = ma$

Solving Eqs. (i) , (ii) and (iii),we get

$T = T_0\left(1+\frac{a}{g}\right)$

Example 14. A metal piece of mass 10 g is suspended by a vertical spring. The spring elongates 10 cm over its natural length to keep the piece in equilibrium. A beaker containing water is now placed below the piece so as to immerse the piece completely in water. Find the elongation of the spring. Density of metal = 9000 kg/m^3. Take g = 10 m/s^2.

Solution : Let the spring constant be k. When the piece is hanging in air, the equilibrium condition gives

k (10 cm) = (0.01 kg) (10 m/s^2)

or k (10 cm) = 0.1 N. .....(i)

The volume of the metal piece

$= \frac{0.01\text{kg}}{9000\text{kg/m}^3} = \frac{1}{9} \times 10^{-5}$ m^3.

This is also the volume of water displaced when the piece is immersed in water. The force of buoyancy

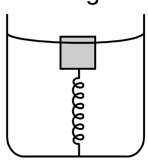
= weight of the liquid displaced = $\frac{1}{9} \times 10^{-5}$ m^3 × (1000 kg/m^3) × (10 m/s^2) = 0.011 N.

If the elongation of the spring is x when the piece is immersed in water, the equilibrium condition of the piece gives,

kx = 0.1 N – 0.011 N = 0.089 N. .....(ii)

By (i) and (ii), $x = \frac{0.089}{10}$ cm = 0.0089 cm.

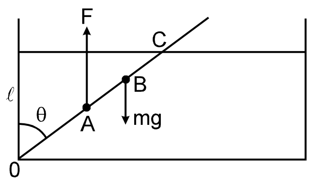
Example 15. A cubical block of plastic of edge 3 cm floats in water. The lower surface of the cube just touches the free end of a vertical spring fixed at the bottom of the pot. Find the maximum weight that can be put on the block without wetting it. Density of plastic = 800 kg/m^3 and spring constant of the spring = 100 N/m. Take g = 10 m/s^2.



Solution : The specific gravity of the block = 0.8. Hence the height inside water = 3 cm × 0.8 = 2.4 cm. The height outside water = 3 cm – 2.4 = 0.6 cm. Suppose the maximum weight that can be put

without wetting it is W. The block in this case is completely immersed in the water. The volume of the displaced water
= volume of the block = 27×10^{-6} m^3.
Hence, the force of buoyancy
= $(27 \times 10^{-6}$ m$^3) \times 1(1000$ kg/m3$) \times (10$ m/s$^2) = 0.27$ N.
The spring is compressed by 0.6 cm and hence the upward force exerted by the spring
= 100 N/m × 0.6 cm = 0.6 N.
The force of buoyancy and the spring force taken together balance the weight of the block plus the weight W put on the block. The weight of the block is
$W' = (27 \times 10^{-6}$ m$) \times (800$ kg/m$^3) \times (10$ m/s$^2) = 0.22$ N.
Thus, W = 0.27 N + 0.6 N – 0.22 N = 0.65 N.

Example 16. A wooden plank of length 2ℓ m and uniform cross-section is hinged at one end to the bottom of a tank as shown in figure. The tank is filled with water up to a height of ℓ m. The specific gravity of the plank is 0.5. Find the angle θ that the plank makes with the vertical in the equilibrium position. (Exclude the case $\theta = 0$)



Solution : The forces acting on the plank are shown in the figure. The height of water level is ℓ. The length of the plank is 2ℓ. The weight of the plank acts through the centre B of the plank. We have OB = ℓ. The buoyant force F acts through the point A which is the middle point of the dipped part OC of the plank.

We have OA = $\frac{OC}{2} = \frac{\ell}{2\cos\theta}$.

Let the mass per unit length of the plank be ρ.

Its weight mg = $2\ell\rho g$.

The mass of the part OC of the plank = $\left(\frac{\ell}{\cos\theta}\right)\rho$.

The mass of water displaced = $\frac{1}{0.5}\frac{\ell}{\cos\theta}\rho = \frac{2\ell\rho}{\cos\theta}$.

The buoyant force F is, therefore, F = $\frac{2\ell\rho g}{\cos\theta}$.

Now, for equilibrium, the torque of mg about O should balance the torque of F about O.

So, mg (OB) $\sin\theta$ = F(OA) $\sin\theta$

or, $(2\ell\rho)\ell = \left(\frac{2\ell\rho}{\cos\theta}\right)\left(\frac{\ell}{2\cos\theta}\right)$ or, $\cos^2\theta = \frac{1}{2}$ or, $\cos\theta = \frac{1}{\sqrt{2}}$, or, $\theta = 45°$.

Example 17. A cylindrical block of wood of mass m, radius r & density ρ is floating in water with its axis vertical. It is depressed a little and then released. If the motion of the block is simple harmonic. Find its frequency.

Solution : Suppose a height h of the block is dipped in the water in equilibrium position. If r be the radius of the cylindrical block, the volume of the water displaced = $\pi r^2 h$. For floating in equilibrium,

$\pi r^2 h\rho g = W$ ....(i)

where ρ is the density of water and W the weight of the block.

Now suppose during the vertical motion, the block is further dipped through a distance x at some instant. The volume of the displaced water is $\pi r^2 (h + x)$. The forces acting on the block are, the weight W vertically downward and the buoyancy $\pi r^2(h + x) \rho g$ vertically upward.

Net force on the block at displacement x from the equilibrium position is

$F = W - \pi r^2 (h + x)\rho g = W - \pi r^2 h\rho g - \pi r^2 \rho x g$

Using (i) $F = -\pi r^2 \rho g x = -kx$,

where $k = \pi r^2 \rho g$.

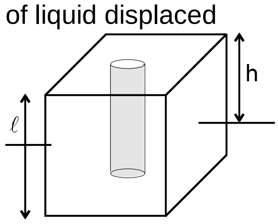
Thus, the block executes SHM with frequency.

$$v = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{\pi r^2 \rho g}{m}}.$$

Example 18. A large block of ice cuboid of height 'ℓ' and density $\rho_{ice} = 0.9\,\rho_w$, has a large vertical hole along its axis. This block is floating in a lake. Find out the length of the rope required to raise a bucket of water through the hole.

Solution : Let area of ice-cuboid excluding hole = A

weight of ice block = weight of liquid displaced



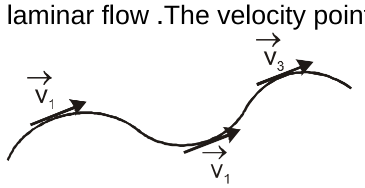
$A \rho_{ice}\, \ell\, g = A \rho_w (\ell - h)\, g$

$$\frac{9\ell}{10} = \ell - h \Rightarrow h = \ell - \frac{9\ell}{10} = \left(\frac{\ell}{10}\right)$$

11. Flow of Fluids

Steady Flow

If the velocity of fluid particles at any point does not vary with time, the flow is said to be steady. Steady flow is also called streamlined or laminar flow .The velocity points may be different. Hence in the figure,

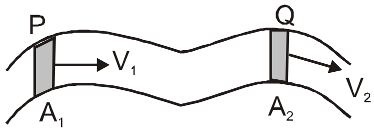


$\vec{v}_1$ = constant, $\vec{v}_2$ = constant, $\vec{v}_3$ = constant

but $\vec{v}_1 \neq \vec{v}_2 \neq \vec{v}_3$

12. Principle of Continuity

It states that, when an incompressible and non-viscous liquid flows in a stream lined motion through a tube of non- uniform cross section ,then the product of the area of cross section and the velocity of flow is same at every point in the tube .



Thus, $A_1 v_1 = A_2 v_2$

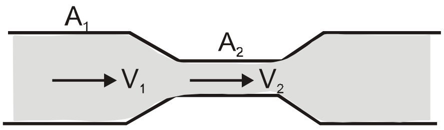
or Av = constant or $v \propto \frac{1}{A}$

This is basically the law of conservation of mass in fluid dynamics.

Proof

Let us consider two cross sections P and Q of area A_1 and A_2 of a tube through which a fluid is flowing. Let v_1 and v_2 be the speeds at these two cross sections. Then being an incompressible fluid, mass of fluid going through P in a time interval Δt = mass of fluid passing through Q in the same interval of time Δt

$\therefore \quad A_1v_1\rho\,\Delta t = A_2v_2\rho\,\Delta t \quad$ or $\quad A_1 v_i = A_2v_2$



Therefore, the velocity of the liquid is smaller in the wider part of the tube and larger in the narrower parts.

or $v_2 > v_1$ as $A_2 < A_1$

Note : The product Av is the volume flow rate $\frac{dV}{dt}$, the rate at which volume crosses a section of the tube. Hence $\frac{dV}{dt}$ = volume flow rate = Av

The mass flow rate is the mass flow per unit time through a cross section. This is equal to density (ρ) times the volume flow rate $\frac{dV}{dt}$.

we can generalize the continuity equation for the case in which the fluid is not incompressible . If ρ_1 and ρ_2 are the densities at sections1 and 2 then,

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

so, this is the continuity equation for a compressible fluid

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13. Energy of a flowing fluid

There are following three types of energies in a flowing fluid.

(i) Pressure energy

if P is the pressure on the area A of a fluid, and the liquid moves through a distance due to this pressure, then pressure energy of liquid = work done

= force ×displacement

= PAl

The volume of the liquid is Al.

$\therefore$ Pressure energy per unit volume of liquid = $\frac{PAl}{Al}$ = P

(ii) Kinetic energy

If a liquid of mass m and volume V is flowing with velocity v, then the kinetic energy is $\frac{1}{2}mv^2$

$\therefore$ kinetic energy per unit volume of liquid = $\frac{1}{2}\left(\frac{m}{v}\right)v^2 = \frac{1}{2}\rho v^2$

Here, ρ is the density of liquid.

(iii) Potential energy

If a liquid of mass m is at a height h from the reference line (h = 0), then its potential energy is mgh.

$\therefore$ Potential energy per unit volume of the liquid = $\left(\frac{m}{v}\right)gh = \rho gh$

14. Bernoulli's Equation

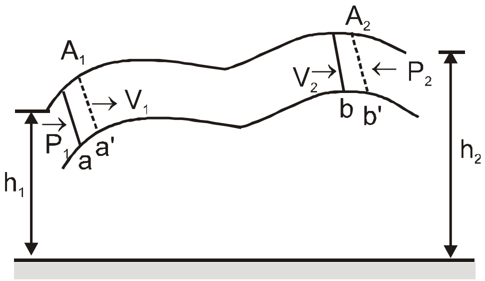
The Bernoulli's equation is **"Sum of total energy per unit volume (pressure + kinetic + potential) is constant for an Ideal fluid".**

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant } (J/m^3)$$

Bernoulli's equation relates the pressure, flow speed and height for flow of an ideal (incompressible and nonviscous) fluid .The pressure of a fluid depends on height as the static situation, and it also depends on the speed of flow.

To derive Bernoulli's equation, we apply the work- energy theorem to the fluid in a section of the fluid element Consider the element of fluid that at some initial time lies between two cross sections a and b. The speeds at the lower and upper ends are v_1 and v_2 In a small time interval, the fluid that is initially at a moves to aa' distance aa' = $ds_1 = v_1$ dt and the fluid that is initially at b moves to b' distance bb' = $ds_2 = v_2 dt$, The cross- section areas at the two ends are A_1 and A_2 as shown. The fluid is incompressible hence, by the continuity equation, the volume of fluid dV passing through and cross-section during time dt is the same.

That is, $dv = A_1\, ds_1 = A_2\, ds_2$



Work done on the Fluid Element

Let us calculate the work done on this element during interval dt. The pressure at the two ends are P_1 and P_2, the force on the cross section at a is $P_1 A_1$ and the force at b is $P_2 A_2$. The net work done dW on the element by the surrounding fluid during this displacement is,

$$dW = P_1A_1\, ds_1 - P_2A_2\, ds_2 = (P_1 - P_2)dV$$

Change in Potential Energy

At the beginning of dt the potential energy for the mass between a and a' is $dmgh_1 = \rho\,(dV)gh_1\ v_1^2$. At the end of dt the potential energy for the mass between b and b' is $(dm)gh_2 = \rho(dv)gh_2$ .The net change in potential energy dU during dt is ,

$$dU = \rho\,(dV)\,g\,(h_2 - h_1)$$

Change in Kinetic Energy

At the beginning of dt the fluid between a and a' has volume A_1ds_1, mass $\rho A_1\, ds_1$ and kinetic energy $\frac{1}{2}\rho(A_1\, ds_1)\ v_1^2$. At the end of dt the fluid between b and b' has kinetic energy $\frac{1}{2}\rho(A_2\, ds_2)\ v_2^2$. The net change in kinetic energy dK during time dt is.

$$dK = \frac{1}{2}\rho\,(dV)(v_2^2 - v_1^2)$$

Combining Eqs. (i),(ii) and (iii) in the energy equation,

$$dW = dK + dU$$

We obtain,

$$(P_1 - P_2)\,dV = \frac{1}{2}\rho dV\,(v_2^2 - v_1^2) + \rho\,(h_2 - h_1)$$

or $$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g\,(h_2 - h_1)$$

This is Bernoulli's equation. It states that the work done on a unit Volume of fluid by the surrounding fluid is equal to the sum of the changes in kinetic and potential energies per unit volume that occur during the flow. We can also express Eq. (iv) in a more convenient form as.

$$P_1 + \rho gh_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho gh_2 + \frac{1}{2}\rho v_2^2$$

The subscripts 1and 2 refer to any two points along the flow tube, so we can also write

$$\rho + \rho gh + \rho v^2 = \text{constant}$$

Note: When the fluid is not moving ($v_1 = 0 = v_2$) Bernoulli' s equation reduces to ,

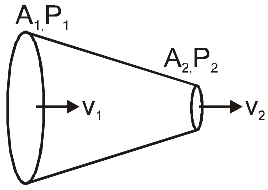
$$P_1 + \rho gh_1 = \rho_2 + \rho gh_2$$

$\therefore$ $$P_1 - P_2 = \rho g(h_2 - h_1)$$

This is the pressure relation we derived for a fluid at rest.

Example 19. Calculate the rate of flow of glycerine of density 1.25×10^3 kg/m^3 through the conical section of a pipe, if the radii of its ends are 0.1m and 0.04 m and the pressure drop across its length is 10 N/m.

Solution : From continuity equation,



$A_1 v_1 = A_2 v_2$

or $\frac{v_1}{v_2} = \frac{A_2}{A_1} = \frac{\pi r_2^2}{\pi r_1^2} = \left(\frac{r_2}{r_1}\right) = \left(\frac{0.04}{0.1}\right)^2 = \frac{4}{25}$

From Bernoulli's equation ,

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

or $v_2^2 - v_1^2 = \frac{2(P_1 - P_2)}{\rho}$

or $v_2^2 - v_1^2 = \frac{2 \times 10}{1.25 \times 10^3} = 1.6 \times 10^{-2}$ m ^{2} /s

Solving Eqs. (i) and (ii), we get

$v_2 \approx 0 .128$ m/s

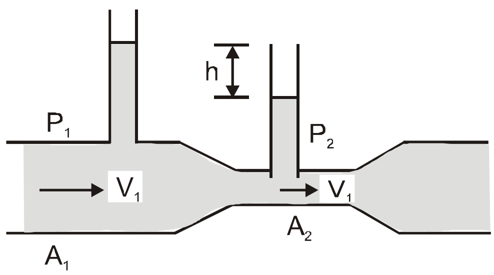
$\therefore$ Rate of volume flow through the tube

$Q = A_2 v_2 = \left(\pi r_2^2\right) v_2 = \pi\,(0.04)^2\,(0.128) = 6.43 \times 10^{-4}$ m^3/s

15. Applications Based on Bernoulli's Equation

(a) Venturimeter

Figure shows a venturimeter used to measure flow speed in a pipe of non- uniform cross-section. We apply Bernoulli's equation to the wide (point 1) and narrow (point 2) parts of the pipe, with $h_1 = h_2$



$P_1 + \frac{1}{2}\rho\ v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

From the continuity equation $v_2 = \frac{A_1 v_1}{A_2}$

Substituting and rearranging, we get

$P_1 - P_2 = \frac{1}{2}\rho v_1^2 \left(\frac{A_1^2}{A_2^2} - 1\right)$

The pressure difference is also equal to pgh, where h is the difference in liquid level in the two tubes. Substituting in Eq. (i) we get

$$v_1 = \sqrt{\frac{2gh}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

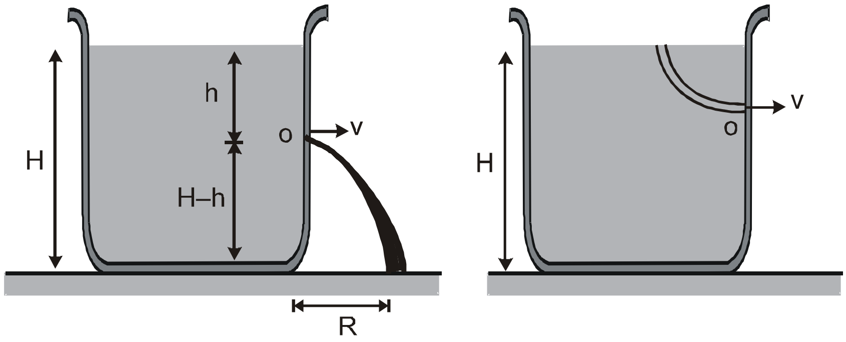
The discharge or volume flow rate can be obtained as,

$$\frac{dV}{dt} = A_1\, v_1 = A_1 \sqrt{\frac{2gh}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

(b) Speed of Efflux

Suppose, the surface of a liquid in a tank is at a height h from the orifice O on its sides, through which the liquid issues out with velocity v. The speed of the liquid coming called the speed of efflux.

If the dimensions of the tank be sufficiently large, the velocity of the liquid at its surface may be taken to be zero and since the pressure there as well as at the orifice O is the same viz atmospheric it plays no part in the flow of the liquid, which thus occurs purely in consequence of the hydrostatic pressure of the liquid itself. So that, considering a tube of flow, starting at the liquid surface and ending at the orifice, as shown in figure .Applying Bernoulli's equation we have



Total energy per unit volume of the liquid at the surface

$= KE + PE + \text{pressure energy} = 0 + \rho gh + P_0$

and total energy per unit volume at the orifice

$= KE + PE + \text{pressure} = \frac{1}{2}\rho v^2 + 0 + P_0$

Since total energy of the liquid must remain constant in steady flow, in accordance with Bernoulli's equation we have

$$\rho gh + P_0 = \frac{1}{2}\rho v^2 + P_0 \quad \text{or} \quad v = \sqrt{2gh}$$

Evangellista Torricelli showed that this velocity is the same as the liquid will attain in falling freely through the height (h) from the surface to the orifice. This is known as Torricelli's theorem and may be stated as. "The velocity of efflux of a liquid issuing out of an orifice is the same as it would attain if allowed to fall freely through the vertical height between the liquid surface and orifice.

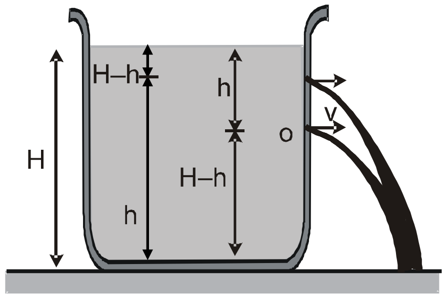
16. Range (R)

Let us find the range R on the ground.

Considering the vertical motion of the liquid, $(H - h) = \frac{1}{2}gt^2$ or $t = \sqrt{\frac{2(H-h)}{g}}$

Now, considering the horizontal motion, $R = vt$ $\quad R = \sqrt{2gh}\left(\sqrt{\frac{2(H-h)}{g}}\right)$ or $R = 2\sqrt{h(H-h)}$

From the expression of R, following conclusions can be drawn,



(i) $R_h = R_{H-h}$

as $R_h = 2\sqrt{h(H-h)}$ and $R_{H-h} = 2\sqrt{h(H-h)}$

This can be maximum at $h = \frac{H}{2}$ and $R_{max} = H$.

Proof : $R^2 = 4(Hh - h^2)$

For R to be maximum, $\frac{dR^2}{dh} = 0$

or $H - 2h = 0$ or $h = \frac{H}{2}$

That is, R is maximum at $h = \frac{H}{2}$ and $R_{max} = 2\sqrt{\frac{H}{2}\left(H - \frac{H}{2}\right)} = H$

Time taken to empty a tank

We are here interested in finding the time required to empty a tank if a hole is made at the bottom of the tank.

Consider a tank filled with a liquid of density ρ upto a height H. A small hole of area of cross section a is made at the bottom of the tank. The area of cross-section of the tank is A.

Let at some instant of time the level of liquid in the tank is y. Velocity of efflux at this instant of time would be

$$v = \sqrt{2gy}$$

Now, at this instant volume of liquid coming out of the hole per second is $\left(\frac{dV_1}{dt}\right)$.

Volume of liquid coming down in the tank per second is $\left(\frac{dV_2}{dt}\right)$.

To calculate time taken to empty a tank $\frac{dV_1}{dt} = \frac{dV_2}{dt}$

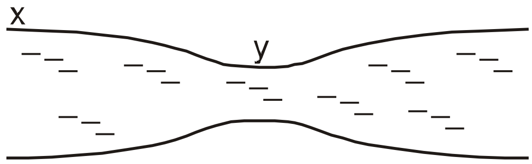
$$\therefore \quad av = A\left(-\frac{dy}{dt}\right) \qquad \therefore \quad a\sqrt{2gy} = A\left(-\frac{dy}{dt}\right)$$

or $$\int_0^t dt = -\frac{A}{a\sqrt{2g}} \int_H^0 y^{-1/2}\, dy$$

$$\therefore \quad t = \frac{2A}{a\sqrt{2g}}\,[\sqrt{y}]_0^H$$

$$\therefore \quad t = \frac{A}{a}\sqrt{\frac{2H}{g}}$$

Example 20. Water flows in a horizontal tube as shown in figure. The pressure of water changes by 600 N/m^2 between x and y where the areas of cross-section are 3cm^2 and 1.5cm^2 respectively. Find the rate of flow of water through the tube.



Solution : Let the velocity at x = v_x and that at y = v_y.

By the equation of continuity, $\frac{v_y}{v_x} = \frac{3cm^2}{1.5cm^2} = 2$.

By Bernoulli's equation,

$P_x + \frac{1}{2}\rho v_x^2 = P_y + \frac{1}{2}\rho v_y^2$

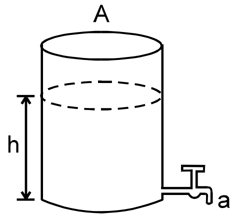
or, $P_x - P_y = \frac{1}{2}\rho(2v_y)^2 - \frac{1}{2}\rho v_y^2 = \frac{3}{2}\rho v_y^2$

or, $600\frac{N}{m^2} = \frac{3}{2}\left(1000\frac{kg}{m^3}\right)v_x^2$

or, $v_x = \sqrt{0.4m^2/s^2} = 0.63$ m/s.

The rate of flow = (3 cm^2) (0.63 m/s) = 189 cm^3/s.

Example 21. A cylindrical container of cross-section area, A is filled up with water upto height 'h'. Water may exit through a tap of cross section area 'a' in the bottom of container. Find out

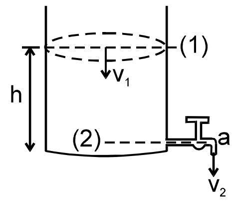


(a) Velocity of water just after opening of tap.

(b) The area of cross-section of water stream coming out of tape at depth h_0 below tap in terms of 'a' just after opening of tap.

(c) Time in which container becomes empty. (Given : $\left(\frac{a}{A}\right)^{1/2} = 0.02$, h = 20 cm, h_0 = 20 cm)

Solution : **(a)** Applying Bernoulli's equation between (1) and (2) -



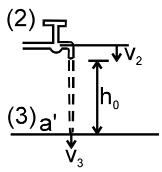
$P_a + \rho gh + \frac{1}{2}\rho v_1^2 = P_a + \frac{1}{2}\rho v_2^2$

Through continuity equation :

$Av_1 = av_2$, $v_1 = \frac{av_2}{a}$ $\quad \rho gh + \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_2^2$

on solving - $v_2 = \sqrt{\frac{2gh}{1-\frac{a^2}{A^2}}}$ = 2m/sec. ....(1)

(b) Applying Bernoulli's equation between (2) and (3)



$\frac{1}{2}\rho v_2^2 + \rho gh_0 = \frac{1}{2}\rho v_3^2$

Through continuity equation –

$av_2 = a' v_3 \Rightarrow \quad v_3 = \frac{av_2}{a'}$

$\Rightarrow \frac{1}{2}\rho v_2^2 + \rho gh_0 = \frac{1}{2}\rho\left(\frac{av_2}{a'}\right)^2$

$\frac{1}{2} \times 2 \times 2 + gh_0 = \frac{1}{2}\left(\frac{a}{a'}\right)^2 \times 2 \times 2$

$\left(\frac{a}{a'}\right)^2 = 1 + \frac{9.8 \times .20}{2} \Rightarrow \left(\frac{a}{a'}\right)^2 = 1.98$

$\Rightarrow \quad a' = \frac{a}{\sqrt{1.98}}$

(c) From (1) at any height 'h' of liquid level in container, the velocity through tap,

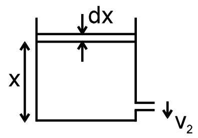
$v = \sqrt{\frac{2gh}{0.98}} = \sqrt{20h}$

we know, volume of liquid coming out of tap = decrease in volume of liquid in container.

For any small time interval 'dt'

$av_2dt = - A \cdot dx$

$a\sqrt{20x}\,dt = - A\,dx \quad \Rightarrow \int_0^t dt = -\frac{A}{a}\int_h^0 \frac{dx}{\sqrt{20x}}$



$t = \frac{A}{a\sqrt{20}}\left[2\sqrt{x}\right]_h^0 \quad \Rightarrow t = \frac{A}{a\sqrt{20}}\,2\sqrt{h}$

$= \frac{A}{a} \times 2 \times \sqrt{\frac{h}{20}} = \frac{2A}{a}\sqrt{\frac{0.20}{20}} = \frac{2A}{a} \times 0.1$

Given $\left(\frac{a}{A}\right)^{1/2} = 0.02$ or $\frac{A}{a} = \frac{1}{0.0004} = 2500$

Thus t = 2 × 2500 × 0.1 = **500 second**.

Example 22. A tank is filled with a liquid upto a height H .A small hole is made at the bottom of this tank Let t_1 be the time taken to empty first half of the tank and t_2 is the time taken to empty rest half of the tank then find $\frac{t_1}{t_2}$.

Solution : Substituting the proper limits in Eq. (i), derived in the theory, we have

$\int_0^{t_1} dt = -\frac{A}{a\sqrt{2g}}\int_H^{H/2} y^{-1/2} dy$

or $t_1 = \frac{2A}{a\sqrt{2g}}[\sqrt{y}]_{H/2}^{H}$ or $t_1 = \frac{2A}{a\sqrt{2g}}\left[\sqrt{H} - \sqrt{\frac{H}{2}}\right]$ or $t_1 = \frac{A}{a}\sqrt{\frac{H}{g}}\left(\sqrt{2}-1\right)$

Similarly $\int_0^{t_2} dt = -\frac{A}{a\sqrt{2g}}\int_{H/2}^{0} y^{-1/2} = dy$

or $t_2 = \frac{A}{a}\sqrt{\frac{H}{g}}$

We get

$\frac{t_1}{t_2} = \sqrt{2} - 1$ or $\frac{t_1}{t_2} = 0.414$

Note : From here we see that $t_1 < t_2$ This is because initially the pressure is high and the liquid comes out with