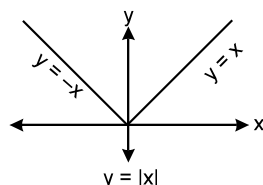


Fundamentals of Mathematics-II

He is unworthy of the name of man who is ignorant of the fact that the diagonal of square is incommensurable with its sidePlato

Absolute value function / modulus function :

The symbol of modulus function is $f(x) = |x|$ and is defined as: $y = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$.



Properties of modulus : For any $a, b \in \mathbb{R}$

- (i) $|a| \geq 0$ (ii) $|a| = |-a|$
- (iii) $|a| \geq a, |a| \geq -a$ (iv) $|ab| = |a| |b|$
- (v) $\left| \frac{a}{b} \right| = \frac{|a|}{|b|}$ (vi) $|a + b| \leq |a| + |b|$; Equality holds when $ab \geq 0$
- (vii) $|a - b| \geq ||a| - |b||$; Equality holds when $ab \geq 0$

Example # 1 : Solve the following linear equations

- (i) $x|x| = 4$ (ii) $|x - 3| + 2|x + 1| = 4$

Solution :

- (i) $x|x| = 4$
 If $x > 0$
 $\therefore x^2 = 4 \Rightarrow x = \pm 2$
 $\therefore x = 2$ ($\because x \geq 0$)
 If $x < 0 \Rightarrow -x^2 = 4 \Rightarrow x^2 = -4$ which is not possible
- (ii) $|x - 3| + 2|x + 1| = 4$

case I : If $x \leq -1$

$$\begin{aligned} \therefore -(x - 3) - 2(x + 1) &= 4 \\ \Rightarrow -x + 3 - 2x - 2 &= 4 \Rightarrow -3x + 1 = 4 \\ \Rightarrow -3x &= 3 \Rightarrow x = -1 \end{aligned}$$

case II : If $-1 < x \leq 3$

$$\begin{aligned} \therefore -(x - 3) + 2(x + 1) &= 4 \\ \Rightarrow -x + 3 + 2x + 2 &= 4 \Rightarrow x = -1 \text{ which is not possible} \end{aligned}$$

case III : If $x > 3$

$$\begin{aligned} x - 3 + 2(x + 1) &= 4 \\ 3x - 1 &= 4 \Rightarrow x = 5/3 \text{ which is not possible} \therefore x = -1 \text{ Ans.} \end{aligned}$$

Rational function :

A rational function is a function of the form, $y = f(x) = \frac{g(x)}{h(x)}$, where $g(x)$ & $h(x)$ are polynomial functions.

Irrational function :

An irrational function is a function $y = f(x)$ in which the operations of addition, subtraction, multiplication, division and raising to a fractional power are used.

For example $y = \frac{x^3 + x^{1/3}}{2x + \sqrt{x}}$ is an irrational function

- (a) The equation $\sqrt{f(x)} = g(x)$, is equivalent to the following system
 $f(x) = g^2(x)$ & $g(x) \geq 0$
- (b) The inequation $\sqrt{f(x)} < g(x)$, is equivalent to the following system

- $f(x) < g^2(x)$ & $f(x) \geq 0$ & $g(x) \geq 0$
 (c) The inequation $\sqrt{f(x)} > g(x)$, is equivalent to the following system
 $g(x) \leq 0$ & $f(x) \geq 0$ or $g(x) \geq 0$ & $f(x) > g^2(x)$

Example # 2 : Solve : $x + 2 > 2\sqrt{1-x^2}$

Solution : $4(1-x^2) < (x+2)^2$ and $x+2 \geq 0$ & $1-x^2 \geq 0$

$$x \in \left(-\infty, -\frac{4}{5}\right) \cup (0, \infty) \quad \dots(1)$$

$$x \in [-2, \infty) \quad \dots(2)$$

$$x \in [-1, 1] \quad \dots(3)$$

$$(1) \cap (2) \cap (3)$$

$$\left[-1, -\frac{4}{5}\right) \cup (0, 1]$$

Self Practice Problem :

(1) $\sqrt{2x^2 + x - 6} < x$

(2) $\sqrt{5-x} > x+1$

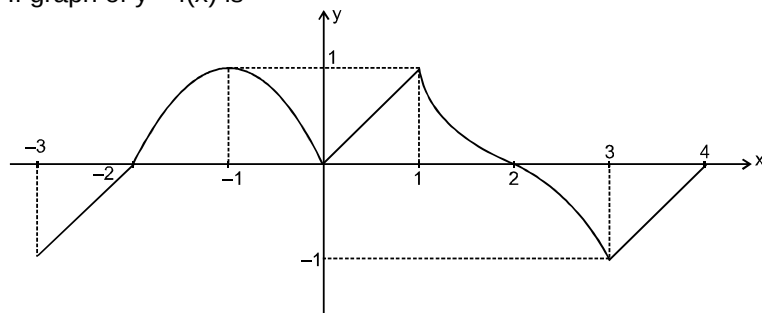
(3) $x+3 + \sqrt{x^2+4x-5} > 0$

(4) $\sqrt{x} - \sqrt{4-x} \geq 1$

Ans. (1) $\left[\frac{3}{2}, 2\right)$ (2) $(-\infty, 1)$ (3) $(-\infty, -1] \cup [5, \infty)$ (4) $\left[\frac{4+\sqrt{7}}{2}, 4\right]$

Graphs Related to modulus :

If graph of $y = f(x)$ is



then draw graph of

(a) $y = -f(x)$

(b) $y = f(-x)$

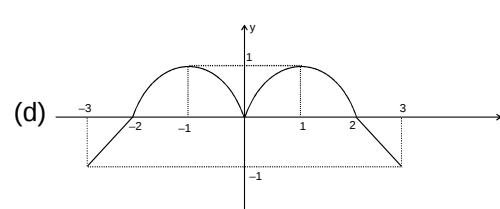
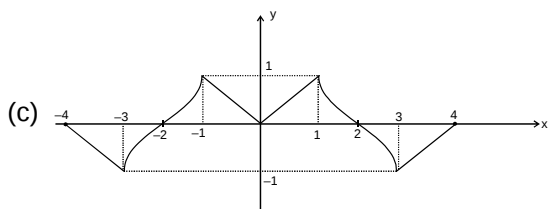
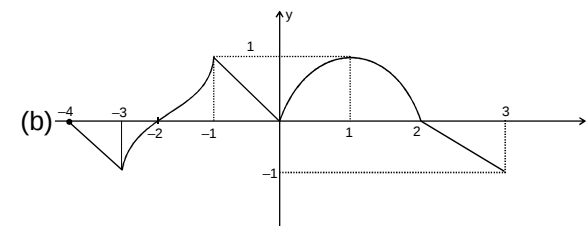
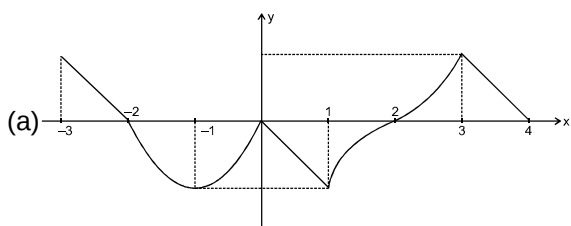
(c) $y = f(|x|)$

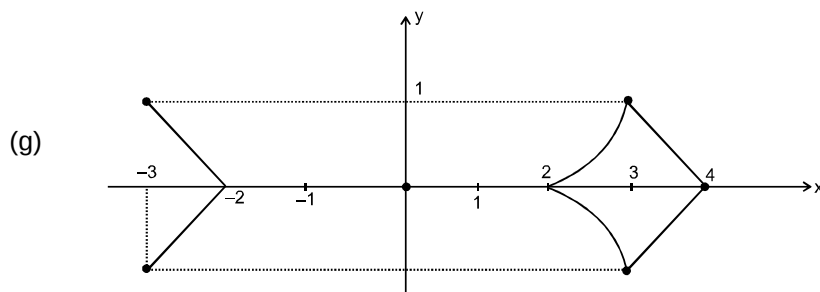
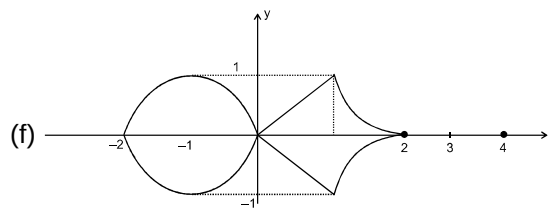
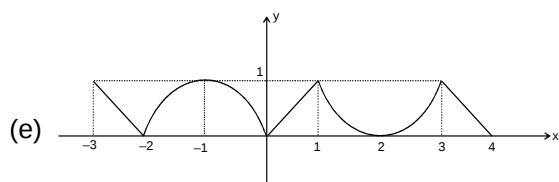
(d) $y = f(-|x|)$

(e) $y = |f(x)|$

(f) $|y| = f(x)$

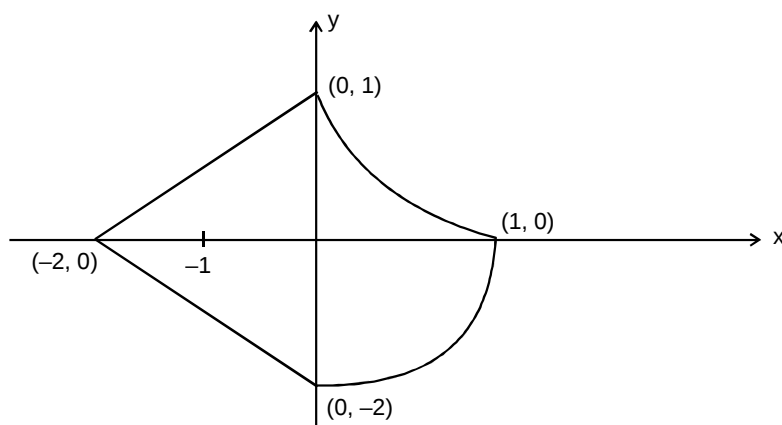
(g) $|y| = -f(x)$





Graphical Trasformation :

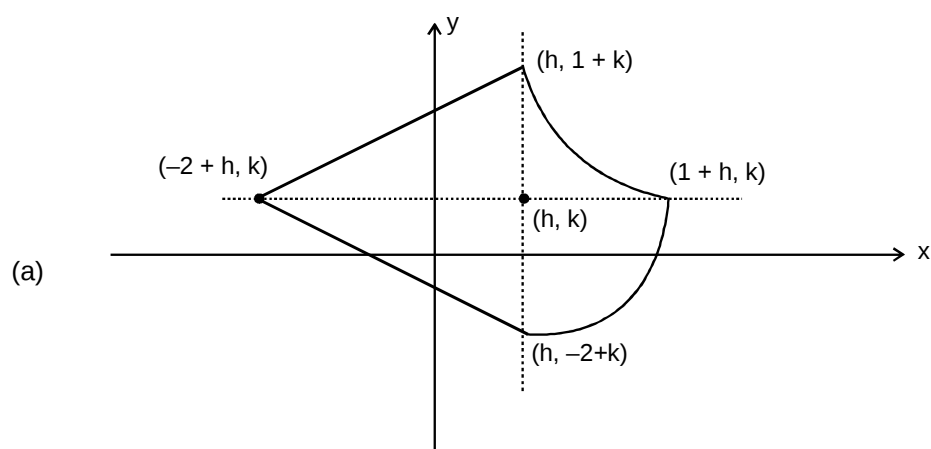
If graph of $y = f(x)$ is

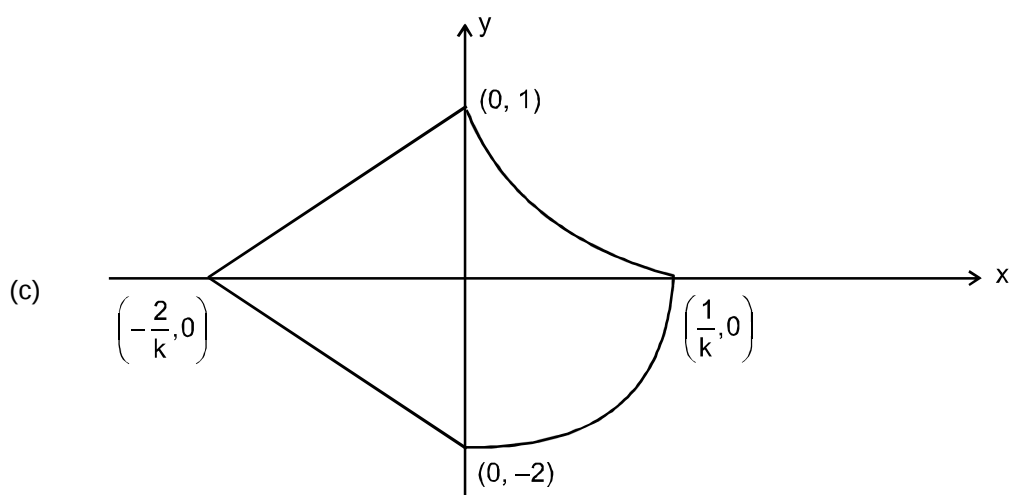
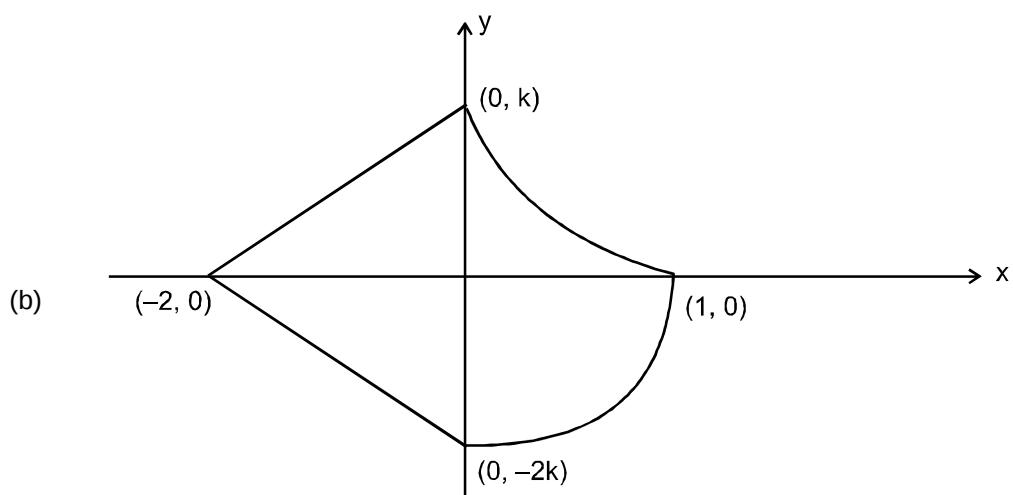


then graph of (a) $y - k = f(x - h)$

(b) $y = kf(x)$, ($k > 0$)

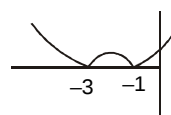
(c) $y = f(kx)$, ($k > 0$)





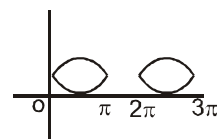
Example # 3 : $y = |x^2 + 4x + 3|$

Solution :



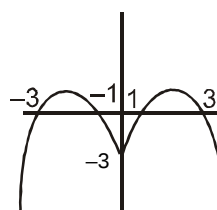
Example # 4 : $|y-1| = \sin x$

Solution :



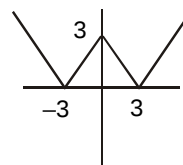
Example # 5 : $y = -x^2 + 4|x| - 3$

Solution :



Example # 6 : $y = ||x| - 3|$

Solution :



Example # 7 : $y = \sin\left(\frac{x}{3}\right)$

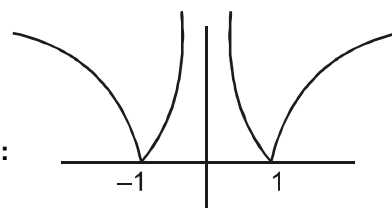
Solution : period is 6π

Example # 8 : $y = |\sin x - 3|$

Solution : Graphical Transformation

Example # 9 : $y = |-\ln|-x||$

Solution :



Example # 10 : $|y| = x^2 - 3x + 2$

Solution :

