INTRODUCTION TO ALGEBRA

INTRODUCTION

In Arithmetic, numerals 1, 2, 3, 4,..... etc. and four fundamental operation : addition, subtraction, multiplication and division are used to deal with various problems. In Algebra, in addition to numerals, we use letters such as x, y, z in various situations to solve the problems.

MATCHSTICK PATTERN

Salman and Aamir are making pattern with matchsticks. They decide to make simple patterns of the letters of the english alphabet. Salman takes two matchsticks and form the letter L as shown in figure 1. The Aamir also picks two sticks form another letter L and puts it next to the one made by salman as shown in figure 2. The Salman adds one more L and this goes on as shown in figure 3.

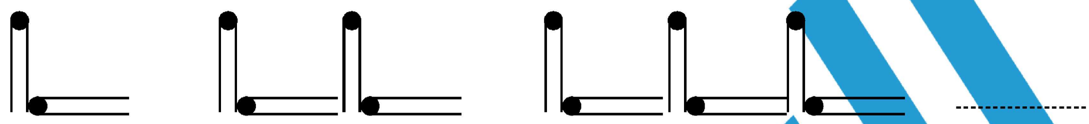

Figure 1 Figure 2 Figure 3

If we tabulate this result we obtained the following table as given below

Number of Ls formed	1	2	3	4	5	6	7	8	....	....
Number of matchsticks required	2	4	6	8	10	12	14	16	....	....

Here we observe that number of matchstick required is twice the number's of L

So, we can say number of matchstick required = 2 × n, where n represent the number of L's

Consider the following statements :

I think of a number and when I subtract 9 from it the result is 23.

If $\square$ is used to represent the number I think of, then the above statement using mathematical symbols, is simply written as $\square - 9 = 23$.

Ex. Use $\square$, Δ etc. and mathematical symbols and rewrite the following statements :

(a) I think of a number, subtract 3 from it and the result is 34.

(b) I think of two numbers, add these numbers, multiply the result by 2 to get the final answer as 14.

(c) I think of two numbers. Twice difference is added by 3 gives result 15.

Sol. (a) $\square - 3 = 34$

(b) $2 \times (\square + \Delta) = 14$

(c) $2 \times (\square - \Delta) + 3 = 15$

Ex. Rewrite each of the following statements without using symbols, beginning each statement with : I think of .....

(a) $\square + 9 = 28$

(b) $2 \times \square - 23 < 25$

(c) $(\square + \Delta) \div 5 = 25$

Sol. (a) I think of a number, when 9 is added to it gives result 28.

(b) I think of a number, when 23 is subtracted from the twice the number gives the result is less than 25.

(c) I think of two numbers, when their sum is divided by 5 gives quotient 25.

LITERAL NUMBERS

We have mentioned earlier that the letters represent the numbers. These letters are called literal numbers and obey all the rules of arithmetic.

NOTE

$5 \times p \times q = 5pq$. 5, p and q are factors of 5pq, 5 is a numerical factor and p, q are literal factors.

Ex. Give expressions for the following cases.

(a) Rita scores x marks in Maths and 46 marks in English. What is her total score in Maths and English.

(b) The difference of x and 9 where $x > 9$.

(c) The product of a and b added to the difference of a and b$(a > b)$.

(d) One-half of a multiplied by the sum of x and y.

(e) The total distance travelled by a car in x hours at a constant speed of y km/h.

(f) The total number of eggs in n cartons if each carton contains k eggs.

Sol. (a) Total marks = $x + 46$

(b) $x - 9$

(c) $ab + a - b$

(d) $\frac{a}{2} \times (x + y)$

(e) distance = $(x \times y)$ Km.

(f) Number of eggs = $(n \times k)$ eggs

Ex. Ali is x years old. Express the following in algebraic form :

(a) Ali's age 5 years ago

(b) Ali's age 2 years from now

(c) 4 times Ali's age 3 years hence.

(d) the present age of Ali's aunt who is four times as old as Ali will be 5 years from today.

(e) The present age of Ali's father who is 5 times as old as Ali was 3 years ago.

Sol. (a) Ali's age 5 years ago = $(x - 5)$ years

(b) Ali's age 2 years from now = $(x + 2)$ years

(c) Ali's age 3 years hence = $(x + 3)$ years 4 times Ali's age 3 years hence = $4(x + 3)$ years

(d) Ali's age after 5 years = $(x + 5)$ years Ali's aunt age = $4(x + 5)$ years

(e) Ali's age before 3 years = $(x - 3)$ years Ali's father age = $5(x - 3)$ years

POWERS OF LITERAL NUMBERS

We have read earlier that $2 \times 2 \times 2 = 2^3$ and $(-3) \times (-3) = (-3)^2$

Similarly, $a \times a \times a = a^3$ and $(-y) \times (-y) \times (-y) \times (-y) = (-y)^4$

a^3 is read as 'a to the power three' or 'a raised to the power three' or 'a cube' or 'third power of a' and $(-y)^4$ is read as '– y to the power four' or '– y raised to the power four' or fourth power of – y'.

In a^3, a is called base and 3 is called exponent or index.

COEFFICIENT

The number expressed in figures or symbols, standing before an algebraic term as a multiplier is called its coefficient. Thus in 3abc, 3 is the coefficient of abc, 3a is the coefficient of bc and 3ab is the coefficient of c. When one of the factors is an Airthmetic number it is always written first and is called a numerical coefficient. Thus in 3xyz, 3 is the numerical coefficient of xyz. When the coefficient is expressed in letter, it is called a literal coefficient is expressed in letter, it is called a literal coefficient. Thus in axy a is the literal coefficient of xy. When the coefficient is 1 or – 1, the number 1 is usually omitted. Thus 1 x is written as x and – 1 x as – x, 1xy is written as xy and – 1 xy as – xy.

VARIABLES AND CONSTANTS

In algebra we come across two types of symbols, namely variables and constants. A symbol having a fixed value is called a constant whereas a symbol which takes on various numerical values is called a variable.

For example, the perimeter p of rectangle is given by the formula $p = 2(l + b)$, where l and b are its length and breadth. Here 2 is a fixed number and hence a constant but the literal numbers p, l and b depends on different sizes of the rectangle and hence they are variables.

Ex. Write down the coefficient of :

(a) x in 3xy (b) abc in –5abc (c) y in 2xyz (d) a^2 in $-a^2bc$

Sol. (a) 3y (b) –5 (c) 2xz (d) – bc

Ex. Write down the numerical coefficinet in each of the following :

(a) 5 ab (b) – 3xyz (c) px (d) – y

Sol. (a) 5 (b) – 3 (c) 1 (d) – 1

ALGEBRAIC EXPRESSION

Any combination of letters or of numerals and letters connected by the symbols +, –, ×, ÷ is called an Algebraic expression. For example, 2x – 3y + 5z is an algebraic expression.

The several parts of an expression connected by the signs + and – are called the terms of the expression.

NOTE :

Only the signs + and – separate the terms of an expression. 5xy is one term whereas x – y are two terms x and –y.

An expression consisting of one term is called a monomial. 2x, 5ab, – 7xy, 20 are all examples of monomials. An expression consisting of two terms is called a Binomial. Some example of the binomials are 2x – 3y, 5a – 2b, p + 2q etc. An expression consisting of three terms is called a trinomial. The expresion consisiting of several terms is called multinomial or polynomial expression.

Ex. Ruchika buys 5 copies for Maths, 2x copies for English and y^2 copies for Hindi. Express the total number of copies she buys as an algebraic expression.

Sol. Number of copies for Maths = 5

Number of copies for English = 2x

Number of copies for Hindi = y^2

Total number of copies = $5 + 2 + y^2$

Operation	+	–	×	÷
Algebraic Expression	n + 14	n – 5	12n	$4 + y$ or $\boxed{\frac{4}{y}}$
Verbal Expression	* 14 added to n * n plus 14 * Sum of n and 14 * 14 more than n * n increased by 14	* 5 subtracted from n * the difference of n and 5 * n decreased by 5 * 5 less than n * n less 5 * take away 5 from n	* 12 times n * product of 12 and n * n multiplled by 12 * 12 groups of n	* 4 divided by y * quotient of 4 and y * 4 divided into y groups

Ex. Write the following, using symbols :

(i) a increased by twice b

(ii) three times the difference of 30 and c

(iii) 70 increased by the quotient of x and y

(iv) length in centimetres that is 4 cm longer than y metres

Sol. (i) a + 2b

(ii) 3(30 – c)

(iii) $70 + \frac{x}{y}$

(iv) 100y + 4 ($\because$ y metres = 100 × y = 100 y cm)

Ex. Write down separately the terms of the following algebraic expression :

(a) $3x - 4y$

(b) $2ab + 4ac^2 - 6z$

(c) $7xyz + 2yz - 8x^2y^3$

(d) $-2pq^2 + 7qr^4 - 3p + 4p^2q^3r^4$

Sol. (a) $3x, -4y$

(b) $2ab, 4ac^2, -6z$

(c) $7xyz, 2yz, -8x^2y^3$

(d) $-2pq^2, 7qr^4, -3p, 4p^2q^3r^4$

Ex. Write down the algebraic expression whose terms are given below :

(a) $2a, -3b, -4c$

(b) $5bc^2, -2ab, 7a^2c$

(c) $3t^2uw^3, 7t^2w^2 - 2p^2q + 7$

Sol. (a) $2a - 3b - 4c$

(b) $5bc^2 - 2ab + 7a^2c$

(c) $3t^2uw^3 + 7t^2w^2 - 2p^2q + 7$

Ex. Separate monomials, binomials and trinomials from the following :

$5x^2 - 3xy$, $2x^4$, $3x - 2y + 4z$, $-3y^2$, $5x^3y^2 + 4y^2z - z^5$, $ax - by + cz^2$, $pq + rst$

Sol. Monomials are $2x^4, -3y^2$

Binomials are $5x^2 - 3xy$, $pq + rst$

Trinomials are : $3x - 2y + 4z$, $5x^3y^2 + 4y^2z - z^5$, $ax - by + cz^2$

Ex. If a = 2, b = 3, c = 4, find the value of :

(a) $a + b + c$

(b) $2c - b - a$

(c) $3a - b + 2c$

(d) $a^2 - b^2 + c^2$

(e) $ab - 3abc - 2ac$

(f) $a^2b + bc - 3c^3 - 2abc$

Sol. (a) $a + b + c = 2 + 3 + 4 = 9$

(b) $2c - b - a = 2 \times 4 - 3 - 2$

$= 8 - 3 - 2 = 3$

(c) $3a - b + 2c = 3 \times 2 - 3 + 2 \times 4$

$= 6 - 3 + 8 = 11$

(d) $a^2 - b^2 + c^2 = 2^2 - 3^2 + 4^2$

$= 4 - 9 + 16 = 11$

(e) $ab - 3abc - 2ac = 2 \times 3 - 3 \times 2 \times 3 \times 4 - 2 \times 2 \times 4$

$= 6 - 72 - 16 = -82$

(f) $a^2b + bc - 3c^3 - 2abc$

$= 2^2 \times 3 + 3 \times 4 - 3 \times 4^3 - 2 \times 2 \times 3 \times 4$

$= 12 + 12 - 192 - 48 = -216$

ALGEBRAIC EQUATIONS

An equation is a mathematical statement equating two quantities. The expressions on either side of the equal sign (=) are called members of the equation. In an equation, the value of the quantity which is not known is referred to as the unknown member or the unknown. Here are some examples of equations.

$2x + 3 = 11$ $x - 3 = 8$ $y + 3 = 21$

Ex. Rewrite the following statements by using symbols wherever needed.

(i) a exceeds b by 10

(ii) Twice the product of p and q divided by r.

(iii) x is not equal to two times y.

(iv) Four times m is greater than seven.

(v) The excess of 15 over 10 is 5.

(vi) Since two times x equal sixteen, therefore x is equal to eight.

(vii) Since four into y equal forty, therefore y equals ten.

(viii) Twice the product of p and q upon the sum of a and b equal five.

(ix) The difference of x and y is less the sum of two and ten.

(x) Nine times two is greater than ten.

Sol. (i) $a - b = 10$ (ii) $\frac{2pq}{r}$

(iii) $x \neq 2y$ (iv) $4m > 7$

(v) $15 - 10 = 5$ (vi) Since $2x = 16 \therefore x = 8$

(vii) Since $4y = 40 \therefore y = 10$ (viii) $\frac{2pq}{a+b} = 5$

(ix) $x - y < 2 + 10$ (x) $9 \times 2 > 10$

SOLVING AN EQUATION

To solve an equation is to determine the value (s) of the variable (or unknown) that balances the equation. That value(s) is called the root (s) of the equation or solution of the equation.

Let us take an example.

$x + 7 = 15$;

We have to find the value of x which will satisfy the equation. And we observe that if we put $x = 8$ in this equation it will satisfy the equation. So $x = 8$ is the solution or root of this equation.

SOLUTION OF AN EQUATION BY TRIAL AND ERROR

One of the simplest ways of solving an equation is by the trial-and-error method. In this, a guess is made about the value of x, and this value is then substituted in the equation to check if it is the root of the equation.

Consider the following example : $4x + 3 = 23$

Our equation is $4x + 3 = 23$. So we substitute different values for x and try to find out which value of x will satisfy the equation. Make a chart as follow.

x	L.H.S.	R.H.S. = 23
1	$4 \times 1 + 3 = 7$	23
2	$4 \times 2 + 3 = 11$	23
3	$4 \times 3 + 3 = 15$	23
4	$4 \times 4 + 3 = 19$	23
5	$4 \times 5 + 3 = 23$	23

When $x = 5$, $4x + 3 = 23$ so the root of the equation or the solution of the equation $4x + 3 = 23$ is 5.

Ex. Determine if 3 is the root of the equation $5x - 10 = 5$.

Sol. If we put $x = 3$, then L.H.S. $= 5x - 10$

$= 5 \times 3 - 10$

$= 15 - 10 = 5$

R.H.S. = 5

$\therefore$ L.H.S. = R.H.S.

Thus, 3 is a root of the given equation.

Ex. Express the following as algebraic equation and solve.

(a) Twice a number increased by 7 is 13. What is the number?

(b) Seven times a number decreased by 4 is 10. Find the number

Sol. (a) Let the number be x

$\therefore$ Twice the number = $2x$

$\therefore 2x + 7 = 13$

x	L.H.S. = 2x + 7	R.H.S. = 13
1	$2 \times 1 + 7 = 9$	13
2	$2 \times 2 + 7 = 11$	13
3	$2 \times 3 + 7 = 13$	13

Thus for $x = 3$, L.H.S. = R.H.S.

So the required number is 3.

(b) Let the numbers be x.

$\therefore 7x - 4 = 10$

x	L.H.S. = 7x – 4	R.H.S. = 10
x = 1	7 × 1 – 4 = 3	10
x = 2	7 × 2 – 4 = 10	10

Here for x = 2, L.H.S. = R.H.S.

∴ The required number is 2.

Ex. If 20 is subtracted from a number, is result is 45.

Convert this statement into an algebraic equation.

Sol. Let us suppose that x is the unknown number,

Then x – 20 stands for 20 subtracted from the number x. This is equal to 45.

Hence, x – 20 = 45

Once you convert statement into an algebraic equation, it is easier to solve and find the root.

Ex. Sunny is twice as old as Manoj. Convent this statement into an algebraic equation.

Sol. Let sunny's age be s and Manoj's age be m. Twice Manoj's age is 2m.

Hence, the equation is s = 2m.

SYSTEMATIC METHOD

A much better method of solving an equation is the systematic method as the trial and error method could take a lot of time.

Property -1 : We can add the same number to both sides of the equation;

Ex. Solve the equation x – 7 = –2 and check the result.

Sol. We have, x – 7 = –2.

In order to solve this equation, we have to get x by itself on the L.H.S., We need to shift – 7. This can be done by adding 7 to both sides of the given equation.

Thus,

$x - 7 = -2$

$\Rightarrow$ $x - 7 + 7 = -2 + 7$ [Adding 7 to both sides]

$\Rightarrow$ $x + 0 = 5$ [$\because$ $-7 + 7 = 0$ and $-2 + 7 = 5$]

$\Rightarrow$ $x = 5$

Thus, x = 5 is the solution of the given equation.

L.H.S. = 5 – 7 = –2 and R.H.S. = – 2

Thus, when x = 5, we have L.H.S. = R.H.S.

Property-2 : We can subtract the same number from both sides of the equation.

Ex. Solve the equation x + 4 = –2 and check the result

Sol. In order to solve this equation, we have to obtain x by itself on L.H.S. To get x by itself on L.H.S., we need to shift 4. This can be done by subtracting 4 from both sides of the given equation.

Thus, $x + 4 = -2$

$\Rightarrow$ $x + 4 - 4 = -2 - 4$ [Subtracting 4 from both sides]

$\Rightarrow$ $x + 0 = -6$ [$\because$ $4 - 4 = 0$ and $-2 - 4 = -6$]

$\Rightarrow$ $x = -6$

Thus, x = –6 is the solution of the given equation.

Property-3 : We can multiply both sides of the equation by same non-zero number.

Ex. Solve the equation $\frac{y}{12}$ = 48 and check the result.

Sol. In order to solve this equation, we have to get y by itself on L.H.S. To get y by itself on L.H.S., we have to remove 12 from L.H.S. This can be done by multiplying both sides of the equation by 12 thus, we have

$$\frac{y}{12} = 48$$

$$\Rightarrow \quad \frac{y}{12} \times 12 = 48 \times 12 \quad \text{[Multiplying both sides by 12]}$$

$$\Rightarrow \quad y = 576$$

Check : Putting, y = 576 in the given equation, we get L.H.S. = $\frac{576}{12}$ = 48 and R.H.S. = 48.

Thus, for y = 567, we have L.H.S. = R.H.S.

(iv) We can divide both sides of the equation by the same non-zero number.

Ex. Solve the equation $\frac{2}{3}$ x = 18 and check the result.

Sol. We have,

$$\frac{2}{3}x = 18$$

Multiplying both sides by $\frac{3}{2}$

$$\Rightarrow \quad \frac{2}{3} \times \frac{3}{2} \times x = \frac{3}{2} \times 18$$

Thus, for x = 27 is solution of given equation.

Property -4 : In an equation, we can drop a term from one side and put it on the other side with the opposite sign. This process is known as transposition.

Ex. Solve : 3(x + 3) – 2 (x – 1) = 5 (x – 5).

Sol. We have,

$$3(x + 3) - 2(x - 1) = 5(x - 5)$$

$$\Rightarrow \quad 3x + 9 - 2x + 2 = 5x - 25 \quad \text{[Expanding brackets on both side]}$$

$$\Rightarrow \quad 3x - 2x + 9 + 2 = 5x - 25$$

$$\Rightarrow \quad x + 11 = 5x - 25 \quad \text{[Taking 5x to the L.H.S. and 11 to the R.H.S.]}$$

$$\Rightarrow \quad -4x = -36$$

$$\therefore \quad x = 9$$