INTEGRALS

INTEGRATION BY PARTS

Integration by parts :

Product of two functions f(x) and g(x) can be integrate using formula :

$$\int (f(x)\, g(x))\ dx = f(x) \int (g(x))\ dx - \int \left(\frac{d}{dx}(f(x)) \int (g(x))\, dx \right) dx$$

(i) when you find integral $\int g(x)\,dx$ then it will not contain arbitarary constant.

(ii) $\int g(x)\,dx$ should be taken as same at both places.

(iii) The choice of f(x) and g(x) can be decided by ILATE guideline.

the function will come later is taken an integral function (g(x)).

I $\rightarrow$ Inverse function

L $\rightarrow$ Logarithmic function

A $\rightarrow$ Algebraic function

T $\rightarrow$ Trigonometric function

E $\rightarrow$ Exponential function

Ex.1 Evaluate : $\int x \log_e x\, dx$

Sol. Let I = $\int x \log_e x\, dx$

$$\log_e x \int x\, dx - \int \left\{ \frac{d}{dx}(\log x) \int x dx \right\} dx$$

$$= \log_e x \left(\frac{x^2}{2} \right) - \int \frac{1}{x} \times \frac{x^2}{2}\ dx \quad = \frac{x^2}{2} \log_e x - \frac{x^2}{4} + C$$

Ex.2 Evaluate : $\int x\, \ell n(1+x)\, dx$

Sol. Let I = $\int x\, \ell n(1+x)\, dx$

$$= \lambda n\ (x+1) \cdot \frac{x^2}{2} - \int \frac{1}{x+1} \cdot \frac{x^2}{2}\ dx$$

$= \frac{x^2}{2} \lambda n\,(x+1) - \frac{1}{2}\int \frac{x^2}{x+1}\,dx = \frac{x^2}{2}\ \lambda n\,(x+1) - \frac{1}{2}\int \frac{x^2-1+1}{x+1}dx$

$= \frac{x^2}{2}\ \lambda n\,(x+1) - \frac{1}{2}\int\left(\frac{x^2-1}{x+1}+\frac{1}{x+1}\right)\ dx$

$= \frac{x^2}{2}\ \lambda n\,(x+1) - \frac{1}{2}\int\left((x-1)+\frac{1}{x+1}\right)dx$

$= \frac{x^2}{2}\ \lambda n\,(x+1) - \frac{1}{2}\left[\frac{x^2}{2} - x + \ell n\,|x+1|\right] + C$

Ex.3 Evaluate : $\int e^{2x}\sin 2x\,dx\ \ dx$

Sol. We know that $\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}$ (asinbx – bcosbx) + C

a = 2 and b = 2 = $\frac{e^{2x}}{8}$ (2sin2x – 2cos2x) + C

Note : (i) $\int e^{x}\,[f(x) + f'(x)]\,dx = e^{x}\cdot f(x) + C$

(ii) $\int [f(x) + xf'(x)]\,dx = x\,f(x) + C$

Ex.4 Evaluate : $\int\left[\ell n\ (\ell nx) + \frac{1}{(\ell nx)^2}\right] dx$

Sol. Let $I = \int\left[\ell n\ (\ell nx) + \frac{1}{(\ell nx)^2}\right]\ dx$ {put $x = e^t$ $dx = e^t\,dt$}

$\therefore$ $I = \int e^t\left(\ell nt\ +\ \frac{1}{t^2}\right)\ dt \int e^t\left(\ell nt\ -\ \frac{1}{t}+\frac{1}{t}+\frac{1}{t^2}\right) =\ dt$

$= e^t\left(\ell nt\ -\ \frac{1}{t}\right) + C$

$= x\left[\ell n\ (\ell nx)\ -\ \frac{1}{\ell nx}\right] + C$