INTEGRALS

INTEGRALS BY SUBSTITUTION

Evaluation of Definite Integral by change of variable

When the variable (x) in a definite integral is changed with (t) then, substitution affects at three places

(1) Integrand is changed

(2) d(x) is changed with d f(t)

(3) Upper and lower limits are changed

Ex.1 Evaluate $\int_0^{\infty} \frac{x}{(1+x)\left(1+x^2\right)} dx$

Sol. Substitute $x = \tan\theta, \quad dx = \sec^2\theta d\theta$ (limits of integration are changed to 0 and $\pi/2$)

$$\therefore \quad I = \int_0^{\infty} \frac{x}{(1+x)\left(1+x^2\right)} dx$$

$$= \int_0^{\pi/2} \frac{\tan\theta}{(1+\tan\theta)\sec^2\theta} \sec^2\theta d\theta$$

$$= \int_0^{\pi/2} \frac{\tan\theta}{1+\tan\theta} d\theta$$

$$\Rightarrow \quad I = \int_0^{\pi/2} \frac{\sin\theta}{\sin\theta + \cos\theta} d\theta$$

$$\Rightarrow \quad I = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2}-\theta\right)}{\sin\left(\frac{\pi}{2}-\theta\right) + \cos\theta\left(\frac{\pi}{2}-\theta\right)} d\theta$$

$$\Rightarrow \quad I = \int_0^{\pi/2} \frac{\cos\theta}{\cos\theta + \sin\theta} d\theta$$

Adding (1) and (2), we get

$$2l = \int_0^{\pi/2} d\theta = \pi / 2$$

$$\Rightarrow \quad l = \frac{\pi}{4}$$

Some more properties of Definite Integral

Property 9 If $f(x) \geq 0 \forall x \in [a,b]$, then $\int_a^b f(x)dx \geq 0$

Property10 If $f(x)$ is an odd function of x, then $\int_0^{\hat{f}} f(t)dt$ is an even function of x.

Ex.8 Show that if f(t) is an odd function then $F(x) = \int_a^x$ dt is an even function.

Sol. We have, $F(x) = \int_a^0 f(t)dt + \int_0^x f(t)dt$

$$\Rightarrow F(-x) = \int_a^0 f(t)dt + \int_0^{-x} f(t)dt$$

Substituting $t = -u$ in the 2^{nd} integral

$$F(-x) = \int_a^0 f(t)dt + \int_0^x f(-u)(-du)$$

$$= \int_a^0 f(t)dt + \int_0^x f(u)(du) \quad (\because f(-u) = -f(u))$$

$$= \int_a^x f(t)dt + F(x)$$

Therefore $F(x)$ is an even function