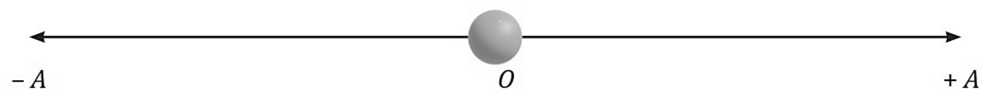
OSCILLATIONS

FORCE LAW FOR SIMPLE HARMONIC MOTION

STANDARD EQUATION OF SHM

Let us examine a particle engaged in periodic and oscillatory motions, moving within the range of -A to A, as illustrated in the diagram.



The force acting on the particle is given by,
F = - kx .....(i)

Here, the presence of the negative sign signifies the restorative characteristic of the force. When the particle's mass is denoted by 'm,' the equation (i) can be expressed as follows.

$ma = -kx$

$a = -\left(\frac{k}{m}\right)x$

Where is the centripetal acceleration of the corresponding UCM

$v\frac{dv}{dx} = -\left(\frac{k}{m}\right)x$ (Where v is the velocity of the particle)

By integrating equation (ii) within the limits($v = 0 \rightarrow v_x$, $x = A \rightarrow x$), we get,

$\int_0^{v_x} v\,dv = -\int_A^x \left(\frac{k}{m}\right)x\,dx$

$\left[\frac{v^2}{2}\right]_0^{v_x} = -\left(\frac{k}{m}\right)\left[\frac{x^2}{2}\right]_A^x$

$\frac{v^2{}_x}{2} = -\left(\frac{k}{m}\right)\left[\frac{x^2}{2} - \frac{A^2}{2}\right]$

$v^2{}_x = \left(\frac{k}{m}\right)(A^2 - x^2)$

$v_x = \pm\sqrt{\frac{k}{m}}\sqrt{A^2 - x^2}$

This equation tells us the speed of the object when it's at a certain distance from the middle position, x.

In this context, the use of positive and negative signs signifies the direction of the velocity, whether it's along the positive x-axis or the negative x-axis.

You will learn later that $\sqrt{\frac{k}{m}} = \omega$ and the above equation can be re-written as,

$v_x = \pm\, \omega\sqrt{A^2 + x^2}$

Standard Equation of SHM (Continued)

In the last lesson, we found out that if a particle is heading in the direction of the positive x-axis and is located at a distance x from the middle point, its speed is determined by this equation.

$v = \omega\sqrt{A^2 - x^2}$

$\frac{dx}{dt} = \omega\sqrt{A^2 - x^2}$

$\frac{dx}{\sqrt{A^2 - x^2}} = \omega dt$

By taking the indefinite integral of the equation, we get,

$\int \frac{dx}{\sqrt{A^2 - x^2}} = \int \omega dt$

$sin^{-1}\left(\frac{x}{A}\right) = \omega t + \emptyset$

$\frac{x}{A} = \sin(\omega t + \emptyset)$

$x = A\sin(\omega t + \emptyset)$

This formula represents the position of a particle in Simple Harmonic Motion (SHM) with an angular frequency ω and an amplitude A. The amplitude, denoted as 'A,' also serves as a measure of the energy involved in the SHM.

The term $\omega t + \phi$ is referred to as the phase angle of the SHM and is commonly represented by the Greek letter δ. The specific value of ϕ is contingent upon the initial position of the particle when time is $t = 0$.

When we find the rate of change of position with respect to time, we obtain the general formula for a particle's velocity. Additionally, when we find the rate of change of velocity with respect to time, we arrive at the general formula for the acceleration of the particle.

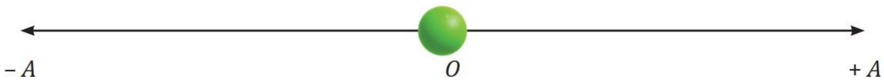
$v = \frac{dx}{dt} = A\omega\cos(\omega t + \emptyset)$

In this context, it's evident that the highest velocity magnitude, denoted as $v_{max} = A\omega$, occurs when the particle's phase is either zero or π, signifying the particle's presence at the midpoint.

$a = \frac{dv}{dt} = -A\omega^2\sin(\omega t + \emptyset) = -\omega^2 x$

Based on the acceleration equation, it becomes apparent that the acceleration is proportional to the negative of the displacement, $a \propto -x$, highlighting the behavior of the restoring force exerted on the particle. At the farthest points of the motion, the maximum acceleration's magnitude reaches$A\omega^2$.

Equation of the SHM of a particle starting from the mean position



In the contemplation of a particle embarking upon a SHM trajectory, commencing its journey in the direction of the affirmative x-axis from its equilibrium position, it is imperative to note that at the temporal point of t = 0, the particle's positional displacement, denoted as x, attains a state of absolute quiescence, resting at the point of origin.

The equation of displacement for a particle executing SHM is given by

$x = A\sin(\omega t + \emptyset)$

By putting t = 0 and x = 0 in the equation of displacement, we get,

$0 = A\sin\emptyset$

$\emptyset = 0, \pi$

As the particle initiates its motion toward the positive x-axis at the moment t = 0,

$x = A\sin\omega t$

The sine and cosine functions have a repeating pattern every 2π units. So, the formulas for displacement, velocity, and acceleration will start over after a phase shift of 2π, which means when ωt + ϕ equals 2π.

In this case, ϕ = 0

$\omega T = 2\pi$

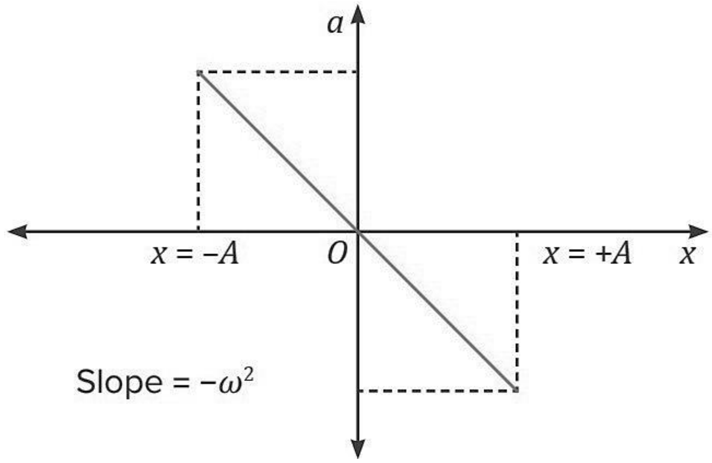
Where T is the time period of the SHM.

$T = \frac{2\pi}{\omega}$

Acceleration vs Displacement

The way acceleration and displacement are connected for a particle in SHM is described by this equation:

$a = -\omega^2 x$, Where $\omega = \sqrt{\frac{k}{m}}$ This constitutes an equation representing a linear function that intersects the origin when plotted on a graph depicting acceleration against displacement. The gradient of this line corresponds to -ω^2.



a
x = −A
O
x = +A
x
Slope = $-\omega^2$